Learning Objectives

- **LO1** Understand the three time horizons and which models apply for each 104
- **LO2** Explain when to use each of the four qualitative models 107
- **LO3** Apply the naive, moving-average, exponential smoothing, and trend methods 109
- **LO4** Compute three measures of forecast accuracy 114
- **LO5** Develop seasonal indices 122
- **LO6** Conduct a regression and correlation analysis 126
- **LO7** Use a tracking signal 133

What Is Forecasting?

Every day, managers like those at Disney make decisions without knowing what will happen in the future. They order inventory without knowing what sales will be, purchase new equipment despite uncertainty about demand for products, and make investments without knowing what profits will be. Managers are always trying to make better estimates of what will happen in the future in the face of uncertainty. Making good estimates is the main purpose of forecasting.

Student tip

An increasingly complex world economy makes forecasting challenging.

In this chapter, we examine different types of forecasts and present a variety of forecasting models. Our purpose is to show that there are many ways for managers to forecast. We also provide an overview of business sales forecasting and describe how to prepare, monitor, and judge the accuracy of a forecast. Good forecasts are an *essential* part of efficient service and manufacturing operations.

Forecasting is the art and science of predicting future events. Forecasting may involve taking historical data (such as past sales) and projecting them into the future with a mathematical model. It may be a subjective or an intuitive prediction (e.g., "this is a great new product and will sell 20% more than the old one"). It may be based on demand-driven data, such as customer plans to purchase, and projecting them into the future. Or the forecast may involve a combination of these, that is, a mathematical model adjusted by a manager's good judgment.

Forecasting

The art and science of predicting future events.

As we introduce different forecasting techniques in this chapter, you will see that there is seldom one superior method. Forecasts may be influenced by a product's position in its life cycle—whether sales are in an introduction, growth, maturity, or decline stage. Other products can be influenced by the demand for a related product—for example, navigation systems may track with new car sales. Because there are limits as to what can be expected from forecasts, we develop error measures. Preparing and monitoring forecasts can also be costly and time-consuming.

Few businesses, however, can afford to avoid the process of forecasting by just waiting to see what happens and then taking their chances. Effective planning in both the short run and long run depends on a forecast of demand for the company's products.

Forecasting Time Horizons

A forecast is usually classified by the *future time horizon* that it covers. Time horizons fall into three categories:

LO1

Understand the three time horizons and which models apply for each

- **1.** *Short-range forecast:* This forecast has a time span of up to 1 year but is generally less than 3 months. It is used for planning purchasing, job scheduling, workforce levels, job assignments, and production levels.
- **2.** *Medium-range forecast:* A medium-range, or intermediate, forecast generally spans from 3 months to 3 years. It is useful in sales planning, production planning and budgeting, cash budgeting, and analysis of various operating plans.
- **3.** *Long-range forecast:* Generally 3 years or more in time span, long-range forecasts are used in planning for new

products, capital expenditures, facility location or expansion, and research and development.

Medium and long-range forecasts are distinguished from short-range forecasts by three features:

- **1.** First, intermediate and long-range forecasts *deal with more comprehensive issues* supporting management decisions regarding planning and products, plants, and processes. Implementing some facility decisions, such as GM's decision to open a new Brazilian manufacturing plant, can take 5 to 8 years from inception to completion.
- **2.** Second, short-term forecasting usually *employs different methodologies* than longer-term forecasting. Mathematical techniques, such as moving averages, exponential smoothing, and trend extrapolation (all of which we shall examine shortly), are common to short-run projections. Broader, *less* quantitative methods are useful in predicting such issues as whether a new product, like the optical disk recorder, should be introduced into a company's product line.
- **3.** Finally, as you would expect, short-range forecasts *tend to be more accurate* than longer-range forecasts. Factors that influence demand change every day. Thus, as the time horizon lengthens, it is likely that forecast accuracy will diminish. It almost goes without saying, then, that sales forecasts must be updated regularly to maintain their value and integrity. After each sales period, forecasts should be reviewed and revised.

Types of Forecasts

Organizations use three major types of forecasts in planning future operations:

- **1.** **Economic forecasts** address the business cycle by predicting inflation rates, money supplies, housing starts, and other planning indicators.

  Economic forecasts

  Planning indicators that are valuable in helping organizations prepare medium- to long-range forecasts.

- **2.** **Technological forecasts** are concerned with rates of technological progress, which can result in the birth of exciting new products, requiring new plants and equipment.

  Technological forecasts

  Long-term forecasts concerned with the rates of technological progress.

- **3.** **Demand forecasts** are projections of demand for a company's products or services. Forecasts drive decisions, so managers need immediate and accurate information about real demand. They need *demand-driven forecasts*, where the focus is on rapidly identifying and tracking customer desires. These forecasts may use recent point-of-sale (POS) data, retailer-generated reports of customer preferences, and any other information that will help to forecast with the most current data possible. Demand-driven forecasts drive a company's production, capacity, and scheduling systems and serve as inputs to financial, marketing, and personnel planning. In addition, the payoff in reduced inventory and obsolescence can be huge.

Demand forecasts

Projections of a company's sales for each time period in the planning horizon.

Economic and technological forecasting are specialized techniques that may fall outside the role of the operations manager. The emphasis in this chapter will therefore be on demand forecasting.

The Strategic Importance of Forecasting

Good forecasts are of critical importance in all aspects of a business: *The forecast is the only estimate of demand until actual demand becomes known.* Forecasts of demand therefore drive decisions in many areas. Let's look at the impact of product demand forecast on three activities: (1) supply-chain management, (2) human resources, and (3) capacity.

Supply-Chain Management

Good supplier relations and the ensuing advantages in product innovation, cost, and speed to market depend on accurate forecasts. Here are just three examples:

- ▸ Apple has built an effective global system where it controls nearly every piece of the supply chain, from product design to retail store. With rapid communication and accurate data shared up and down the supply chain, innovation is enhanced, inventory costs are reduced, and speed to market is improved. Once a product goes on sale, Apple tracks demand by the hour for each store and adjusts production forecasts daily. At Apple, forecasts for its supply chain are a strategic weapon.

- Toyota develops sophisticated car forecasts with input from a variety of sources, including dealers. But forecasting the demand for accessories such as navigation systems, custom wheels, spoilers, and so on is particularly difficult. And there are over 1,000 items that vary by model and color. As a result, Toyota not only reviews reams of data with regard to vehicles that have been built and wholesaled but also looks in detail at vehicle forecasts before it makes judgments about the future accessory demand. When this is done correctly, the result is an efficient supply chain and satisfied customers.
- Walmart collaborates with suppliers such as Sara Lee and Procter & Gamble to make sure the right item is available at the right time in the right place and at the right price. For instance, in hurricane season, Walmart's ability to analyze 700 million store–item combinations means it can forecast that not only flashlights but Pop-Tarts and beer sell at seven times the normal demand rate. These forecasting systems are known as *collaborative planning, forecasting, and replenishment* (CPFR). They combine the intelligence of multiple supply-chain partners. The goal of CPFR is to create significantly more accurate information that can power the supply chain to greater sales and profits.

Human Resources

Hiring, training, and laying off workers all depend on anticipated demand. If the human resources department must hire additional workers without warning, the amount of training declines and the quality of the workforce suffers. A large Louisiana chemical firm almost lost its biggest customer when a quick expansion to around-the-clock shifts led to a total breakdown in quality control on the second and third shifts.

Capacity

When capacity is inadequate, the resulting shortages can lead to loss of customers and market share. This is exactly what happened to Nabisco when it underestimated the huge demand for its new Snackwell Devil's Food Cookies. Even with production lines working overtime, Nabisco could not keep up with demand, and it lost customers. Nintendo faced this problem when its Wii was introduced in 2007 and exceeded all forecasts for demand. And as the photo below shows, Amazon made the same error with its Kindle. On the other hand, when excess capacity exists, costs can skyrocket.

Seven Steps in the Forecasting System

Forecasting follows seven basic steps. We use Disney World, the focus of this chapter's *Global Company Profile*, as an example of each step:

- **1.** *Determine the use of the forecast:* Disney uses park attendance forecasts to drive decisions about staffing, opening times, ride availability, and food supplies.
- **2.** *Select the items to be forecasted:* For Disney World, there are six main parks. A forecast of daily attendance at each is the main number that determines labor, maintenance, and scheduling.

© David Robertson/Alamy

Even vaunted Amazon can make a major forecasting error, as it did in the case of its much-hyped Kindle e-book reader. With the holiday shopping season at hand, Amazon's Web page announced "Due to heavy customer demand, Kindle is sold out … ships in 11 to 13 weeks." Underforecasting demand for the product was the culprit, according to the Taiwanese manufacturer Prime View, which has since ramped up production.

- **3.** *Determine the time horizon of the forecast:* Is it short, medium, or long term? Disney develops daily, weekly, monthly, annual, and 5-year forecasts.
- **4.** *Select the forecasting model(s):* Disney uses a variety of statistical models that we shall discuss, including moving averages, econometrics, and regression analysis. It also employs judgmental, or nonquantitative, models.
- **5.** *Gather the data needed to make the forecast:* Disney's forecasting team employs 35 analysts and 70 field personnel

to survey 1 million people/businesses every year. Disney also uses a firm called Global Insights for travel industry forecasts and gathers data on exchange rates, arrivals into the U.S., airline specials, Wall Street trends, and school vacation schedules.
- **6.** *Make the forecast.*
- **7.** *Validate and implement the results:* At Disney, forecasts are reviewed daily at the highest levels to make sure that the model, assumptions, and data are valid. Error measures are applied; then the forecasts are used to schedule personnel down to 15-minute intervals.

These seven steps present a systematic way of initiating, designing, and implementing a forecasting system. When the system is to be used to generate forecasts regularly over time, data must be routinely collected. Then actual computations are usually made by computer.

Regardless of the system that firms like Disney use, each company faces several realities:

- ‣ Outside factors that we cannot predict or control often impact the forecast.
- ‣ Most forecasting techniques assume that there is some underlying stability in the system. Consequently, some firms automate their predictions using computerized forecasting software, then closely monitor only the product items whose demand is erratic.
- ‣ Both product family and aggregated forecasts are more accurate than individual product forecasts. Disney, for example, aggregates daily attendance forecasts by park. This approach helps balance the over- and underpredictions for each of the six attractions.

Forecasting Approaches

There are two general approaches to forecasting, just as there are two ways to tackle all decision modeling. One is a quantitative analysis; the other is a qualitative approach. **Quantitative forecasts** use a variety of mathematical models that rely on historical data and/or associative variables to forecast demand. Subjective or **qualitative forecasts** incorporate such factors as the decision maker's intuition, emotions, personal experiences, and value system in reaching a forecast. Some firms use one approach and some use the other. In practice, a combination of the two is usually most effective.

Quantitative forecasts

Forecasts that employ mathematical modeling to forecast demand.

Qualitative forecasts

Forecasts that incorporate such factors as the decision maker's intuition, emotions, personal experiences, and value system.

Overview of Qualitative Methods

In this section, we consider four different *qualitative* forecasting techniques:

- **1. Jury of executive opinion:** Under this method, the opinions of a group of high-level experts or managers, often in combination with statistical models, are pooled to arrive at a group estimate of demand. Bristol-Myers Squibb Company, for example, uses 220 well-known research scientists as its jury of executive opinion to get a grasp on future trends in the world of medical research.

  Jury of executive opinion

  A forecasting technique that uses the opinion of a small group of high-level managers to form a group estimate of demand.

- **2. Delphi method:** There are three different types of participants in the Delphi method: decision makers, staff personnel, and respondents. Decision makers usually consist of a group of 5 to 10 experts who will be making the actual forecast. Staff personnel assist decision makers by preparing, distributing, collecting, and summarizing a series of questionnaires and survey results. The respondents are a group of people, often located in different places, whose judgments are valued. This group provides inputs to the decision makers before the forecast is made.

  Delphi method

  A forecasting technique using a group process that allows experts to make forecasts.

