LEARNING OBJECTIVES

- **LO1** Use a Gantt chart for scheduling 63
- **LO2** Draw AOA and AON networks 66
- **LO3** Complete forward and backward passes for a project 69
- **LO4** Determine a critical path 73
- **LO5** Calculate the variance of activity times 75
- **LO6** Crash a project 80

The Importance of Project Management

STUDENT TIP

Wherever your career takes you, one of the most useful tools you can have, as a manager, is the ability to manage a project.

When Bechtel, the subject of the opening Global Company Profile, begins a project, it quickly has to mobilize substantial resources, often consisting of manual workers, construction professionals, cooks, medical personnel, and even security forces. Its project management team develops a supply chain to access materials to build everything from ports to bridges, dams, and monorails. Bechtel is just one example of a firm that faces modern phenomena: growing project complexity and collapsing product/service life cycles. This change stems from awareness of the strategic value of time-based competition and a quality mandate for continuous improvement. Each new product/service introduction is a unique event—a project. In addition, projects are a common part of our everyday life. We may be planning a wedding or a surprise birthday party, remodeling a house, or preparing a semester-long class project.

VIDEO 3.1

Project Management at Hard Rock's Rockfest

Scheduling projects can be a difficult challenge for operations managers. The stakes in project management are high. Cost overruns and unnecessary delays occur due to poor scheduling and poor controls.

Projects that take months or years to complete are usually developed outside the normal production system. Project organizations within the firm may be set up to handle such jobs and are often disbanded when the project is complete. On other occasions, managers find projects just a part of their job. The management of projects involves three phases (see Figure 3.1):

- **1.** *Planning:* This phase includes goal setting, defining the project, and team organization.
- **2.** *Scheduling:* This phase relates people, money, and supplies to specific activities and relates activities to each other.
- **3.** *Controlling:* Here the firm monitors resources, costs, quality, and budgets. It also revises or changes plans and shifts resources to meet time and cost demands.

We begin this chapter with a brief overview of these functions. Three popular techniques to allow managers to plan, schedule, and control—Gantt charts, PERT, and CPM—are also described.

Project Planning

Projects can be defined as a series of related tasks directed toward a major output. In some firms a **project organization** is developed to make sure existing programs continue to run smoothly on a day-to-day basis while new projects are successfully completed.

Project organization

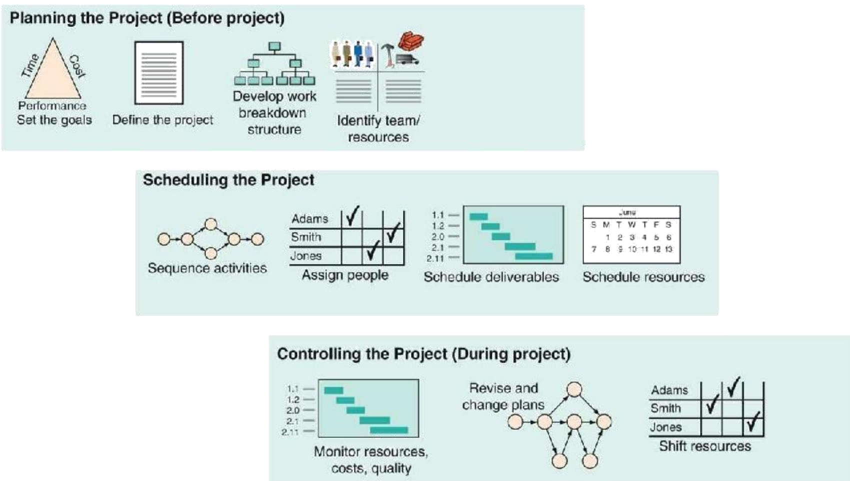
An organization formed to ensure that programs (projects) receive the proper management and attention.

For companies with multiple large projects, such as a construction firm, a project organization is an effective way of assigning the people and physical resources needed. It is a temporary organization structure designed to achieve results by using specialists from throughout the firm.

The project organization may be most helpful when:

- **1.** Work tasks can be defined with a specific goal and deadline.
- **2.** The job is unique or somewhat unfamiliar to the existing organization.
- **3.** The work contains complex interrelated tasks requiring specialized skills.

Figure 3.1 Project Planning, Scheduling, and Controlling



STUDENT TIP

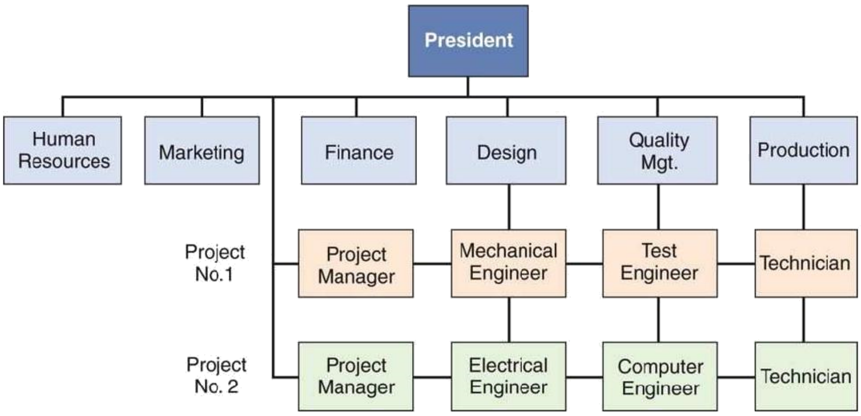
Managers must "make the plan and then work the plan."

- **4.** The project is temporary but critical to the organization.
- **5.** The project cuts across organizational lines.

The Project Manager

An example of a project organization is shown in Figure 3.2. Project team members are temporarily assigned to a project and report to the project manager. The manager heading the project coordinates activities with other departments and reports directly to top management. Project managers receive high visibility in a firm and are responsible for making sure that (1) all necessary activities are finished in proper sequence and on time; (2) the project comes in within budget; (3) the project meets its quality goals; and (4) the people assigned to the project receive the motivation, direction, and information needed to do their jobs. This means that project managers should be good coaches and communicators, and be able to organize activities from a variety of disciplines.

Figure 3.2 A Sample Project Organization



STUDENT TIP

Project organizations can be temporary or permanent. A permanent organization is usually called a *matrix organization.*

Ethical Issues Faced in Project Management

Project managers not only have high visibility but they also face ethical decisions on a daily basis. How they act establishes the code of conduct for the project. Project managers often deal with (1) offers of gifts from contractors, (2) pressure to alter status reports to mask the reality of delays, (3) false reports for charges of time and expenses, and (4) pressures to compromise quality to meet bonuses or avoid penalties related to schedules.

Using the Project Management Institute's (**www.pmi.org**) ethical codes is one means of trying to establish standards. These codes need to be accompanied by good leadership and a strong organizational culture, with its ingrained ethical standards and values.

Work Breakdown Structure

The project management team begins its task well in advance of project execution so that a plan can be developed. One of its first steps is to carefully establish the project's objectives, then break the project down into manageable parts. This **work breakdown structure (WBS)** defines the project by dividing it into its major subcomponents (or tasks), which are then subdivided into more detailed components, and finally into a set of activities and their related costs. The division of the project into smaller and smaller tasks can be difficult, but is critical to managing the project and to scheduling success. Gross requirements for people, supplies, and equipment are also estimated in this planning phase.

Work breakdown structure (WBS)

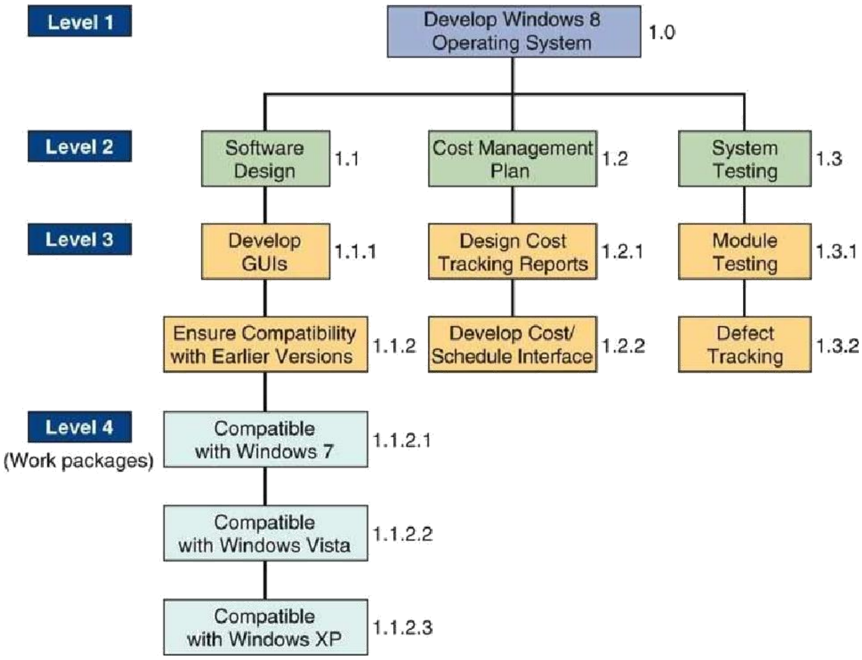
A hierarchical description of a project into more and more detailed components.

The work breakdown structure typically decreases in size from top to bottom and is indented like this:

Level

- 1 Project
- 2 Major tasks in the project
- 3 Subtasks in major tasks
- 4 Activities (or "work packages") to be completed

Figure 3.3 Work Breakdown Structure



This hierarchical framework can be illustrated with the development of Microsoft's operating system Windows 8. As we see in Figure 3.3, the project, creating a new operating system, is labeled 1.0. The first step is to identify the major tasks in the project (level 2). Three examples would be software design (1.1), cost management plan (1.2), and system testing (1.3). Two major subtasks for 1.1 are development of graphical user interfaces (GUIs) (1.1.1) and creating compatibility with previous versions of Windows (1.1.2). The major subtasks for 1.1.2 are level-4 activities, such as creating a team to handle compatibility with Windows 7 (1.1.2.1), creating a team for Windows Vista (1.1.2.2), and creating a team for Windows XP (1.1.2.3). There are usually many level-4 activities.

Project Scheduling

Project scheduling involves sequencing and allotting time to all project activities. At this stage, managers decide how long each activity will take and compute the resources needed at each stage of production. Managers may also chart separate schedules for personnel needs by type of skill (management, engineering, or pouring concrete, for example) and material needs.

One popular project scheduling approach is the Gantt chart. **Gantt charts** are low-cost means of helping managers make sure that (1) activities are planned, (2) order of performance is documented, (3) activity time estimates are recorded, and (4) overall project time is developed. As Figure 3.4 shows, Gantt charts are easy to understand. Horizontal bars are drawn for each project activity along a time line. This illustration of a routine servicing of a Delta jetliner during a 40-minute layover shows that Gantt charts also can be used for scheduling repetitive operations. In this case, the chart helps point out potential delays. The *OM in Action* box on Delta provides additional insights.

Gantt charts

Planning charts used to schedule resources and allocate time.

On simple projects, scheduling charts such as these permit managers to observe the progress of each activity and to spot and tackle problem areas. Gantt charts, though, do not adequately illustrate the interrelationships between the activities and the resources.

STUDENT TIP

Gantt charts are simple and visual, making them widely used.