  The state of Alaska, for example, has used the Delphi method to develop its long-range economic forecast. A large part of the state's budget is derived from the million-plus barrels of oil pumped daily through a pipeline at Prudhoe Bay. The large Delphi panel of experts had to represent all groups and opinions in the state and all geographic areas.

LO2

Explain when to use each of the four qualitative models

- **3. Sales force composite:** In this approach, each salesperson estimates what sales will be in his or her region. These forecasts are then reviewed to ensure that they are realistic. Then they are combined at the district and national levels to reach an overall forecast. A variation of this approach occurs at Lexus, where every quarter Lexus dealers have a "make meeting." At this meeting, they talk about what is selling, in what colors, and with what options, so the factory knows what to build.

  Sales force composite

  A forecasting technique based on salespersons' estimates of expected sales.

- **4. Market survey:** This method solicits input from customers or potential customers regarding future purchasing plans. It can help not only in preparing a forecast but also in improving product design and planning for new products. The consumer market survey and sales force composite methods can, however, suffer from overly optimistic forecasts that arise from customer input.

Market survey

A forecasting method that solicits input from customers or potential customers regarding future purchasing plans.

Overview of Quantitative Methods[1]

Five quantitative forecasting methods, all of which use historical data, are described in this chapter. They fall into two categories:

$$\left.\begin{array}{l}\textbf{1.}\ \text{Naive approach}\\ \textbf{2.}\ \text{Moving averages}\\ \textbf{3.}\ \text{Exponential smoothing}\end{array}\right\}\ \textbf{Time-series models}$$

$$\left.\begin{array}{l}\textbf{4.}\ \text{Trend projection}\\ \textbf{5.}\ \text{Linear regression}\end{array}\right\}\ \textbf{Associative model}$$

Time-Series Models

Time-series models predict on the assumption that the future is a function of the past. In other words, they look at what has happened over a period of time and use a series of past data to make a forecast. If we are predicting sales of lawn mowers, we use the past sales for lawn mowers to make the forecasts.

Time series

A forecasting technique that uses a series of past data points to make a forecast.

Associative Models

Associative models, such as linear regression, incorporate the variables or factors that might influence the quantity being forecast. For example, an associative model for lawn mower sales might use factors such as new housing starts, advertising budget, and competitors' prices.

Student tip

Here is the meat of this chapter. We now show you a wide variety of models that use time-series data.

Time-Series Forecasting

A time series is based on a sequence of evenly spaced (weekly, monthly, quarterly, and so on) data points. Examples include weekly sales of Nike Air Jordans, quarterly earnings reports of Microsoft stock, daily shipments of Coors beer, and annual consumer price indices. Forecasting time-series data implies that future values are predicted *only* from past values and that other variables, no matter how potentially valuable, may be ignored.

Decomposition of a Time Series

Analyzing time series means breaking down past data into components and then projecting them forward. A time series has four components:

Student tip

The peak "seasons" for sales of Frito-Lay chips are the Super Bowl, Memorial Day, Labor Day, and the Fourth of July.

- **1.** *Trend* is the gradual upward or downward movement of the data over time. Changes in income, population, age distribution, or cultural views may account for movement in trend.
- **2.** *Seasonality* is a data pattern that repeats itself after a period of days, weeks, months, or quarters. There are six common seasonality patterns:

PERIOD LENGTH	"SEASON" LENGTH	NUMBER OF "SEASONS" IN PATTERN
Week	Day	7
Month	Week	$4\text{–}4\frac{1}{2}$
Month	Day	28–31
Year	Quarter	4
Year	Month	12
Year	Week	52

[1] For a good review of statistical terms, refer to Tutorial 1, "Statistical Review for Managers," at our Web site, www.pearsonhighered.com/heizer.

OM in Action Forecasting at Olive Garden and Red Lobster

It's Friday night in the college town of Gainesville, Florida, and the local Olive Garden restaurant is humming. Customers may wait an average of 30 minutes for a table, but they can sample new wines and cheeses and admire scenic paintings of Italian villages on the Tuscan-style restaurant's walls. Then comes dinner with portions so huge that many people take home a doggie bag. The typical bill: under $15 per person.

Crowds flock to the Darden restaurant chain's Olive Garden, Red Lobster, Seasons 52, and Bahama Breeze for value and consistency—*and* they get it.

Every night, Darden's computers crank out forecasts that tell store managers what demand to anticipate the next day. The forecasting software generates a total meal forecast and breaks that down into specific menu items. The system tells a manager, for instance, that if 625 meals will be served the next day, "you will serve these items in these quantities. So before you go home, pull 25 pounds of shrimp and 30 pounds of crab out, and tell your operations people to prepare 42 portion packs of chicken, 75 scampi dishes, 8 stuffed flounders, and so on." Managers often fine-tune the quantities based on local conditions, such as weather or a convention, but they know what their customers are going to order.

By relying on demand history, the forecasting system has cut millions of dollars of waste out of the system. The forecast also reduces labor costs by providing the necessary information for improved scheduling. Labor costs decreased almost a full percent in the first year, translating into additional millions in savings for the Darden chain. In the low-margin restaurant business, every dollar counts.

Source: Interviews with Darden executives.

Restaurants and barber shops, for example, experience weekly seasons, with Saturday being the peak of business. See the *OM in Action* box "Forecasting at Olive Garden and Red Lobster." Beer distributors forecast yearly patterns, with monthly seasons. Three "seasons"—May, July, and September—each contain a big beer-drinking holiday.

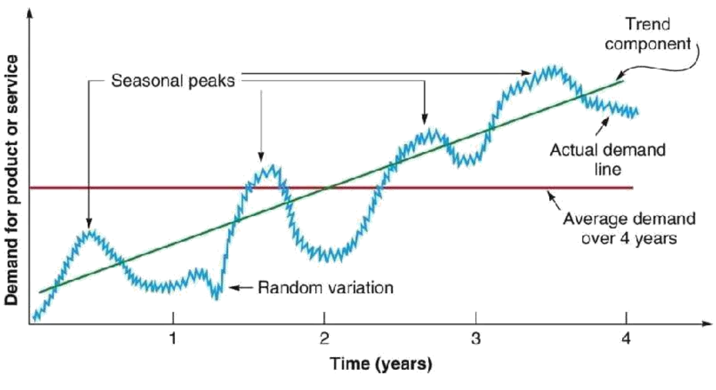
- **3.** *Cycles* are patterns in the data that occur every several years. They are usually tied into the business cycle and are of major importance in short-term business analysis and planning. Predicting business cycles is difficult because they may be affected by political events or by international turmoil.
- **4.** *Random variations* are "blips" in the data caused by chance and unusual situations. They follow no discernible pattern, so they cannot be predicted.

Figure 4.1 illustrates a demand over a 4-year period. It shows the average, trend, seasonal components, and random variations around the demand curve. The average demand is the sum of the demand for each period divided by the number of data periods.

LO3

Apply the naive, moving-average, exponential smoothing, and trend methods

Figure 4.1 Demand Charted over 4 Years, with a Growth Trend and Seasonality Indicated



Student tip

Forecasting is easy when demand is stable. But with trend, seasonality, and cycles considered, the job is a lot more interesting.

Naive Approach

The simplest way to forecast is to assume that demand in the next period will be equal to demand in the most recent period. In other words, if sales of a product—say, Nokia cell phones—were 68 units in January, we can forecast that February's sales will also be 68 phones. Does this make any sense? It turns out that for some product lines, this **naive approach** is the most cost-effective and efficient objective forecasting model. At least it provides a starting point against which more sophisticated models that follow can be compared.

Naive approach

A forecasting technique which assumes that demand in the next period is equal to demand in the most recent period.

Moving Averages

A **moving-average** forecast uses a number of historical actual data values to generate a forecast. Moving averages are useful *if we can assume that market demands will stay fairly steady over time.* A 4-month moving average is found by simply summing the demand during the past 4 months and dividing by 4. With each passing month, the most recent month's data are added to the sum of the previous 3 months' data, and the earliest month is dropped. This practice tends to smooth out short-term irregularities in the data series.

Moving averages

A forecasting method that uses an average of the *n* most recent periods of data to forecast the next period.

Mathematically, the simple moving average (which serves as an estimate of the next period's demand) is expressed as:

$$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n} \quad (4\text{-}1)$$

where n is the number of periods in the moving average—for example, 4, 5, or 6 months, respectively, for a 4-, 5-, or 6-period moving average.

Example 1 shows how moving averages are calculated.

Example 1 DETERMINING THE MOVING AVERAGE

Donna's Garden Supply wants a 3-month moving-average forecast, including a forecast for next January, for shed sales.

APPROACH Storage shed sales are shown in the middle column of the table below. A 3-month moving average appears on the right.

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\frac{2}{3}$
May	19	$(12 + 13 + 16)/3 = 13\frac{2}{3}$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\frac{1}{3}$
August	30	$(19 + 23 + 26)/3 = 22\frac{2}{3}$
September	28	$(23 + 26 + 30)/3 = 26\frac{1}{3}$
October	18	$(26 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\frac{1}{3}$
December	14	$(28 + 18 + 16)/3 = 20\frac{2}{3}$

SOLUTION The forecast for December is $20\frac{2}{3}$. To project the demand for sheds in the coming January, we sum the October, November, and December sales and divide by 3: January forecast = (18 + 16 + 14)/3 = 16.

INSIGHT Management now has a forecast that averages sales for the last 3 months. It is easy to use and understand.

LEARNING EXERCISE If actual sales in December were 18 (rather than 14), what is the new January forecast? [Answer: $17\frac{1}{3}$.]

RELATED PROBLEMS 4.1a, 4.2b, 4.5a, 4.6, 4.8a, b, 4.10a, 4.13b, 4.15, 4.47

EXCEL OM Data File **Ch04Ex1.xls** can be found at www.pearsonhighered.com/heizer.

ACTIVE MODEL 4.1 This example is further illustrated in Active Model 4.1 at www.pearsonhighered.com/heizer.

When a detectable trend or pattern is present, *weights* can be used to place more emphasis on recent values. This practice makes forecasting techniques more responsive to changes because more recent periods may be more heavily weighted. Choice of weights is somewhat arbitrary because there is no set formula to determine them. Therefore, deciding which weights to use requires some experience. For example, if the latest month or period is weighted too heavily, the forecast may reflect a large unusual change in the demand or sales pattern too quickly.

A weighted moving average may be expressed mathematically as:

$$\text{Weighted moving average} = \frac{\sum((\text{Weight for period } n)(\text{Demand in period } n))}{\sum \text{Weights}} \quad (4\text{-}2)$$

Example 2 shows how to calculate a weighted moving average.

Example 2 DETERMINING THE WEIGHTED MOVING AVERAGE

Donna's Garden Supply (see Example 1) wants to forecast storage shed sales by weighting the past 3 months, with more weight given to recent data to make them more significant.

APPROACH Assign more weight to recent data, as follows:

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

Forecast for this month =

$$\frac{3 \times \text{Sales last mo.} + 2 \times \text{Sales 2 mos. ago} + 1 \times \text{Sales 3 mos. ago}}{\text{Sum of the weights}}$$

SOLUTION The results of this weighted-average forecast are as follows:

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\frac{1}{6}$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14\frac{1}{3}$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20\frac{1}{2}$
August	30	$[(3 \times 26) + (2 \times 23) + (19)]/6 = 23\frac{5}{6}$
September	28	$[(3 \times 30) + (2 \times 26) + (23)]/6 = 27\frac{1}{2}$
October	18	$[(3 \times 28) + (2 \times 30) + (26)]/6 = 28\frac{1}{3}$
November	16	$[(3 \times 18) + (2 \times 28) + (30)]/6 = 23\frac{1}{3}$
December	14	$[(3 \times 16) + (2 \times 18) + (28)]/6 = 18\frac{2}{3}$

INSIGHT In this particular forecasting situation, you can see that more heavily weighting the latest month provides a much more accurate projection.

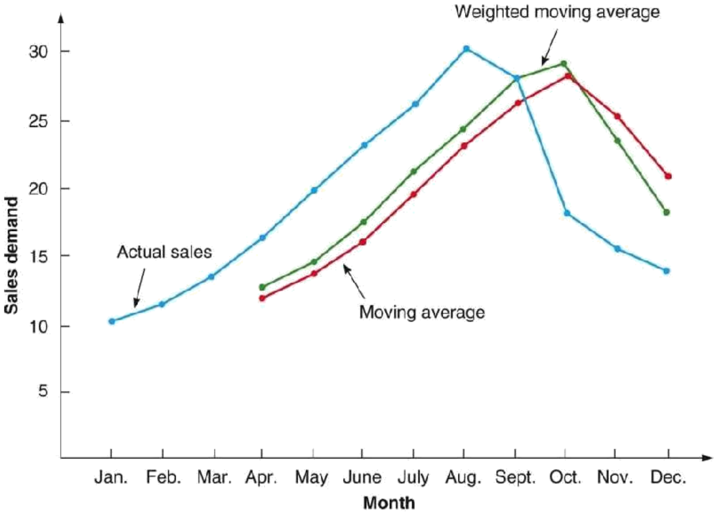
LEARNING EXERCISE If the assigned weights were 0.50, 0.33, and 0.17 (instead of 3, 2, and 1), what is the forecast for January's weighted moving average? Why? [Answer: There is no change. These are the same *relative* weights. Note that Σ weights = 1 now, so there is no need for a denominator. When the weights sum to 1, calculations tend to be simpler.]

RELATED PROBLEMS 4.1b, 4.2c, 4.5c, 4.6, 4.7, 4.10b

EXCEL OM Data File **Ch04Ex2.xls** can be found at www.pearsonhighered.com/heizer.

Figure 4.2 Actual Demand vs. Moving Average and Weighted-Moving-

Average Methods for Donna's Garden Supply



Student tip

Moving average methods always lag behind when there is a trend present, as shown by the blue line (actual sales) for January through August.

Both simple and weighted moving averages are effective in smoothing out sudden fluctuations in the demand pattern to provide stable estimates. Moving averages do, however, present three problems:

- **1.** Increasing the size of n (the number of periods averaged) does smooth out fluctuations better, but it makes the method less sensitive to changes in the data.
- **2.** Moving averages cannot pick up trends very well. Because they are averages, they will always stay within past levels and will not predict changes to either higher or lower levels. That is, they *lag* the actual values.
- **3.** Moving averages require extensive records of past data.

Figure 4.2, a plot of the data in Examples 1 and 2, illustrates the lag effect of the moving-average models. Note that both the moving-average and weighted-moving-average lines lag the actual demand. The weighted moving average, however, usually reacts more quickly to demand changes. Even in periods of downturn (see November and December), it more closely tracks the demand.

Exponential Smoothing

Exponential smoothing is another weighted-moving-average forecasting method. It involves very *little* record keeping of past data and is fairly easy to use. The basic exponential smoothing formula can be shown as follows:

Exponential smoothing

A weighted-moving-average forecasting technique in which data points are weighted by an exponential function.

New forecast = Last period's forecast + α (Last period's actual demand − Last period's forecast) (4-3)

where α is a weight, or **smoothing constant**, chosen by the forecaster, that has a value greater than or equal to 0 and less than or equal to 1. Equation (4-3) can also be written mathematically as:

Smoothing constant

The weighting factor used in an exponential smoothing forecast, a number greater than or equal to 0 and less than or equal to 1.

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \quad (4\text{-}4)$$

where

F_t = new forecast

F_{t-1} = previous period's forecast

α = smoothing (or weighting) constant ($0 \leq \alpha \leq 1$)

A_{t-1} = previous period's actual demand

The concept is not complex. The latest estimate of demand is equal to the old forecast adjusted by a fraction of the difference between the last period's actual demand and last period's forecast. Example 3 shows how to use exponential smoothing to derive a forecast.

Example 3 DETERMINING A FORECAST VIA EXPONENTIAL SMOOTHING

In January, a car dealer predicted February demand for 142 Ford Mustangs. Actual February demand was 153 autos. Using a smoothing constant chosen by management of α = .20, the dealer wants to forecast March demand using the exponential smoothing model.

APPROACH The exponential smoothing model in Equations (4-3) and (4-4) can be applied.

SOLUTION Substituting the sample data into the formula, we obtain:

$$\text{New forecast (for March demand)} = 142 + .2(153 - 142) = 142 + 2.2$$

$$= 144.2$$

Thus, the March demand forecast for Ford Mustangs is rounded to 144.

INSIGHT Using just two pieces of data, the forecast and the actual demand, plus a smoothing constant, we developed a forecast of 144 Ford Mustangs for March.

LEARNING EXERCISE If the smoothing constant is changed to .30, what is the new forecast? [Answer: 145.3]

RELATED PROBLEMS 4.1c, 4.3, 4.4, 4.5d, 4.6, 4.9d, 4.11, 4.12, 4.13a, 4.17, 4.18, 4.37, 4.43, 4.47, 4.49

The *smoothing constant,* α, is generally in the range from .05 to .50 for business applications. It can be changed to give more weight to recent data (when α is high) or more weight to past data (when α is low). When α reaches the extreme of 1.0, then in Equation (4-4), $F_t = 1.0\,A_{t-1}$. All the older values drop out, and the forecast becomes identical to the naive model mentioned earlier in this chapter. That is, the forecast for the next period is just the same as this period's demand.

The following table helps illustrate this concept. For example, when α = .5, we can see that the new forecast is based almost entirely on demand in the last three or four periods. When α = .1, the forecast places little weight on recent demand and takes many periods (about 19) of historical values into account.

	WEIGHT ASSIGNED TO				
SMOOTHING CONSTANT	MOST RECENT PERIOD (α)	2ND MOST RECENT PERIOD $\alpha(1-\alpha)$	3RD MOST RECENT PERIOD $\alpha(1-\alpha)^2$	4TH MOST RECENT PERIOD $\alpha(1-\alpha)^3$	5TH MOST RECENT PERIOD $\alpha(1-\alpha)^4$

$\alpha = .1$	.1	.09	.081	.073	.066
$\alpha = .5$	.5	.25	.125	.063	.031

Selecting the Smoothing Constant

Exponential smoothing has been successfully applied in virtually every type of business. However, the appropriate value of the smoothing constant, α, can make the difference between an accurate forecast and an inaccurate forecast. High values of α are chosen when the underlying average is likely to change. Low values of α are used when the underlying average is fairly stable. In picking a value for the smoothing constant, the objective is to obtain the most accurate forecast.

Student tip

Forecasts tend to be more accurate as they become shorter. Therefore, forecast error also tends to drop with shorter forecasts.

Measuring Forecast Error

The overall accuracy of any forecasting model—moving average, exponential smoothing, or other—can be determined by comparing the forecasted values with the actual or observed values. If F_t denotes the forecast in period t, and A_t denotes the actual demand in period t, the *forecast error* (or deviation) is defined as:

$$\text{Forecast error} = \text{Actual demand} - \text{Forecast value}$$

$$= A_t - F_t$$

Several measures are used in practice to calculate the overall forecast error. These measures can be used to compare different forecasting models, as well as to monitor forecasts to ensure they are performing well. Three of the most popular measures are mean absolute deviation (MAD), mean squared error (MSE), and mean absolute percent error (MAPE). We now describe and give an example of each.

Mean Absolute Deviation

The first measure of the overall forecast error for a model is the **mean absolute deviation (MAD)**. This value is computed by taking the sum of the absolute values of the individual forecast errors (deviations) and dividing by the number of periods of data (n):

$$\text{MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{n} \quad (4\text{-}5)$$

Mean absolutedeviation (MAD)

A measure of the overall forecast error for a model.

Example 4 applies MAD, as a measure of overall forecast error, by testing two values of α.

LO4

Compute three measures of forecast accuracy

Example 4 DETERMINING THE MEAN ABSOLUTE DEVIATION (MAD)

During the past 8 quarters, the Port of Baltimore has unloaded large quantities of grain from ships. The port's operations manager wants to test the use of exponential smoothing to see how well the technique works in predicting tonnage unloaded.

He guesses that the forecast of grain unloaded in the first quarter was 175 tons. Two values of α are to be examined: α = .10 and α = .50.

APPROACH Compare the actual data with the data we forecast (using each of the two α values) and then find the absolute deviation and MADs.

SOLUTION The following table shows the *detailed* calculations for α = .10 only:

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH α = .10	FORECAST WITH α = .50
1	180	175	175
2	168	175.50 = 175.00 + .10(180 − 175)	177.50
3	159	174.75 = 175.50 + .10(168 − 175.50)	172.75
4	175	173.18 = 174.75 + .10(159 − 174.75)	165.88
5	190	173.36 = 173.18 + .10(175 − 173.18)	170.44
6	205	175.02 = 173.36 + .10(190 − 173.36)	180.22
7	180	178.02 = 175.02 + .10(205 − 175.02)	192.61
8	182	178.22 = 178.02 + .10(180 − 178.02)	186.30
9	?	178.59 = 178.22 + .10(182 − 178.22)	184.15

To evaluate the accuracy of each smoothing constant, we can compute forecast errors in terms of absolute deviations and MADs:

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH α = .10	ABSOLUTE DEVIATION FOR α = .10	FORECAST WITH α = .50	ABSOLUTE DEVIATION FOR α = .50
1	180	175	5.00	175	5.00
2	168	175.50	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78

7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
	Sum of absolute deviations:		82.45		
	$\text{MAD} = \frac{\sum \lvert \text{Deviation} \rvert}{n}$		98.62	10.31	12.33

INSIGHT On the basis of this comparison of the two MADs, a smoothing constant of α = .10 is preferred to α = .50 because its MAD is smaller.

LEARNING EXERCISE If the smoothing constant is changed from α = .10 to α = .20, what is the new MAD? [Answer: 10.21.]

RELATED PROBLEMS 4.5b, 4.8c, 4.9c, 4.14, 4.23, 4.37a

EXCEL OM Data File **Ch04Ex4a.xls** and **Ch04Ex4b.xls** can be found at **www.pearsonhighered.com/heizer**.

ACTIVE MODEL 4.2 This example is further illustrated in Active Model 4.2 at **www.pearsonhighered.com/heizer**.

Most computerized forecasting software includes a feature that automatically finds the smoothing constant with the lowest forecast error. Some software modifies the α value if errors become larger than acceptable.

Mean Squared Error

The **mean squared error (MSE)** is a second way of measuring overall forecast error. MSE is the average of the squared differences between the forecasted and observed values. Its formula is:

Mean squared error (MSE)

The average of the squared differences between the forecasted and observed values.

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n} \quad (4\text{-}6)$$

Example 5 finds the MSE for the Port of Baltimore introduced in Example 4.

Example 5 DETERMINING THE MEAN SQUARED ERROR (MSE)

The operations manager for the Port of Baltimore now wants to compute MSE for α = .10.

APPROACH Using the same forecast data for α = .10 from Example 4, compute the MSE with Equation (4-6).

SOLUTION

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR α = .10	$(\text{ERROR})^2$
1	180	175	$5^2 = 25$
2	168	175.50	$(-7.5)^2 = 56.25$

3	159	174.75	$(-15.75)^2 = 248.06$
4	175	173.18	$(1.82)^2 = 3.31$
5	190	173.36	$(16.64)^2 = 276.89$
6	205	175.02	$(29.98)^2 = 898.80$
7	180	178.02	$(1.98)^2 = 3.92$
8	182	178.22	$(3.78)^2 = 14.29$
			Sum of errors squared = 1,526.52

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n} = 1,526.52 / 8 = 190.8$$

INSIGHT Is this MSE = 190.8 good or bad? It all depends on the MSEs for other forecasting approaches. A low MSE is better because we want to minimize MSE. MSE exaggerates errors because it squares them.

LEARNING EXERCISE Find the MSE for $\alpha = .50$. [Answer: MSE = 195.24. The result indicates that $\alpha = .10$ is a better choice because we seek a lower MSE. Coincidentally, this is the same conclusion we reached using MAD in Example 4.]