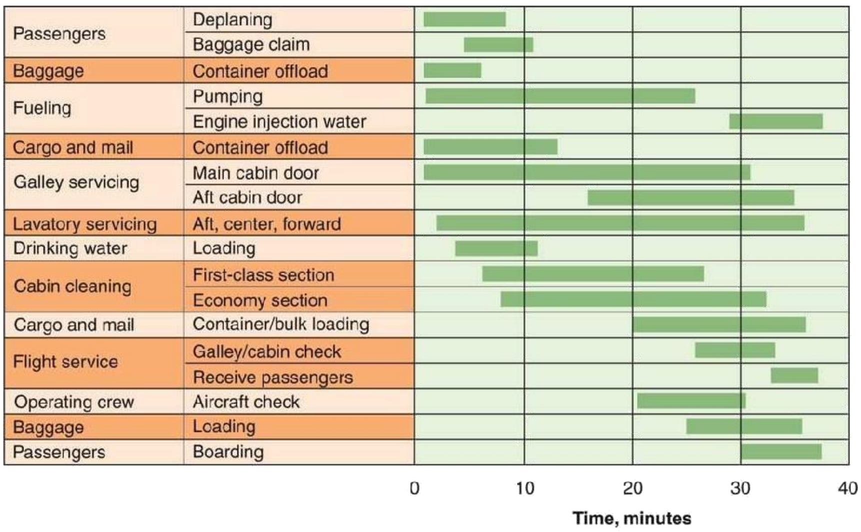
PERT and CPM, the two widely used network techniques that we shall discuss shortly, *do* have the ability to consider precedence relationships and interdependency of activities. On complex projects, the scheduling of which is almost always computerized, PERT and CPM thus have an edge over the simpler Gantt charts. Even on huge projects, though, Gantt charts can be used as summaries of project status and may complement the other network approaches.

LO1 *Use* a Gantt chart for scheduling

To summarize, whatever the approach taken by a project manager, project scheduling serves several purposes:

- **1.** It shows the relationship of each activity to others and to the whole project.
- **2.** It identifies the precedence relationships among activities.
- **3.** It encourages the setting of realistic time and cost estimates for each activity.
- **4.** It helps make better use of people, money, and material resources by identifying critical bottlenecks in the project.

Figure 3.4 Gantt Chart of Service Activities for a Delta Jet during a 40-Minute Layover



Delta saves $50 million a year with this turnaround time, which is a reduction from its traditional 60-minute routine.

OM in Action: Delta's Ground Crew Orchestrates a Smooth Takeoff

Flight 574's engines screech its arrival as the jet lumbers down Richmond's taxiway with 140 passengers arriving from Atlanta. In 40 minutes, the plane is to be airborne again.

However, before this jet can depart, there is business to attend to: passengers, luggage, and cargo to unload and load; thousands of gallons of jet fuel and countless drinks to restock; cabin and restrooms to clean; toilet holding tanks to drain; and engines, wings, and landing gear to inspect.

The 10-person ground crew knows that a miscue anywhere—a broken cargo loader, lost baggage, misdirected passengers—can mean a late departure and trigger a chain reaction of headaches from Richmond to Atlanta to every destination of a connecting flight.

Carla Sutera, the operations manager for Delta's Richmond International Airport, views the turnaround operation like a pit boss awaiting a race car. Trained crews are in place for Flight 574 with baggage carts and tractors, hydraulic cargo loaders, a truck to load food and drinks, another to lift the cleanup crew, another to put fuel on, and a fourth to take water off. The "pit crew" usually performs so smoothly that most passengers never suspect the proportions of the effort. Gantt charts, such as the one in Figure 3.4, aid Delta and other airlines with the staffing and scheduling that are needed for this task.

Jeff Topping/Getty Images

Sources: Knight Ridder Tribune Business News (July 16, 2005) and (November 21, 2002).

Project Controlling

The control of projects, like the control of any management system, involves close monitoring of resources, costs, quality, and budgets. Control also means using a feedback loop to revise the project plan and having the ability to shift resources to where they are needed most. Computerized PERT/CPM reports and charts are widely available today from scores of competing software firms. Some of the more popular of these programs are Primavera (by Primavera Systems, Inc.), MacProject (by Apple Computer Corp.), MindView (by Match Ware), HP Project (by Hewlett-Packard), Fast Track (by AEC Software), and Microsoft Project (by Microsoft Corp.), which we illustrate in this chapter.

VIDEO 3.2

Project Management at Arnold Palmer Hospital

These programs produce a broad variety of reports, including (1) detailed cost breakdowns for each task, (2) total program labor curves, (3) cost distribution tables, (4) functional cost and hour summaries, (5) raw material and expenditure forecasts, (6) variance reports, (7) time analysis reports, and (8) work status reports.

☆ STUDENT TIP

To use project management software, you first need to understand the next two sections in this chapter.

Jonathan Bailey Associates

Construction of the new 11-story building at Arnold Palmer Hospital in Orlando, Florida, was an enormous project for the hospital administration. The photo on the left shows the first six floors under construction. The photo on the right shows the building as completed two years later. Prior to beginning actual construction, regulatory and funding issues added, as they do with most projects, substantial time to the overall project. Cities have zoning and parking issues; the EPA has drainage and waste issues; and regulatory authorities have their own requirements, as do issuers of bonds. The $100 million, 4-year project at Arnold Palmer Hospital is discussed in the Video Case Study at the end of this chapter.

Pia Gandolfo/Jonathan Bailey Associates

Project Management Techniques: PERT and CPM

Program evaluation and review technique (PERT) and the **critical path method (CPM)** were both developed in the 1950s to help managers schedule, monitor, and control large and complex projects. CPM arrived first, as a tool developed to assist in the building and maintenance of chemical plants at duPont. Independently, PERT was developed in 1958 for the U.S. Navy.