RELATED PROBLEMS 4.8d, 4.11c, 4.14, 4.15c, 4.16c, 4.20

A drawback of using the MSE is that it tends to accentuate large deviations due to the squared term. For example, if the forecast error for period 1 is twice as large as the error for period 2, the squared error in period 1 is four times as large as that for period 2. Hence, using MSE as the measure of forecast error typically indicates that we prefer to have several smaller deviations rather than even one large deviation.

Mean Absolute Percent Error

A problem with both the MAD and MSE is that their values depend on the magnitude of the item being forecast. If the forecast item is measured in thousands, the MAD and MSE values can be very large. To avoid this problem, we can use the **mean absolute percent error (MAPE)**. This is computed as the average of the absolute difference between the forecasted and actual values, expressed as a percentage of the actual values. That is, if we have forecasted and actual values for *n* periods, the MAPE is calculated as:

Mean absolute percenterror (MAPE)

The average of the absolutedifferences between the forecast and actual values, expressed as a percent of actual values.

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100 \left| \text{Actual}_i - \text{Forecast}_i \right| / \text{Actual}_i}{n} \quad (4\text{-}7)$$

Example 6 illustrates the calculations using the data from Examples 4 and 5.

Example 6 DETERMINING THE MEAN ABSOLUTE PERCENT ERROR (MAPE)

The Port of Baltimore wants to now calculate the MAPE when $\alpha = .10$.

APPROACH Equation (4-7) is applied to the forecast data computed in Example 4.

SOLUTION

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	ABSOLUTE PERCENT ERROR100 (|ERROR|/ACTUAL)
1	180	175.00	100(5/180) = 2.78%
2	168	175.50	100(7.5/168) = 4.46%
3	159	174.75	100(15.75/159) = 9.90%
4	175	173.18	100(1.82/175) = 1.05%
5	190	173.36	100(16.64/190) = 8.76%
6	205	175.02	100(29.98/205) = 14.62%
7	180	178.02	100(1.98/180) = 1.10%
8	182	178.22	100(3.78/182) = 2.08%
			Sum of % errors = 44.75%

$$\text{MAPE} = \frac{\sum \text{absolute percent error}}{n} = \frac{44.75\ \%}{8} = 5.59\ \%$$

INSIGHT MAPE expresses the error as a percent of the actual values, undistorted by a single large value.

LEARNING EXERCISE What is MAPE when α is .50? [Answer: MAPE = 6.75%. As was the case with MAD and MSE, the α = .1 was preferable for this series of data.]

RELATED PROBLEMS 4.8e, 4.33c

The MAPE is perhaps the easiest measure to interpret. For example, a result that the MAPE is 6% is a clear statement that is not dependent on issues such as the magnitude of the input data.

Exponential Smoothing with Trend Adjustment

Simple exponential smoothing, the technique we just illustrated in Examples 3 to 6, is like any other moving-average technique: It fails to respond to trends. Other forecasting techniques that can deal with trends are certainly available. However, because exponential smoothing is such a popular modeling approach in business, let us look at it in more detail.

Here is why exponential smoothing must be modified when a trend is present. Assume that demand for our product or service has been increasing by 100 units per month and that we have been forecasting with α = 0.4 in our exponential smoothing model. The following table shows a severe lag in the second, third, fourth, and fifth months, even when our initial estimate for month 1 is perfect:

MONTH	ACTUAL DEMAND	FORECAST (F_t) FOR MONTHS 1–5
1	100	$F_1 = 100$ (given)
2	200	$F_2 = F_1 + \alpha(A_1 - F_1) = 100 + .4(100 - 100) = 100$
3	300	$F_3 = F_2 + \alpha(A_2 - F_2) = 100 + .4(200 - 100) = 140$
4	400	$F_4 = F_3 + \alpha(A_3 - F_3) = 140 + .4(300 - 140) = 204$
5	500	$F_5 = F_4 + \alpha(A_4 - F_4) = 204 + .4(400 - 204) = 282$

To improve our forecast, let us illustrate a more complex exponential smoothing model, one that adjusts for trend. The idea is to compute an exponentially smoothed average of the data and then adjust for positive or negative lag in trend. The new formula is:

Forecast including trend (FIT_t) = Exponentially smoothed forecast average (F_t) + Exponentially smoothed trend (T_t) (4-8)

With trend-adjusted exponential smoothing, estimates for both the average and the trend are smoothed. This procedure requires two smoothing constants: α for the average and β for the trend. We then compute the average and trend each period:

F_t = α(Actual demand last period) + (1 − α)(Forecast last period + Trend estimate last period) or:

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1}) \quad (4\text{-}9)$$

T_t = β(Forecast this period − Forecast last period) + (1 − β)(Trend estimate last period) or:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1} \quad (4\text{-}10)$$

where F_t = exponentially smoothed forecast average of the data series in period t

T_t = exponentially smoothed trend in period t

A_t = actual demand in period t

α = smoothing constant for the average ($0 \le \alpha \le 1$)

β = smoothing constant for the trend ($0 \le \beta \le 1$)

So the three steps to compute a trend-adjusted forecast are:

STEP 1: Compute F_t, the exponentially smoothed forecast average for period t, using Equation (4-9).

STEP 2: Compute the smoothed trend, T_t, using Equation (4-10).

STEP 3: Calculate the forecast including trend, FIT_t, by the formula $FIT_t = F_t + T_t$ [from Equation (4-8)].

Example 7 shows how to use trend-adjusted exponential smoothing.

Example 7 COMPUTING A TREND-ADJUSTED EXPONENTIAL SMOOTHING FORECAST

A large Portland manufacturer wants to forecast demand for a piece of pollution-control equipment. A review of past sales, as shown below, indicates that an increasing trend is present:

MONTH (t)	ACTUAL DEMAND (A_t)	MONTH (t)	ACTUAL DEMAND (A_t)
1	12	6	21
2	17	7	31
3	20	8	28
4	19	9	36
5	24	10	?

Smoothing constants are assigned the values of $\alpha = .2$ and $\beta = .4$. The firm assumes the initial forecast average for month 1 (F_1) was 11 units and the trend over that period (T_1) was 2 units.

APPROACH A trend-adjusted exponential smoothing model, using Equations (4-9), (4-10), and (4-8) and the three steps above, is employed.

SOLUTION

- **Step 1:** Forecast average for month 2:

$$F_2 = \alpha A_1 + (1 - \alpha)(F_1 + T_1)$$

$$F_2 = (.2)(12) + (1 - .2)(11 + 2)$$

$$= 2.4 + (.8)(13) = 2.4 + 10.4 = 12.8 \text{ units}$$

- **Step 2:** Compute the trend in period 2:

$$T_2 = \beta(F_2 - F_1) + (1 - \beta)T_1$$

$$= .4(12.8 - 11) + (1 - .4)(2)$$

$= (.4)(1.8) + (.6)(2) = .72 + 1.2 = 1.92$

- **Step 3:** Compute the forecast including trend (FIT_t:

$FIT_2 = F_2 + T_2$

$= 12.8 + 1.92$

$= 14.72$ units

We will also do the same calculations for the third month:

- **Step 1:**

$F_3 = \alpha A_2 + (1 - \alpha)(F_2 + T_2) = (.2)(17) + (1 - .2)(12.8 + 1.92)$

$= 3.4 + (.8)(14.72) = 3.4 + 11.78 = 15.18$

- **Step 2:**

$T_3 = \beta(F_3 - F_2) + (1 - \beta)T_2 = (.4)(15.18 - 12.8) + (1 - .4)(1.92)$

$= (.4)(2.38) + (.6)(1.92) = .952 + 1.152 = 2.10$

- **Step 3:**

$FIT_3 = F_3 + T_3$

$= 15.18 + 2.10 = 17.28.$

Table 4.1 completes the forecasts for the 10-month period.

TABLE 4.1 Forecast with α = .2 and β = .4

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89

7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10	—	32.48	2.68	35.16

INSIGHT Figure 4.3 compares actual demand (A_t) to an exponential smoothing forecast that includes trend (FIT_t). *FIT* picks up the trend in actual demand. A simple exponential smoothing model (as we saw in Examples 3 and 4) trails far behind.

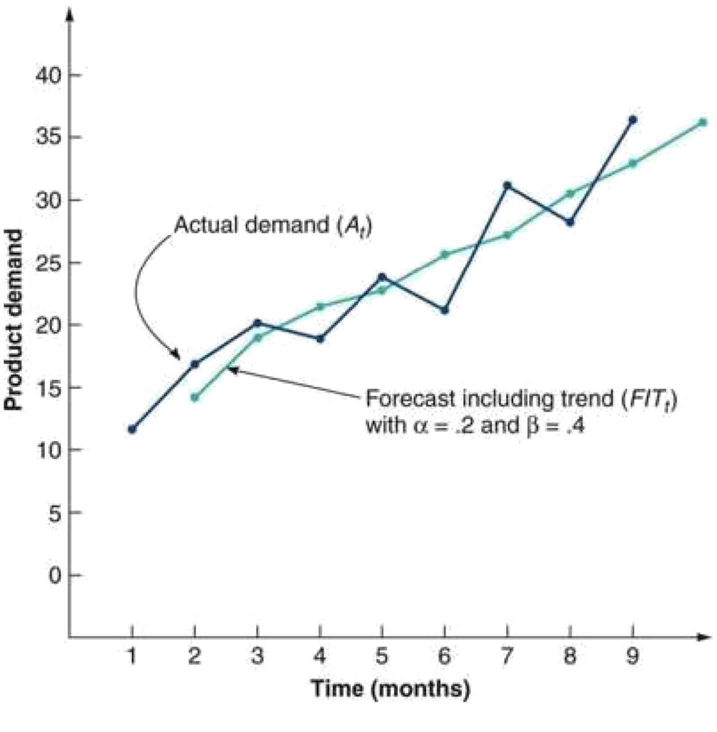
LEARNING EXERCISE Using the data for actual demand for the 9 months, compute the exponentially smoothed forecast average *without* trend [using Equation (4-4) as we did earlier in Examples 3 and 4]. Apply $\alpha = .2$ and assume an initial forecast average for month 1 of 11 units. Then plot the months 2–10 forecast values on Figure 4.3. What do you notice? [Answer: Month 10 forecast = 24.65. All the points are below and lag the trend-adjusted forecast.]

RELATED PROBLEMS 4.19, 4.20, 4.21, 4.22, 4.44

ACTIVE MODEL 4.3 This example is further illustrated in Active Model 4.3 at **www.pearsonhighered.com/heizer**.

EXCEL OM Data File **Ch04Ex7.xls** can be found at **www.pearsonhighered.com/heizer**.

Figure 4.3 Exponential Smoothing with Trend-Adjustment Forecasts Compared to Actual Demand Data



The value of the trend-smoothing constant, β, resembles the α constant because a high β is more responsive to recent changes in trend. A low β gives less weight to the most recent trends and tends to smooth out the present trend. Values of β can be found by the trial-and-error approach or by using sophisticated commercial forecasting software, with the MAD used as a measure of comparison.

Simple exponential smoothing is often referred to as *first-order smoothing*, and trend-adjusted smoothing is called *second-order*, or *double smoothing*. Other advanced exponential-smoothing models are also used, including seasonal-adjusted and triple smoothing.

Trend Projections

The last time-series forecasting method we will discuss is **trend projection**. This technique fits a trend line to a series of historical data points and then projects the slope of the line into the future for medium to long-range forecasts. Several mathematical trend equations can be developed (for example, exponential and quadratic), but in this section, we will look at *linear* (straight-line) trends only.

Trend projection

A time-series forecasting method that fits a trend line to a series of historical data points and then projects the line into the future for forecasts.

If we decide to develop a linear trend line by a precise statistical method, we can apply the *least-squares method*. This approach results in a straight line that minimizes the sum of the squares of the vertical differences or deviations from the line to each of the actual observations. Figure 4.4 illustrates the least-squares approach.

A least-squares line is described in terms of its *y*-intercept (the height at which it intercepts the *y*-axis) and its expected change (slope). If we can compute the *y*-intercept and slope, we can express the line with the following equation:

$$\hat{y} = a + bx \quad (4\text{-}11)$$

where $\hat{y}$ (called "*y* hat") = computed value of the variable to be predicted (called the *dependent variable*)

a = *y*-axis intercept

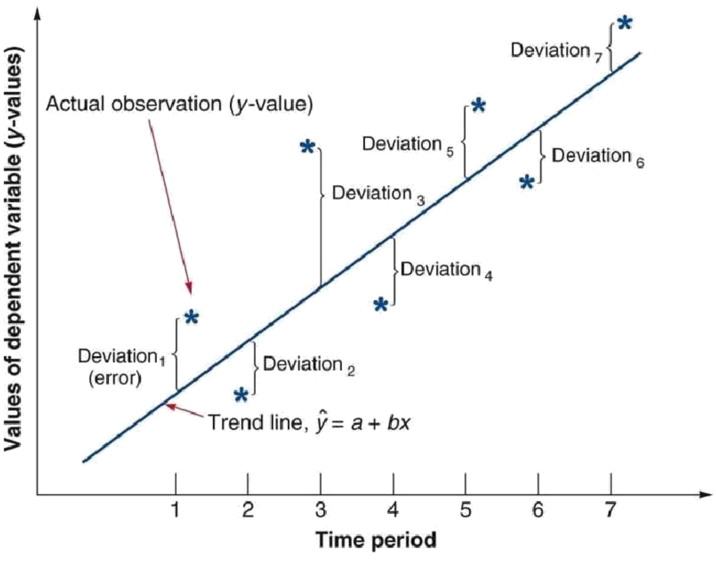
b = slope of the regression line (or the rate of change in *y* for given changes in *x*)

x = the independent variable (which in this case is *time*)

Statisticians have developed equations that we can use to find the values of *a* and *b* for any regression line. The slope *b* is found by:

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} \quad (4\text{-}12)$$

Figure 4.4 The Least-Squares Method for Finding the Best-Fitting Straight line, Where the Asterisks Are the Locations of the Seven Actual Observations or Data points



where b = slope of the regression line

Σ = summation sign

x = known values of the independent variable

y = known values of the dependent variable

$\bar{x}$ = average of the x-values

$\bar{y}$ = average of the y-values

n = number of data points or observations

We can compute the y-intercept a as follows:

$$a = \bar{y} - b\bar{x} \quad (4\text{-}13)$$

Example 8 shows how to apply these concepts.

Example 8 FORECASTING WITH LEAST SQUARES

The demand for electric power at N.Y. Edison over the past 7 years is shown in the following table, in megawatts. The firm wants to forecast next year's demand by fitting a straight-line trend to these data.

YEAR	ELECTRICAL POWER DEMAND	YEAR	ELECTRICAL POWER DEMAND
1	74	5	105
2	79	6	142
3	80	7	122
4	90		

APPROACH Equations (4-12) and (4-13) can be used to create the trend projection model.

SOLUTION

YEAR (x)	ELECTRIC POWER DEMAND (y)	x^2	xy
1	74	1	74
2	79	4	158

3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3{,}063$

$$\bar{x} = \frac{\sum x}{n} = \frac{28}{7} = 4 \quad \bar{y} = \frac{\sum y}{n} = \frac{692}{7} = 98.86$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3,063 - (7)(4)(98.86)}{140 - (7)(4^2)} = \frac{295}{28} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

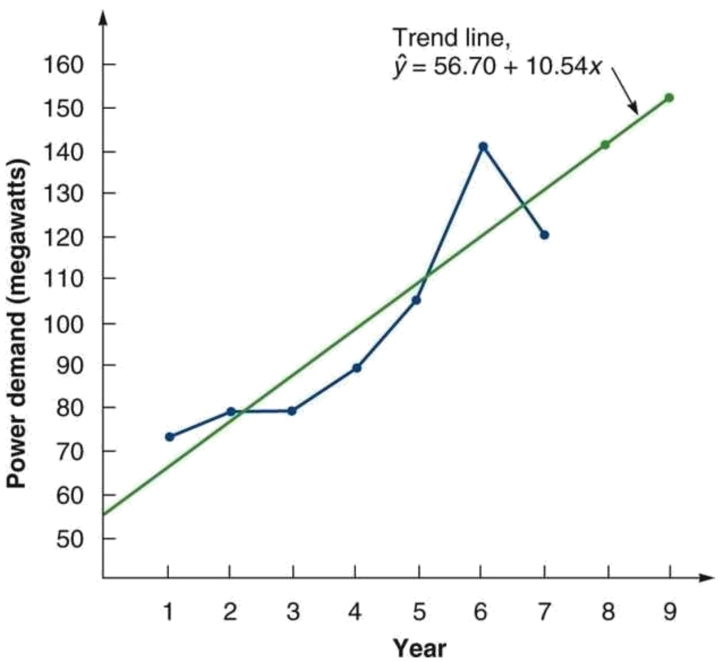
Thus, the least-squares trend equation is $\hat{y} = 56.70 + 10.54x$. To project demand next year, $x = 8$:

Demand in year 8 = 56.70 + 10.54(8)

= 141.02, or 141 megawatts

INSIGHT To evaluate the model, we plot both the historical demand and the trend line in Figure 4.5. In this case, we may wish to be cautious and try to understand the year 6 to year 7 swing in demand.

Figure 4.5 Electrical Power and the Computed Trend Line



LEARNING EXERCISE Estimate demand for year 9. [Answer: 151.56, or 152 megawatts.]

RELATED PROBLEMS 4.6, 4.13c, 4.16, 4.25, 4.39, 4.49

EXCEL OM Data File **Ch04Ex8.xls** can be found at **www.pearsonhighered.com/heizer**.

ACTIVE MODEL 4.4 This example is further illustrated in Active Model 4.4 at **www.pearsonhighered.com/heizer**.

Notes on the Use of the Least-Squares Method

Using the least-squares method implies that we have met three requirements:

- **1.** We always plot the data because least-squares data assume a linear relationship. If a curve appears to be present, curvilinear analysis is probably needed.
- **2.** We do not predict time periods far beyond our given database. For example, if we have 20 months' worth of average prices of Microsoft stock, we can forecast only 3 or 4 months into the future. Forecasts beyond that have little statistical validity. Thus, you cannot take 5 years' worth of sales data and project 10 years into the future. The world is too uncertain.
- **3.** Deviations around the least-squares line (see Figure 4.4) are assumed to be random and normally distributed, with most observations close to the line and only a smaller number farther out.

Seasonal Variations in Data

Seasonal variations in data are regular movements in a time series that relate to recurring events such as weather or holidays. Demand for coal and fuel oil, for example, peaks during cold winter months. Demand for golf clubs or sunscreen may be highest in summer.

Seasonal variations

Regular upward or downward movements in a time series that tie to recurring events.

Demand for many products is seasonal. Yamaha, the manufacturer of this jet ski and snowmobile, produces products with complementary demands to address seasonal fluctuations.

Seasonality may be applied to hourly, daily, weekly, monthly, or other recurring patterns. Fast-food restaurants experience *daily* surges at noon and again at 5 p.m. Movie theaters see higher demand on Friday and Saturday evenings. The post office, Toys "Я" Us, The Christmas Store, and Hallmark Card Shops also exhibit seasonal variation in customer traffic and sales.

Similarly, understanding seasonal variations is important for capacity planning in organizations that handle peak loads. These include electric power companies during extreme cold and warm periods, banks on Friday afternoons, and buses and subways during the morning and evening rush hours.

Student tip

John Deere understands seasonal variations: It has been able to obtain 70% of its orders in advance of seasonal useso it can smooth production.

Time-series forecasts like those in Example 8 involve reviewing the trend of data over a series of time periods. The presence of seasonality makes adjustments in trend-line forecasts necessary. Seasonality is expressed in terms of the amount that actual values differ from average values in the time series. Analyzing data in monthly or quarterly terms usually makes it easy for a statistician to spot seasonal patterns. Seasonal indices can then be developed by several common methods.

In what is called a *multiplicative seasonal model*, seasonal factors are multiplied by an estimate of average demand to produce a seasonal forecast. Our assumption in this section is that trend has been removed from the data. Otherwise, the magnitude of the seasonal data will be distorted by the trend.

Here are the steps we will follow for a company that has "seasons" of 1 month:

- **1.** Find the *average historical demand each season* (or month in this case) by summing the demand for that month in each year and dividing by the number of years of data available. For example, if, in January, we have seen sales of 8, 6, and 10 over the past 3 years, average January demand equals (8 + 6 + 10)/3 = 8 units.
- **2.** Compute the *average demand over all months* by dividing the total average annual demand by the number of seasons. For example, if the total average demand for a year is 120 units and there are 12 seasons (each month), the average monthly demand is 120/12 = 10 units.

  LO5

  Develop seasonal indices

- **3.** Compute a *seasonal index* for each season by dividing that *month's* historical average demand (from Step 1) by the average demand over all months (from Step 2). For example, if the average historical January demand over the past 3 years is 8 units and the average demand over all months is 10 units, the seasonal index for January is 8/10 = .80. Likewise, a seasonal index of 1.20 for February would mean that February's demand is 20% larger than the average demand over all months.
- **4.** Estimate next year's total annual demand.
- **5.** Divide this estimate of total annual demand by the number of seasons, then multiply it by the seasonal index for each month. This provides the *seasonal forecast.*

Example 9 illustrates this procedure as it computes seasonal indices from historical data.

Example 9 DETERMINING SEASONAL INDICES

A Des Moines distributor of Sony laptop computers wants to develop monthly indices for sales. Data from the past 3 years, by month, are available.

APPROACH Follow the five steps listed above.

SOLUTION

DEMAND						
MONTH	Year 1	Year 2	Year 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND [a]	SEASONAL INDEX [b]
Jan.	80	85	105	90	94	.957 (= 90/94)
Feb.	70	85	85	80	94	.957 (= 90/94)
Mar.	80	93	82	85	94	.851 (= 80/94)
Apr.	90	95	115	100	94	.904 (= 85/94)
May	113	125	131	123	94	1.064 (= 100/94)
June	110	115	120	115	94	1.309 (= 123/94)
July	100	102	113	105	94	1.223 (= 115/94)
Aug.	88	102	110	100	94	1.117 (= 105/94)

Sept.	85	90	95	90	94	1.064 (= 100/94)
Oct.	77	78	85	80	94	.957 (= 90/94)
Nov.	75	82	83	80	94	.851 (= 80/94)
Dec.	82	78	80	80	94	.851 (= 80/94)

Total average annual demand = 1,128

[a]Average monthly demand = $\frac{1,128}{12 \text{ month}} = 94.$

[b]Seasonal index = $\frac{\text{Average monthly demand for past 3 years}}{\text{Average monthly demand}}$.

If we expect the annual demand for computers to be 1,200 units next year, we would use these seasonal indices to forecast the monthly demand as follows:

MONTH	DEMAND
Jan.	$\frac{1,200}{12} \times .957 = 96$
Feb.	$\frac{1,200}{12} \times .851 = 85$
Mar.	$\frac{1,200}{12} \times .904 = 90$
Apr.	$\frac{1,200}{12} \times 1.064 = 106$
May	$\frac{1,200}{12} \times 1.309 = 131$
June	$\frac{1,200}{12} \times 1.223 = 122$
July	$\frac{1,200}{12} \times 1.117 = 112$

Aug.	$\frac{1,200}{12} \times 1.064 = 106$
Sept.	$\frac{1,200}{12} \times .957 = 96$
Oct.	$\frac{1,200}{12} \times .851 = 85$
Nov.	$\frac{1,200}{12} \times .851 = 85$
Dec.	$\frac{1,200}{12} \times .851 = 85$

INSIGHT Think of these indices as percentages of average sales. The average sales (without seasonality) would be 94, but with seasonality, sales fluctuate from 85% to 131% of average.