Program evaluation and review technique (PERT)

A project management technique that employs three time estimates for each activity.

Critical path method (CPM)

A project management technique that uses only one time factor per activity.

The Framework of PERT and CPM

PERT and CPM both follow six basic steps:

- **1.** Define the project and prepare the work breakdown structure.
- **2.** Develop the relationships among the activities. Decide which activities must precede and which must follow others.
- **3.** Draw the network connecting all the activities.
- **4.** Assign time and/or cost estimates to each activity.
- **5.** Compute the *longest* time path through the network. This is called the **critical path**.

  Critical path

  The computed *longest* time path(s) through a network.

- **6.** Use the network to help plan, schedule, monitor, and control the project.

Step 5, finding the critical path, is a major part of controlling a project. The activities on the critical path represent tasks that will delay the entire project if they are not completed on time. Managers can gain the flexibility needed to complete critical tasks by identifying noncritical activities and replanning, rescheduling, and reallocating labor and financial resources.

Although PERT and CPM differ to some extent in terminology and in the construction of the network, their objectives are the same. Furthermore, the analysis used in both techniques is very similar. The major difference is that PERT employs three time estimates for each activity. These time estimates are used to compute expected values and standard deviations for the activity. CPM makes the assumption that activity times are known with certainty and hence requires only one time factor for each activity.

For purposes of illustration, the rest of this section concentrates on a discussion of PERT. Most of the comments and procedures described, however, apply just as well to CPM.

PERT and CPM are important because they can help answer questions such as the following about projects with thousands of activities:

- **1.** When will the entire project be completed?
- **2.** What are the critical activities or tasks in the project—that is, which activities will delay the entire project if they are late?
- **3.** Which are the noncritical activities—the ones that can run late without delaying the whole project's completion?
- **4.** What is the probability that the project will be completed by a specific date?
- **5.** At any particular date, is the project on schedule, behind schedule, or ahead of schedule?
- **6.** On any given date, is the money spent equal to, less than, or greater than the budgeted amount?
- **7.** Are there enough resources available to finish the project on time?

- **8.** If the project is to be finished in a shorter amount of time, what is the best way to accomplish this goal at the least cost?

Network Diagrams and Approaches

The first step in a PERT or CPM network is to divide the entire project into significant activities in accordance with the work breakdown structure. There are two approaches for drawing a project network: **activity on node (AON)** and **activity on arrow (AOA)**. Under the AON convention, *nodes* designate activities. Under AOA, *arrows* represent activities. Activities consume time and resources. The basic difference between AON and AOA is that the nodes in an AON diagram represent activities. In an AOA network, the nodes represent the starting and finishing times of an activity and are also called *events*. So nodes in AOA consume neither time nor resources.

Activity-on-node (AON)

A network diagram in which nodes designate activities.

Activity-on-arrow (AOA)

A network diagram in which arrows designate activities.

Although both AON and AOA are popular in practice, many of the project management software packages, including Microsoft Project, use AON networks. For this reason, although we illustrate both types of networks in the next examples, we focus on AON networks in subsequent discussions in this chapter.

Example 1 PREDECESSOR RELATIONSHIPS FOR POLLUTION CONTROL AT MILWAUKEE PAPER

Milwaukee Paper Manufacturing had long delayed the expense of installing advanced computerized air pollution control equipment in its facility. But when the board of directors adopted a new proactive policy on sustainability, it did not just *authorize* the budget for the state-of-the-art equipment. It directed the plant manager, Julie Ann Williams, to complete the installation in time for a major announcement of the policy, on Earth Day, exactly 16 weeks away! Under strict deadline from her bosses, Williams needs to be sure that installation of the filtering system progresses smoothly and on time.

Given the following information, develop a table showing activity precedence relationships.

APPROACH Milwaukee Paper has identified the eight activities that need to be performed in order for the project to be completed. When the project begins, two activities can be simultaneously started: building the internal components for the device (activity A) and the modifications necessary for the floor and roof (activity B). The construction of the collection stack (activity C) can begin when the internal components are completed. Pouring the concrete floor and installation of the frame (activity D) can be started as soon as the internal components are completed and the roof and floor have been modified.

After the collection stack has been constructed, two activities can begin: building the high-temperature burner (activity E) and installing the pollution control system (activity F). The air pollution device can be installed (activity G) after the concrete floor has been poured, the frame has been installed, and the high-temperature burner has been built. Finally, after the control system and pollution device have been installed, the system can be inspected and tested (activity H).

SOLUTION Activities and precedence relationships may seem rather confusing when they are presented in this descriptive form. It is therefore convenient to list all the activity information in a table, as shown in Table 3.1. We see in the table that activity A is listed as an *immediate predecessor* of activity C. Likewise, both activities D and E must be performed prior to starting activity G.

TABLE 3.1 Milwaukee Paper Manufacturing's Activities and Predecessors

ACTIVITY	DESCRIPTION	IMMEDIATE PREDECESSORS
A	Build internal components	—
B	Modify roof and floor	—
C	Construct collection stack	A
D	Pour concrete and install frame	A, B
E	Build high-temperature burner	C
F	Install pollution control system	C
G	Install air pollution device	D, E
H	Inspect and test	F, G

INSIGHT To complete a network, all predecessors must be clearly defined.

LEARNING EXERCISE What is the impact on this sequence of activities if Environmental Protection Agency (EPA) approval is required after *Inspect and Test?* [Answer: The immediate predecessor for the new activity would be H, *Inspect and Test*, with *EPA approval* as the last activity.]