LEARNING EXERCISE If next year's annual demand is 1,150 laptops (instead of 1,200), what will the January, February, and March forecasts be? [Answer: 91.7, 81.5, and 86.6, which can be rounded to 92, 82, and 87.]

RELATED PROBLEMS 4.27, 4.28

EXCEL OM Data File **Ch04Ex9.xls** can be found at www.pearsonhighered.com/heizer.

For simplicity, only 3 periods (years) are used for each monthly index in the preceding example. Example 10 illustrates how indices that have already been prepared can be applied to adjust trend-line forecasts for seasonality.

Example 10 APPLYING BOTH TREND AND SEASONAL INDICES

San Diego Hospital wants to improve its forecasting by applying both trend and seasonal indices to 66 months of data it has collected. It will then forecast "patient-days" over the coming year.

APPROACH A trend line is created; then monthly seasonal indices are computed. Finally, a multiplicative seasonal model is used to forecast months 67 to 78.

SOLUTION Using 66 months of adult inpatient hospital days, the following equation was computed:

$$\hat{y} = 8,090 + 21.5x$$

where

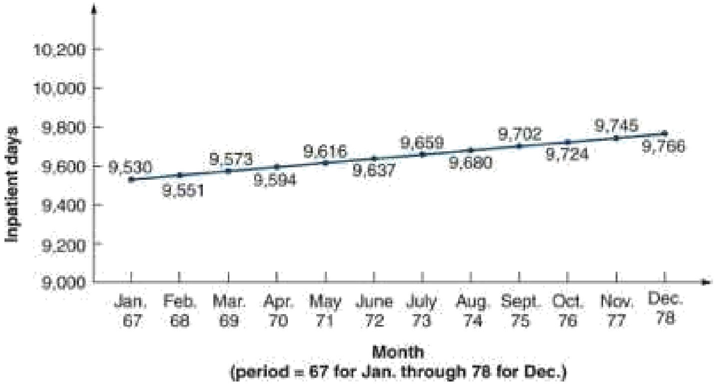
$$\hat{y} = \text{patient days}$$
$$x = \text{time, in months}$$

Based on this model, which reflects only trend data, the hospital forecasts patient days for the next month (period 67) to be:

Patient days = 8,090 + (21.5)(67) = 9,530 (trend only)

While this model, as plotted in Figure 4.6, recognized the upward trend line in the demand for inpatient services, it ignored the seasonality that the administration knew to be present.

Figure 4.6 Trend Data for San Diego Hospital



Source: From "Modern Methods Improve Hospital Forecasting" by W. E. Sterk and E. G. Shryock from *Healthcare Financial Management* 41, no. 3, p. 97. Reprinted by permission of Healthcare Financial Management Association.

The following table provides seasonal indices based on the same 66 months. Such seasonal data, by the way, were found to be typical of hospitals nationwide.

Seasonality Indices for Adult Inpatient Days at San Diego Hospital

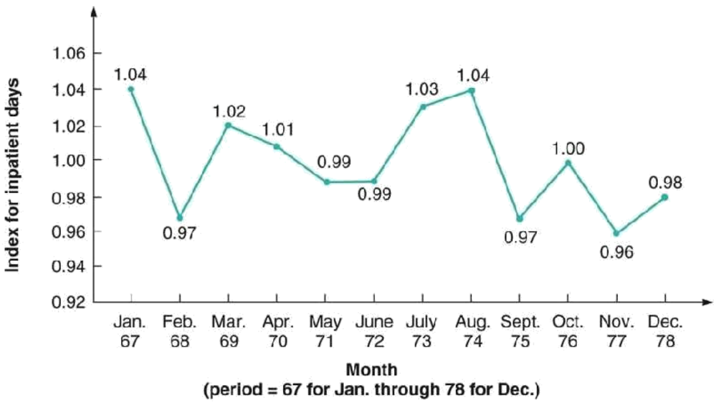
MONTH	SEASONALITY INDEX	MONTH	SEASONALITY INDEX
January	1.04	July	1.03
February	0.97	August	1.04
March	1.02	September	0.97
April	1.01	October	1.00
May	0.99	November	0.96
June	0.99	December	0.98

These seasonal indices are graphed in Figure 4.7. Note that January, March, July, and August seem to exhibit significantly higher patient days on average, while February, September, November, and December experience lower patient days.

However, neither the trend data nor the seasonal data alone provide a reasonable forecast for the hospital. Only when the hospital multiplied the trend-adjusted data by the appropriate seasonal index did it obtain good forecasts. Thus, for period 67 (January):

- Patient days = (Trend-adjusted forecast)(Monthly seasonal index) = (9,530)(1.04) = 9,911

Figure 4.7 Seasonal Index for San Diego Hospital

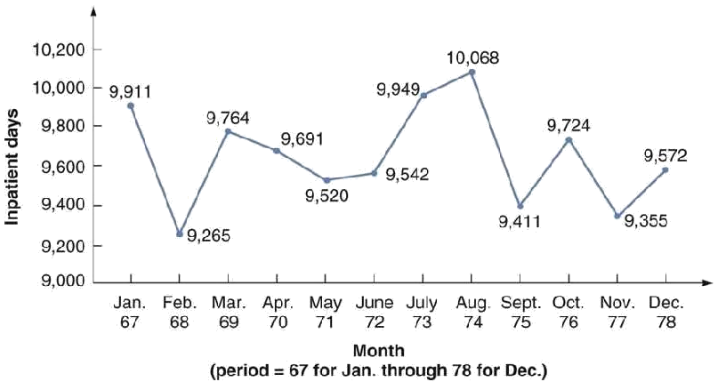


The patient-days for each month are:

Period	67	68	69	70	71	72	73	74	75	76	77	78
Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Forecast with Trend & Seasonality	9,911	9,265	9,764	9,691	9,520	9,542	9,949	10,068	9,411	9,724	9,355	9,572

A graph showing the forecast that combines both trend and seasonality appears in Figure 4.8.

Figure 4.8 Combined Trend and Seasonal Forecast



INSIGHT Notice that with trend only, the September forecast is 9,702, but with both trend and seasonal adjustments, the forecast is 9,411. By combining trend and seasonal data, the hospital was better able to forecast inpatient days and the related staffing and budgeting vital to effective operations.

LEARNING EXERCISE If the slope of the trend line for patient-days is 22.0 (rather than 21.5) and the index for December is .99 (instead of .98), what is the new forecast for December inpatient days? [Answer: 9,708.]

RELATED PROBLEMS 4.26, 4.29

Example 11 further illustrates seasonality for quarterly data at a wholesaler.

Example 11 ADJUSTING TREND DATA WITH SEASONAL INDICES

Management at Jagoda Wholesalers, in Calgary, Canada, has used time-series regression based on point-of-sale data to forecast sales for the next 4 quarters. Sales estimates are \$100,000, \$120,000, \$140,000, and \$160,000 for the respective quarters. Seasonal indices for the four quarters have been found to be 1.30, .90, .70, and 1.10, respectively.

APPROACH To compute a seasonalized or adjusted sales forecast, we just multiply each seasonal index by the appropriate trend forecast:

$$\hat{y}_{\text{seasonal}} = \text{Index} \times \hat{y}_{\text{trend forecast}}$$

SOLUTION

- Quarter I: $\hat{y}_{\text{I}} = (1.30)(\$100{,}000) = \$130{,}000$ Quarter II: $\hat{y}_{\text{II}} = (.90)(\$120{,}000) = \$108{,}000$ Quarter III: $\hat{y}_{\text{III}} = (.70)(\$140{,}000) = \$98{,}000$ Quarter IV: $\hat{y}_{\text{IV}} = (1.10)(\$160{,}000) = \$176{,}000$

INSIGHT The straight-line trend forecast is now adjusted to reflect the seasonal changes.

LEARNING EXERCISE If the sales forecast for Quarter IV was $180,000 (rather than $160,000), what would be the seasonally adjusted forecast? [Answer: $198,000.]

RELATED PROBLEMS 4.26, 4.29

Cyclical Variations in Data

Cycles are like seasonal variations in data but occur every several *years*, not weeks, months, or quarters. Forecasting cyclical variations in a time series is difficult. This is because cycles include a wide variety of factors that cause the economy to go from recession to expansion to recession over a period of years. These factors include national or industrywide overexpansion in times of euphoria and contraction in times of concern. Forecasting demand for individual products can also be driven by product life cycles—the stages products go through from introduction through decline. Life cycles exist for virtually all products; striking examples include floppy disks, video recorders, and the original Game Boy. We leave cyclical analysis to forecasting texts.

Cycles

Patterns in the data that occur every several years.

Developing associative techniques of variables that affect one another is our next topic.

Student tip

We now deal with the same mathematical model that we saw earlier, the least-squares method. But we use any potential "cause-and-effect" variable as x.

Associative Forecasting Methods: Regression and Correlation Analysis

Unlike time-series forecasting, *associative forecasting* models usually consider *several* variables that are related to the quantity being predicted. Once these related variables have been found, a statistical model is built and used to forecast the item of interest. This approach is more powerful than the time-series methods that use only the historical values for the forecast variable.

Many factors can be considered in an associative analysis. For example, the sales of Dell PCs may be related to Dell's advertising budget, the company's prices, competitors' prices and promotional strategies, and even the nation's economy and unemployment rates. In this case, PC sales would be called the *dependent variable*, and the other variables would be called *independent variables*. The manager's job is to develop *the best statistical relationship between PC sales and the independent variables*. The most common quantitative associative forecasting model is **linear-regression analysis**.

Linear-regression analysis

A straight-line mathematical model to describe the functional relationships between independent and dependent variables.

Using Regression Analysis for Forecasting

We can use the same mathematical model that we employed in the least-squares method of trend projection to perform a linear-regression analysis. The dependent variables that we want to forecast will still be $\hat{y}$. But now the independent variable, x, need no longer be time. We use the equation:

$$\hat{y} = a + bx$$

LO6

Conduct a regression and correlation analysis

where $\hat{y}$ = value of the dependent variable (in our example, sales)

a = y-axis intercept

b = slope of the regression line

x = independent variable Example 12 shows how to use linear regression.

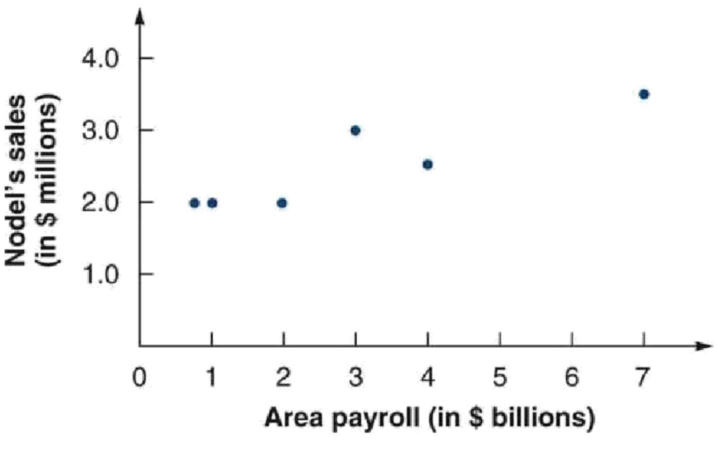
Example 12 COMPUTING A LINEAR REGRESSION EQUATION

Nodel Construction Company renovates old homes in West Bloomfield, Michigan. Over time, the company has found that its dollar volume of renovation work is dependent on the West Bloomfield area payroll. Management wants to establish a mathematical relationship to help predict sales.

APPROACH Nodel's VP of operations has prepared the following table, which lists company revenues and the amount of money earned by wage earners in West Bloomfield during the past 6 years:

NODEL'S SALES(IN $ MILLIONS), y	AREA PAYROLL (IN $ BILLIONS), x	NODEL'S SALES (IN $ MILLIONS), y	AREA PAYROLL (IN $ BILLIONS), x
2.0	1	2.0	2
3.0	3	2.0	1
2.5	4	3.5	7

The VP needs to determine whether there is a straight-line (linear) relationship between area payroll and sales. He plots the known data on a scatter diagram:



Student tip

A scatter diagram is a powerful data analysis tool. It helps quickly size up the relationship between two variables.