Activity-on-Node Example

Note that in Example 1, we only list the *immediate* predecessors for each activity. For instance, in Table 3.1, since activity A precedes activity C, and activity C precedes activity E, the fact that activity A precedes activity E is *implicit*. This relationship need not be explicitly shown in the activity precedence relationships.

When there are many activities in a project with fairly complicated precedence relationships, it is difficult for an individual to comprehend the complexity of the project from just the tabular information. In such cases, a visual representation of the project, using a *project network*, is convenient and useful. A project network is a diagram of all the activities and the precedence relationships that exist between these activities in a project. Example 2 illustrates how to construct an AON project network for Milwaukee Paper Manufacturing.

LO2 *Draw* AOA and AON networks

Example 2 AON GRAPH FOR MILWAUKEE PAPER

Draw the AON network for Milwaukee Paper, using the data in Example 1.

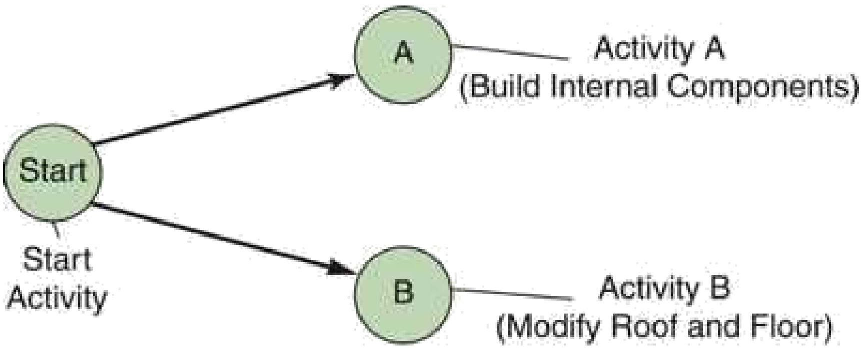
APPROACH In the AON approach, we denote each activity by a node. The lines, or arrows, represent the precedence relationships between the activities.

SOLUTION In this example, there are two activities (A and B) that do not have any predecessors. We draw separate nodes for each of these activities, as shown in Figure 3.5. Although not required, it is usually convenient to have a unique starting activity for a project. We have therefore included a *dummy activity* called *Start* in Figure 3.5. This **dummy activity** does not really exist and takes up zero time and resources. Activity *Start* is an immediate predecessor for both activities A and B, and serves as the unique starting activity for the entire project.

Dummy activity

An activity having no time that is inserted into a network to maintain the logic of the network.

Figure 3.5 Beginning AON Network for Milwaukee Paper

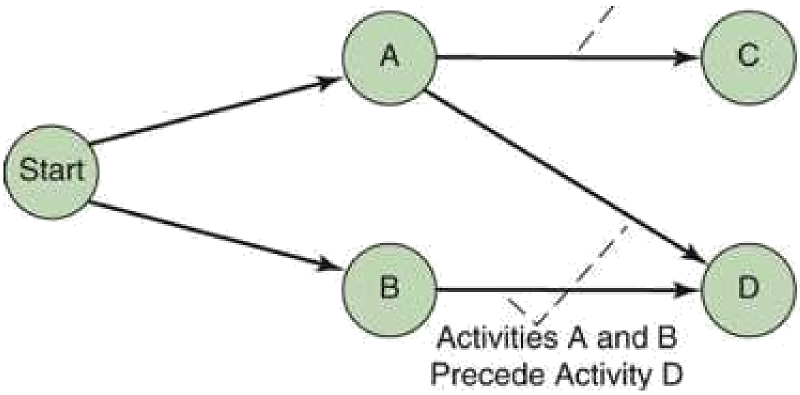


We now show the precedence relationships using lines with arrow symbols. For example, an arrow from activity Start to activity A indicates that Start is a predecessor for activity A. In a similar fashion, we draw an arrow from Start to B.

Next, we add a new node for activity C. Since activity A precedes activity C, we draw an arrow from node A to node C (see Figure 3.6). Likewise, we first draw a node to represent activity D. Then, since activities A and B both precede activity D, we draw arrows from A to D and from B to D (see Figure 3.6).

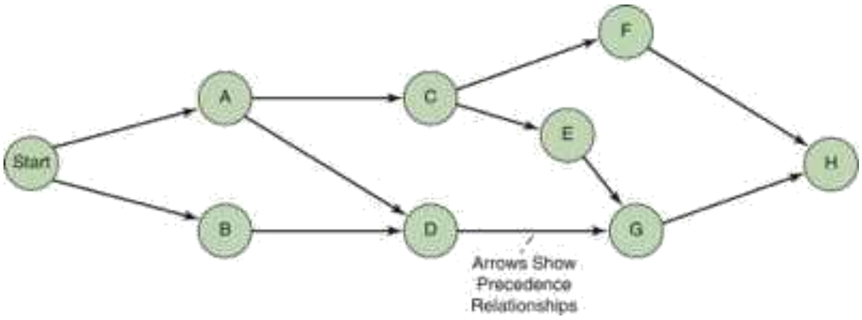
Figure 3.6 Intermediate AON Network for Milwaukee Paper

Activity A Precedes Activity C



We proceed in this fashion, adding a separate node for each activity and a separate line for each precedence relationship that exists. The complete AON project network for the Milwaukee Paper Manufacturing project is shown in Figure 3.7.

Figure 3.7 Complete AON Network for Milwaukee Paper



INSIGHT Drawing a project network properly takes some time and experience. We would like the lines to be straight and arrows to move to the right when possible.

LEARNING EXERCISE If *EPA Approval* occurs after *Inspect and Test*, what is the impact on the graph? [Answer: A straight line is extended to the right beyond H (with a node I added) to reflect the additional activity.]

RELATED PROBLEMS 3.3, 3.6, 3.7, 3.9a, 3.10, 3.12, 3.15a

When we first draw a project network, it is not unusual to place our nodes (activities) in the network in such a fashion that the arrows (precedence relationships) are not straight lines. That is, the lines could be intersecting each other, and even facing in opposite directions. For example, if we had switched the location of the nodes for activities E and F in Figure 3.7, the lines from F to H and E to G would have intersected. Although such a project network is perfectly valid, it is good practice to place the nodes in such a fashion that all arrows point in the same direction.

As with the unique starting node, it is convenient to have the project network finish with a unique ending node. In the Milwaukee Paper example, it turns out that a unique activity, H, is the last activity in the project. We therefore automatically have a unique ending node.

In situations in which a project has multiple ending activities, we include a "dummy" ending activity. We illustrate this type of situation in Solved Problem 3.1 at the end of this chapter.

Activity-on-Arrow Example

In an AOA project network we can represent activities by arrows. A node represents an *event*, which marks the start or completion time of an activity. We usually identify an event (node) by a number.

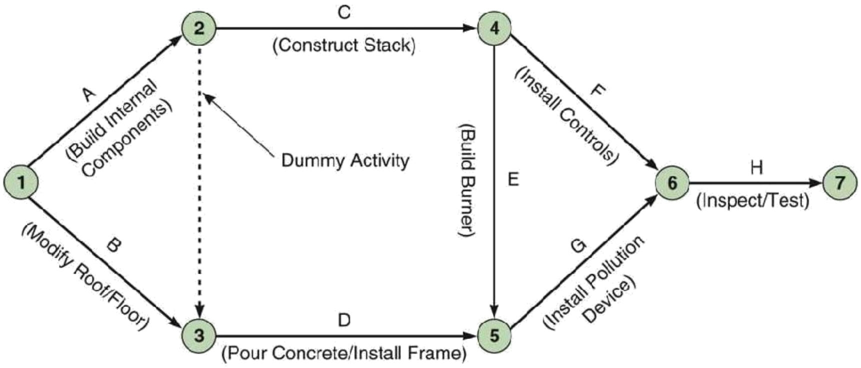
Example 3 ACTIVITY-ON-ARROW FOR MILWAUKEE PAPER

Draw the complete AOA project network for Milwaukee Paper's problem.

APPROACH Using the data from Table 3.1 in Example 1, draw one activity at a time, starting with A.

SOLUTION We see that activity A starts at event 1 and ends at event 2. Likewise, activity B starts at event 1 and ends at event 3. Activity C, whose only immediate predecessor is activity A, starts at node 2 and ends at node 4. Activity D, however, has two predecessors (i.e., A and B). Hence, we need both activities A and B to end at event 3, so that activity D can start at that event. However, we cannot have multiple activities with common starting and ending nodes in an AOA network. To overcome this difficulty, in such cases, we may need to add a dummy line (activity) to enforce the precedence relationship. The dummy activity, shown in Figure 3.8 as a dashed line, is inserted between events 2 and 3 to make the diagram reflect the precedence between A and D. The remainder of the AOA project network for Milwaukee Paper's example is also shown.

Figure 3.8 Complete AOA Network (with Dummy Activity) for Milwaukee Paper



STUDENT TIP

The dummy activity consumes no time, but note how it changes precedence. Now activity D cannot begin until *both* B and the dummy are complete.

INSIGHT Dummy activities are common in AOA networks. They do not really exist in the project and take zero time.

LEARNING EXERCISE A new activity, *EPA Approval*, follows activity H. Add it to Figure 3.8. [Answer: Insert an arrowed line from node 7, which ends at a new node 8, and is labeled I (EPA Approval).]

RELATED PROBLEMS 3.4, 3.5, 3.9b

TABLE 3.2 Time Estimates for Milwaukee Paper Manufacturing

ACTIVITY	DESCRIPTION	TIME (WEEKS)

A	Build internal components	2
B	Modify roof and floor	3
C	Construct collection stack	2
D	Pour concrete and install frame	4
E	Build high-temperature burner	4
F	Install pollution control system	3
G	Install air pollution device	5
H	Inspect and test	2
	Total time (weeks)	25

STUDENT TIP

Does this mean the project will take 25 weeks to complete? No. Don't forget that several of the activities are being performed at the same time. It would take 25 weeks if they were done sequentially.

Determining the Project Schedule

Look back at Figure 3.7 (in Example 2) for a moment to see Milwaukee Paper's completed AON project network. Once this project network has been drawn to show all the activities and their precedence relationships, the next step is to determine the project schedule. That is, we need to identify the planned starting and ending time for each activity.

Let us assume Milwaukee Paper estimates the time required for each activity, in weeks, as shown in Table 3.2. The table indicates that the total time for all eight of the company's activities is 25 weeks. However, since several activities can take place simultaneously, it is clear that the total project completion time may be less than 25 weeks. To find out just how long the project will take, we perform the **critical path analysis** for the network.

Critical path analysis

A process that helps determine a project schedule.

As mentioned earlier, the critical path is the *longest* time path through the network. To find the critical path, we calculate two distinct starting and ending times for each activity. These are defined as follows:

Earliest start (ES)	=	earliest time at which an activity can start, assuming all predecessors have been completed
Earliest finish (EF)	=	earliest time at which an activity can be finished
Latest start (LS)	=	latest time at which an activity can start so as to not delay the completion time of the entire project
Latest finish (LF)	=	latest time by which an activity has to finish so as to not delay the completion time of the entire project

We use a two-pass process, consisting of a forward pass and a backward pass, to determine these time schedules for each activity. The early start and finish times (ES and EF) are determined during the **forward pass**. The late start and finish times (LS and LF) are determined during the backward pass.