From the six data points, there appears to be a slight positive relationship between the independent variable (payroll) and the dependent variable (sales): As payroll increases, Nodel's sales tend to be higher.

SOLUTION We can find a mathematical equation by using the least-squares regression approach:

VideO 4.1

Forecasting Ticket Revenue for Orlando Magic Basketball Games

SALES, y	PAYROLL, x	x^2	xy
2.0	1	1	2.0
3.0	3	9	9.0
2.5	4	16	10.0
2.0	2	4	4.0
2.0	1	1	2.0
3.5	7	49	24.5
$\Sigma y = 15.0$	$\Sigma x = 18$	$\Sigma x^2 = 80$	$\Sigma xy = 51.5$

$$\bar{x} = \frac{\sum x}{6} = \frac{18}{6} = 3$$

$$\bar{y} = \frac{\sum y}{6} = \frac{15}{6} = 2.5$$

$$b = \frac{\sum xy - n\overline{xy}}{\sum x^2 - n\bar{x}^2} = \frac{51.5 - (6)(3)(2.5)}{80 - 6(3^2)} = .25$$

$$a = \bar{y} - b\bar{x} = 2.5 - (2.5)(3) = 1.75$$

The estimated regression equation, therefore, is:

$$\hat{y} = 1.75 + .25x$$

or:

Sales = 1.75 + .25 (payroll)

If the local chamber of commerce predicts that the West Bloomfield area payroll will be \$6 billion next year, we can estimate sales for Nodel with the regression equation:

Sales (in + millions) = 1.75 + .25(6)

= 1.75 + 1.50 = 3.25

or:

Sales = +3,250,000

INSIGHT Given our assumptions of a straight-line relationship between payroll and sales, we now have an indication of the slope of that relationship: on average, sales increase at the rate of $\frac{1}{4}$ million dollars for every billion dollars in the local area payroll. This is because $b = .25$.

LEARNING EXERCISE What are Nodel's sales when the local payroll is \$8 billion? [Answer:\$3.75 million.]

RELATED PROBLEMS 4.24, 4.30, 4.31, 4.32, 4.33, 4.35, 4.38, 4.40, 4.41, 4.46, 4.48, 4.49

EXCEL OM Data File **Ch04Ex12.xls** can be found at www.pearsonhighered.com/heizer.

The final part of Example 12 shows a central weakness of associative forecasting methods like regression. Even when we have computed a regression equation, we must provide a forecast of the independent variable x—in this case, payroll—before estimating the dependent variable y for the next time period. Although this is not a problem for all forecasts, you can imagine the difficulty of determining future values of *some* common independent variables (e.g., unemployment rates, gross national product, price indices, and so on).

Standard Error of the Estimate

The forecast of \$3,250,000 for Nodel's sales in Example 12 is called a *point estimate* of y. The point estimate is really the *mean*, or *expected value*, of a distribution of possible values of sales. Figure 4.9 illustrates this concept.

To measure the accuracy of the regression estimates, we must compute the **standard error of the estimate**, $S_{y,x}$. This computation is called the *standard deviation of the regression:* It measures the error from the dependent variable, y, to the regression line, rather than to the mean. Equation (4-14) is a similar expression to that found in most statistics books for computing the standard deviation of an arithmetic mean:

Standard error of the estimate

A measure of variability around the regression line—its standard deviation.

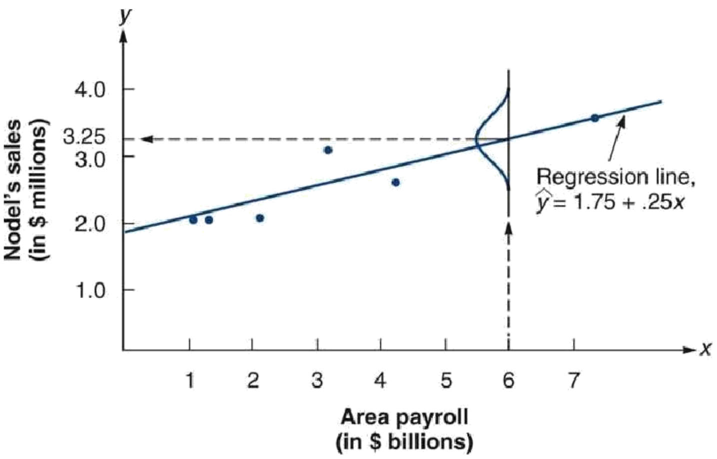
$$S_{y,x} = \sqrt{\frac{\sum \left(y - y_c\right)^2}{n-2}} \quad (4\text{-}14)$$

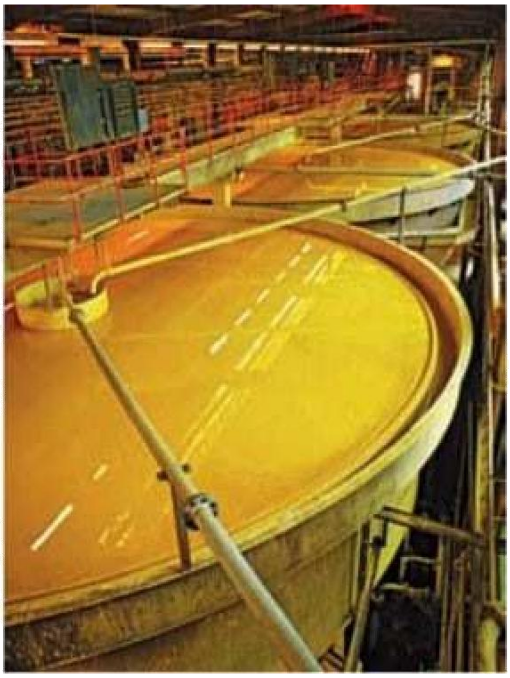
where y = y-value of each data point

y_c = computed value of the dependent variable, from the regression equation

n = number of data points

Figure 4.9 Distribution about the Point Estimate of \$3.25 Million Sales





Glidden Paints' assembly lines require thousands of gallons every hour. To predict demand, the firm uses associative forecasting methods such as linear regression, with independent variables such as disposable personal income and GNP. Although housing starts would be a natural variable, Glidden found that it correlated poorly with past sales. It turns out that most Glidden paint is sold through retailers to customers who already own homes or businesses.

Equation (4-15) may look more complex, but it is actually an easier-to-use version of Equation (4-14). Both formulas provide the same answer and can be used in setting up prediction intervals around the point estimate:[2]

$$S_{y,x} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}} \quad (4\text{-}15)$$

Example 13 shows how we would calculate the standard error of the estimate in Example 12.

Example 13 COMPUTING THE STANDARD ERROR OF THE ESTIMATE

Nodel's VP of operations now wants to know the error associated with the regression line computed in Example 12.

APPROACH Compute the standard error of the estimate, $S_{y,x}$ using Equation (4-15).

SOLUTION The only number we need that is not available to solve for $S_{y,x}$ is Σy^2. Some quick addition reveals $\Sigma y^2 = 39.5$. Therefore:

$$S_{y,x} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}}$$

$$= \sqrt{\frac{39.5 - 1.75(15.0) - 2.5(51.5)}{6-2}}$$

$$= \sqrt{.09375} = .306(\text{in } \$\text{million})$$

The standard error of the estimate is then $306,000 in sales.

INSIGHT The interpretation of the standard error of the estimate is similar to the standard deviation; namely, ±1 standard deviation = .6827. So there is a 68.27% chance of sales being ±$306,000 from the point estimate of $3,250,000.

LEARNING EXERCISE What is the probability sales will exceed $3,556,000? [Answer: About 16%.]

RELATED PROBLEMS 4.41e, 4.48b

Correlation Coefficients for Regression Lines

The regression equation is one way of expressing the nature of the relationship between two variables. Regression lines are not "cause-and-effect" relationships. They merely describe the relationships among variables. The regression equation shows how one variable relates to the value and changes in another variable.

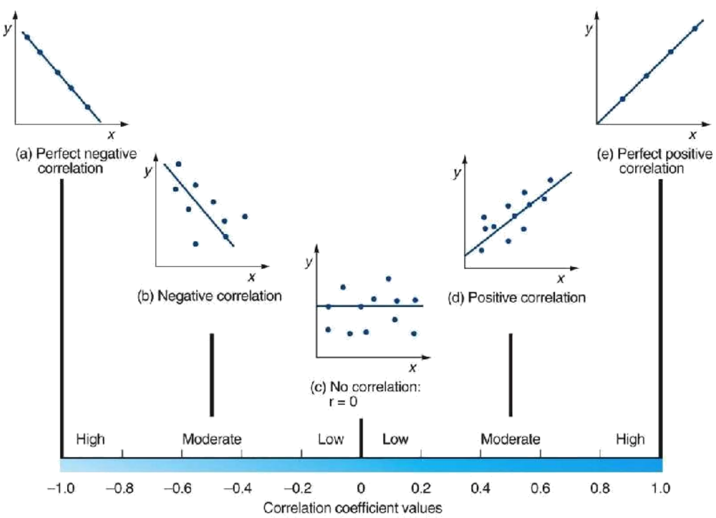
[2]When the sample size is large ($n > 30$), the prediction interval value of y can be computed using normal tables. When the number of observations is small, the t-distribution is appropriate. see D. Groebner et al., *Business Statistics*, 9th ed. (Upper saddle River, NJ: Prentice Hall, 2014).

Another way to evaluate the relationship between two variables is to compute the **coefficient of correlation**. This measure expresses the degree or strength of the linear relationship (but note that correlation does not necessarily imply causality). Usually identified as r, the coefficient of correlation can be any number between +1 and −1. Figure 4.10 illustrates what different values of r might look like.

Coefficient of correlation

A measure of the strength of the relationship between two variables.

Figure 4.10 Five Values of the Correlation Coefficient



To compute r, we use much of the same data needed earlier to calculate a and b for the regression line. The rather lengthy equation for r is:

$$r = \frac{n\Sigma xy - \sum x \sum y}{\sqrt{\left[n\sum x^2 - (\Sigma x)^2\right]\left[n\sum y^2 - (\Sigma y)^2\right]}} \quad \text{(4-16)}$$

Example 14 shows how to calculate the coefficient of correlation for the data given in Examples 12 and 13.

Example 14 DETERMINING THE COEFFICIENT OF CORRELATION

In Example 12, we looked at the relationship between Nodel Construction Company's renovation sales and payroll in its hometown of West Bloomfield. The VP now wants to know the strength of the association between area payroll and sales.

APPROACH We compute the r value using Equation (4-16). We need to first add one more column of calculations—for y^2.

SOLUTION The data, including the column for y^2 and the calculations, are shown here:

y	x	x^2	xy	y^2
2.0	1	1	2.0	4.0
3.0	3	9	9.0	9.0
2.5	4	16	10.0	6.25
2.0	2	4	4.0	4.0
2.0	1	1	2.0	4.0
3.5	7	49	24.5	12.25
$\Sigma y = 15.0$	$\Sigma x = 18$	$\Sigma x^2 = 80$	$\Sigma xy = 51.5$	$\Sigma y^2 = 39.5$

$$r = \frac{(6)(51.5) - (18)(15.0)}{\sqrt{[(6)(80) - (18)^2][(6)(39.5) - (15.0)^2]}}$$

$$= \frac{309 - 270}{\sqrt{(156)(12)}} = \frac{39}{\sqrt{1,872}}$$

$$= \frac{39}{43.3} = .901$$

INSIGHT This r of .901 appears to be a significant correlation and helps confirm the closeness of the relationship between the two variables.

LEARNING EXERCISE If the coefficient of correlation was −.901 rather than +.901, what would this tell you? [Answer: The negative correlation would tell you that as payroll went up, Nodel's sales went down—a rather unlikely occurrence that would suggest you recheck your math.]