Forward pass

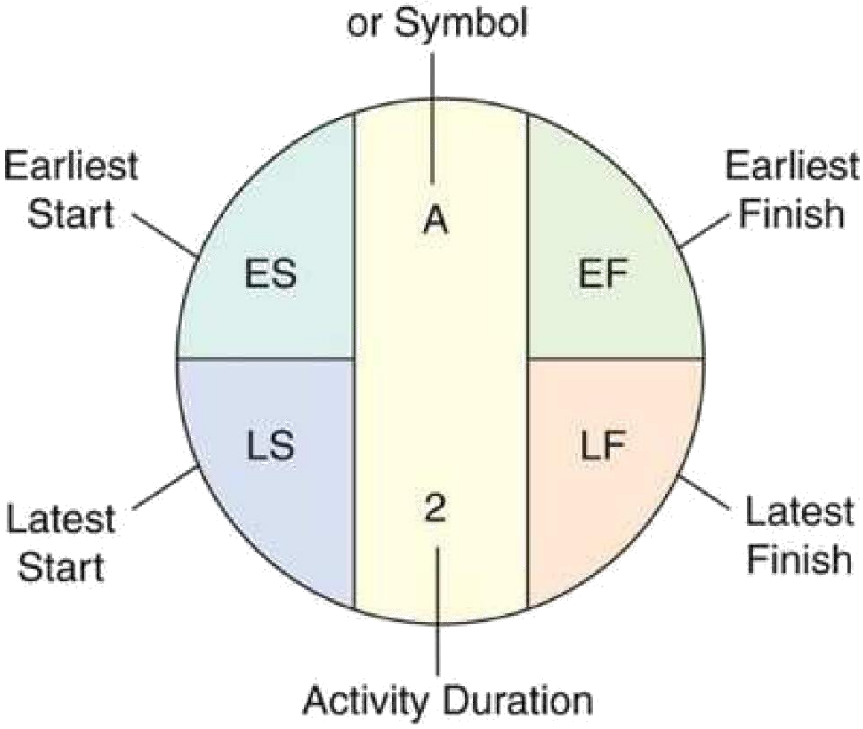
A process that identifies all the early times.

Forward Pass

LO3 *Complete* forward and backward passes for a project

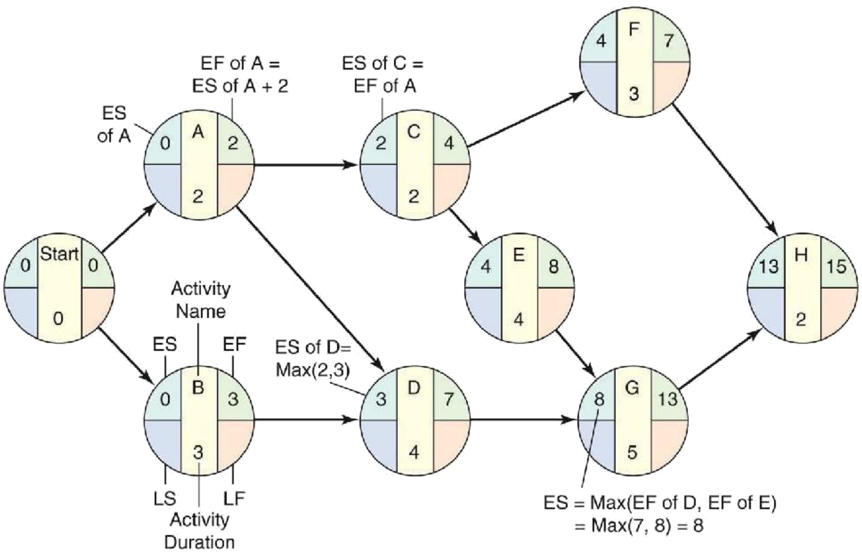
Figure 3.9 Notation Used in Nodes for Forward and Backward Pass

Activity Name



To clearly show the activity schedules on the project network, we use the notation shown in Figure 3.9. The ES of an activity is shown in the top left corner of the node denoting that activity. The EF is shown in the top right corner. The latest times, LS and LF, are shown in the bottom-left and bottom-right corners, respectively.

Figure 3.10 Earliest Start and Earliest Finish Times for Milwaukee Paper



Earliest Start Time Rule

Before an activity can start, *all* its immediate predecessors must be finished:

- ‣ If an activity has only a single immediate predecessor, its ES equals the EF of the predecessor.
- ‣ If an activity has multiple immediate predecessors, its ES is the maximum of all EF values of its predecessors. That is:

  ES = Max {EF of all immediate predecessors} (3-1)

STUDENT TIP

All predecessor activities must be completed before an acitivity can begin.

Earliest Finish Time Rule

The earliest finish time (EF) of an activity is the sum of its earliest start time (ES) and its activity time. That is:

EF = ES 1 Activity time (3-2)

Example 4 COMPUTING EARLIEST START AND FINISH TIMES FOR MILWAUKEE PAPER

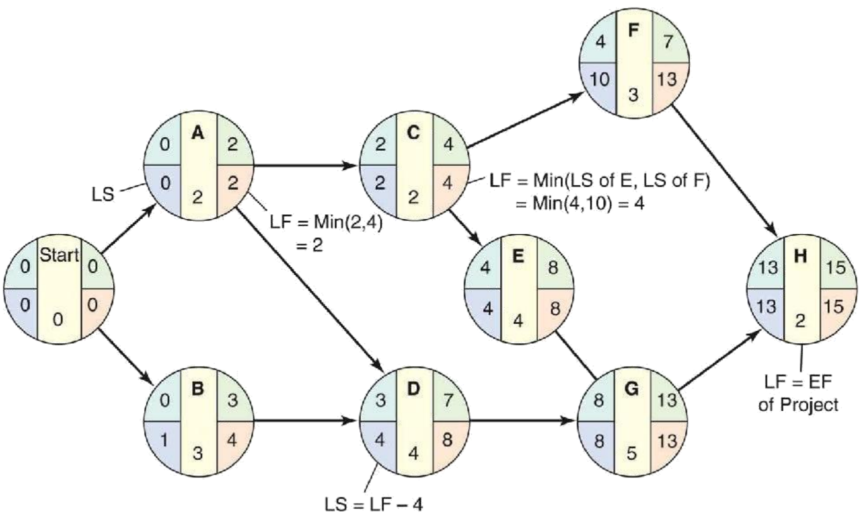
Calculate the earliest start and finish times for the activities in the Milwaukee Paper Manufacturing project.