RELATED PROBLEMS 4.24d, 4.35d, 4.38c, 4.41f, 4.48b

Although the coefficient of correlation is the measure most commonly used to describe the relationship between two variables, another measure does exist. It is called the **coefficient of determination** and is simply the square of the coefficient of correlation—namely, r^2. The value of r^2 will always be a positive number in the range $0 \le r^2 \le 1$. The coefficient of determination is the percent of variation in the dependent variable (y) that is explained by the regression equation. In Nodel's case, the value of r^2 is .81, indicating that 81% of the total variation is explained by the regression equation.

Coefficient of determination

A measure of the amount of variation in the dependent variable about its mean that is explained by the regression equation.

Multiple-Regression Analysis

Multiple regression is a practical extension of the simple regression model we just explored. It allows us to build a model with several independent variables instead of just one variable. For example, if Nodel Construction wanted to include average annual interest rates in its model for forecasting renovation sales, the proper equation would be:

Multiple regression

An associative forecasting method with more than one independent variable.

$$\hat{y} = a + b_1x_1 + b_2x_2 \quad (4\text{-}17)$$

where y = dependent variable, sales

a = a constant, the y intercept

x_1 and x_2 = values of the two independent variables, area payroll and interest rates, respectively

b_1 and b_2 = coefficients for the two independent variables

The mathematics of multiple regression becomes quite complex (and is usually tackled by computer), so we leave the formulas for a, b_1, and b_2 to statistics textbooks. However, Example 15 shows how to interpret Equation (4-17) in forecasting Nodel's sales.

Example 15 USING A MULTIPLE-REGRESSION EQUATION

Nodel Construction wants to see the impact of a second independent variable, interest rates, on its sales.

APPROACH The new multiple-regression line for Nodel Construction, calculated by computer software, is:

$$\hat{y} = 1.80 + .30x_1 - 5.0x_2$$

We also find that the new coefficient of correlation is .96, implying the inclusion of the variable x_2, interest rates, adds even more strength to the linear relationship.

SOLUTION We can now estimate Nodel's sales if we substitute values for next year's payroll and interest rate. If West Bloomfield's payroll will be \$6 billion and the interest rate will be .12 (12%), sales will be forecast as:

$$\text{Sales(\$ millions)} = 1.80 + .30(6) - 5.0(.12)$$

$$= 1.8 + 1.8 - .6$$

$$= 3.00$$

or:

$$\text{Sales} = \$3{,}000{,}000$$

INSIGHT By using both variables, payroll and interest rates, Nodel now has a sales forecast of S3 million and a higher coefficient of correlation. This suggests a stronger relationship between the two variables and a more accurate estimate of sales.

LEARNING EXERCISE If interest rates were only 6%, what would be the sales forecast? [Answer: 1.8 + 1.8 − 5.0(.06) = 3.3, or \$3,300,000.]

RELATED PROBLEMS 4.34, 4.36

Monitoring and Controlling Forecasts

Once a forecast has been completed, it should not be forgotten. No manager wants to be reminded that his or her forecast is horribly inaccurate, but a firm needs to determine why actual demand (or whatever variable is being examined) differed significantly from that projected. If the forecaster is accurate, that individual usually makes sure that everyone is aware of his or her talents. Very seldom does one read articles in *Fortune*, *Forbes*, or *The Wall Street Journal*, however, about money managers who are consistently off by 25% in their stock market forecasts.

One way to monitor forecasts to ensure that they are performing well is to use a tracking signal. A **tracking signal** is a measurement of how well a forecast is predicting actual values. As forecasts are updated every week, month, or quarter, the newly available demand data are compared to the forecast values.

The tracking signal is computed as the cumulative error divided by the *mean absolute deviation (MAD)*:

Tracking signal

A measurement of how well a forecast is predicting actual values.

$$\text{Tracking singal} = \frac{\text{Cumulative error}}{\text{MAD}} = \frac{\sum(\text{Actual demand in period } i - \text{Forecast demand in period } i)}{\text{MAD}} \quad (4\text{-}18)$$

$$\text{where MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{n}$$

as seen earlier, in Equation (4-5).

Student tip

Using a tracking signal is a good way to make sure the forecasting system is continuing to do a good job.

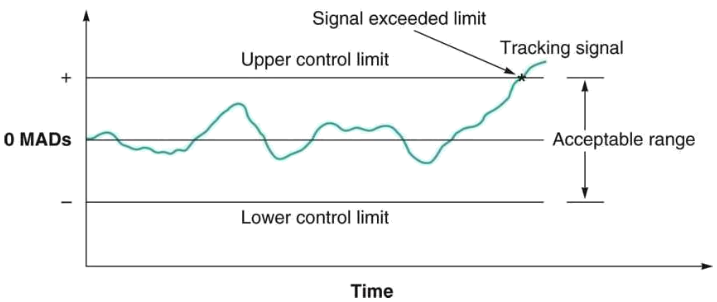
Positive tracking signals indicate that demand is *greater* than forecast. *Negative* signals mean that demand is *less* than forecast. A good tracking signal—that is, one with a low cumulative error—has about as much positive error as it has negative error. In other words, small deviations are okay, but positive and negative errors should balance one another so that the tracking signal centers closely around zero. A consistent tendency for forecasts to be greater or less than the actual values (that is, for a high absolute cumulative error) is called a **bias** error. Bias can occur if, for example, the wrong variables or trend line are used or if a seasonal index is misapplied.

Bias

A forecast that is consistently higher or consistently lower than actual values of a time series.

Once tracking signals are calculated, they are compared with predetermined control limits. When a tracking signal exceeds an upper or lower limit, there is a problem with the forecasting method, and management may want to reevaluate the way it forecasts demand. Figure 4.11 shows the graph of a tracking signal that is exceeding the range of acceptable variation. If the model being used is exponential smoothing, perhaps the smoothing constant needs to be readjusted.

Figure 4.11 A Plot of Tracking Signals



How do firms decide what the upper and lower tracking limits should be? There is no single answer, but they try to find reasonable values—in other words, limits not so low as to be triggered with every small forecast error and not so high as to allow bad forecasts to be regularly overlooked. One MAD is equivalent to approximately .8 standard deviation, ±2 MADs = ±1.6 standard deviations, ±3 MADs = ±2.4 standard deviations, and ±4 MADs = ±3.2 standard deviations. This fact suggests that for a forecast to be "in control," 89% of the errors are expected to fall within ±2 MADs, 98% within ±3 MADs, or 99.9% within ±4 MADs.[3]

LO7

Use a tracking signal

Example 16 shows how the tracking signal and cumulative error can be computed.

Example 16 COMPUTING THE TRACKING SIGNAL AT CARLSON'S BAKERY

Carlson's Bakery wants to evaluate performance of its croissant forecast.

APPROACH Develop a tracking signal for the forecast and see if it stays within acceptable limits, which we define as ±4 MADs.

SOLUTION Using the forecast and demand data for the past 6 quarters for croissant sales, we develop a tracking signal in the table below:

QUARTER	ACTUAL DEMAND	FORECAST DEMAND	ERROR	CUMULATIVE ERROR	ABSOLUTE FORECAST ERROR	CUMULATIVE ABSOLUTE FORECAST ERROR	MAD	TRACKING SIGNAL (CUMULATIVE ERROR/MAD)
1	90	100	−10	−10	10	10	10.0	−10/10 = −1
2	95	100	− 5	−15	5	15	7.5	−15/7.5 = −2
3	115	100	+15	0	15	30	10.0	0/10 = 0
4	100	110	−10	−10	10	40	10.0	−10/10 = −1
5	125	110	+15	+ 5	15	55	11.0	+5/11 = +0.5
6	140	110	+30	+35	30	85	14.2	+35/14.2 = +2.5

At the end of quarter 6,

$$\text{MAD} = \frac{\sum |\text{Forecast errors}|}{n} = \frac{85}{6} = 14.2$$

and Tracking signal =

$$\frac{\text{Cumulative error}}{\text{MAD}} = \frac{35}{14.2} = 2.5 \text{ MADs}$$

INSIGHT Because the tracking signal drifted from −2 MAD to +2.5 MAD (between 1.6 and 2.0 standard deviations), we can conclude that it is within acceptable limits.

LEARNING EXERCISE If actual demand in quarter 6 was 130 (rather than 140), what would be the MAD and resulting tracking signal? [Answer: MAD for quarter 6 would be 12.5, and the tracking signal for period 6 would be 2 MADs.]

RELATED PROBLEMS 4.37, 4.45

[3]To prove these three percentages to yourself, just set up a normal curve for ±1.6 standard deviations (z-values). Using the normal table in Appendix I, you find that the area under the curve is .89. This represents ±2 MADs. Likewise, ±3 MADs = ±2.4 standard deviations encompass 98% of the area,

and so on for ±4 MADs.

Adaptive Smoothing

Adaptive forecasting refers to computer monitoring of tracking signals and self-adjustment if a signal passes a preset limit. For example, when applied to exponential smoothing, the α and β coefficients are first selected on the basis of values that minimize error forecasts and then adjusted accordingly whenever the computer notes an errant tracking signal. This process is called **adaptive smoothing**.

Adaptive smoothing

An approach to exponential smoothing forecasting in which the smoothing constant is automatically changed to keep errors to a minimum.

Focus Forecasting

Rather than adapt by choosing a smoothing constant, computers allow us to try a variety of forecasting models. Such an approach is called focus forecasting. **Focus forecasting** is based on two principles:

Focus forecasting

Forecasting that tries a variety of computer models and selects the best one for a particular application.

- **1.** Sophisticated forecasting models are not always better than simple ones.
- **2.** There is no single technique that should be used for all products or services.

Bernard Smith, inventory manager for American Hardware Supply, coined the term *focus forecasting*. Smith's job was to forecast quantities for 100,000 hardware products purchased by American's 21 buyers.[4] He found that buyers neither trusted nor understood the exponential smoothing model then in use. Instead, they used very simple approaches of their own. So Smith developed his new computerized system for selecting forecasting methods.

Smith chose to test seven forecasting methods. They ranged from the simple ones that buyers used (such as the naive approach) to statistical models. Every month, Smith applied the forecasts of all seven models to each item in stock. In these simulated trials, the forecast values were subtracted from the most recent actual demands, giving a simulated forecast error. The forecast method yielding the least error is selected by the computer, which then uses it to make next month's forecast. Although buyers still have an override capability, American Hardware finds that focus forecasting provides excellent results.

Forecasting in the Service Sector

Forecasting in the service sector presents some unusual challenges. A major technique in the retail sector is tracking demand by maintaining good short-term records. For instance, a barbershop catering to men expects peak flows on Fridays and Saturdays. Indeed, most barbershops are closed on Sunday and Monday, and many call in extra help on Friday and Saturday. A downtown restaurant, on the other hand, may need to track conventions and holidays for effective short-term forecasting. The *OM in Action* box "Forecasting at FedEx's Customer Service Centers" provides an example of a major service-sector industry, the call center.

Student tip

Forecasting at McDonald's, FedEx, and Walmart is as important and complex as it is for manufacturers such as Toyota and Dell.

Specialty Retail Shops

Specialty retail facilities, such as flower shops, may have other unusual demand patterns, and those patterns will differ depending on the holiday. When Valentine's Day falls on a weekend, for example, flowers can't be delivered to offices, and those romantically inclined are likely to celebrate with outings rather than flowers. If a holiday falls on a Monday, some of the celebration may also take place on the weekend, reducing flower sales. However, when Valentine's Day falls in midweek, busy midweek schedules often make flowers the optimal way to celebrate. Because flowers for Mother's Day are to be delivered on Saturday or Sunday, this holiday forecast varies less. Due to special demand patterns, many service firms maintain records of sales, noting not only the day of the week but also unusual events, including the weather, so that patterns and correlations that influence demand can be developed.

VideO 4.2

Forecasting at Hard Rock Cafe

[4]Bernard T. smith, *Focus Forecasting: Computer Techniques for Inventory Control* (Boston: CBI Publishing, 1978).