APPROACH Use Table 3.2, which contains the activity times. Complete the project network for the company's project, along with the ES and EF values for all activities.

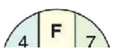
SOLUTION With the help of Figure 3.10, we describe how these values are calculated.

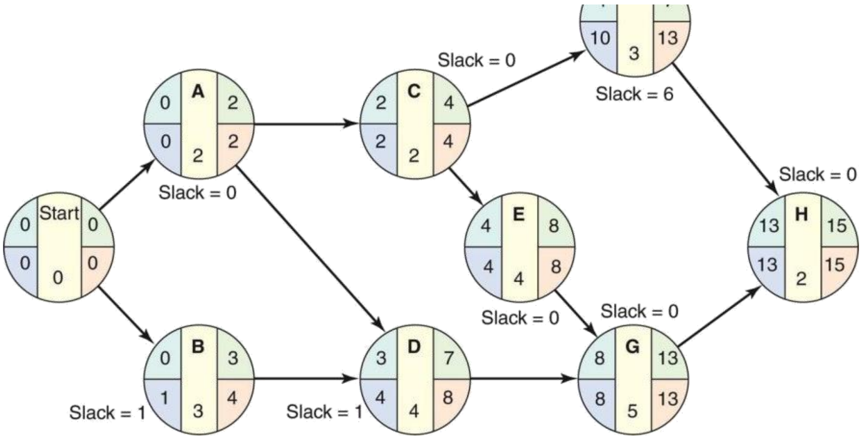
Since activity Start has no predecessors, we begin by setting its ES to 0. That is, activity Start can begin at time 0, which is the same as the beginning of week 1. If activity Start has an ES of 0, its EF is also 0, since its activity time is 0.

Overlay 1: Latest Start and Finish Times Are Now Added.

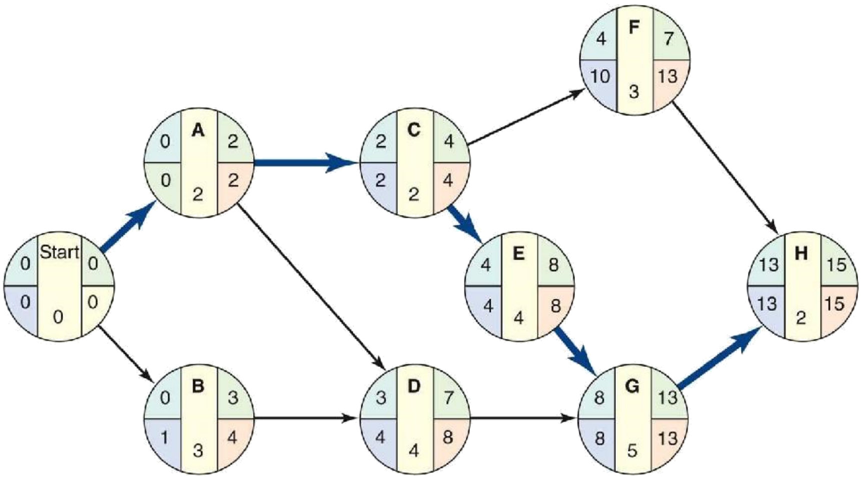


Overlay 2: Slack Times Are Now Computed and Added.





Overlay 3: The Critical Path Is Now Shown in Five Thick Blue Lines.



Next, we consider activities A and B, both of which have only Start as an immediate predecessor. Using the earliest start time rule, the ES for both activities A and B equals zero, which is the EF of activity Start. Now, using the earliest finish time rule, the EF for A is 2 (= 0 + 2), and the EF for B is 3 (= 0 + 3).

Since activity A precedes activity C, the ES of C equals the EF of A (= 2). The EF of C is therefore 4 (= 2 + 2).

We now come to activity D. Both activities A and B are immediate predecessors for D. Whereas A has an EF of 2, activity B has an EF of 3. Using the earliest start time rule, we compute the ES of activity D as follows:

$$\text{ES of D} = \text{Max (EF of A, EF of B)} = \text{Max } (2, 3) = 3$$

The EF of D equals 7 (= 3 + 4). Next, both activities E and F have activity C as their only immediate predecessor. Therefore, the ES for both E and F equals 4 (= EF of C). The EF of E is 8 (= 4 + 4), and the EF of F is 7 (= 4 + 3).

Activity G has both activities D and E as predecessors. Using the earliest start time rule, its ES is therefore the maximum of the EF of D and the EF of E. Hence, the ES of activity G equals 8 (= maximum of 7 and 8), and its EF equals 13 (= 8 + 5).

Finally, we come to activity H. Since it also has two predecessors, F and G, the ES of H is the maximum EF of these two activities. That is, the ES of H equals 13 (= maximum of 13 and 7). This implies that the EF of H is 15 (= 13 + 2). Since H is the last activity in the project, this also implies that the earliest time in which the entire project can be completed is 15 weeks.

INSIGHT The ES of an activity that has only one predecessor is simply the EF of that predecessor. For an activity with more than one predecessor, we must carefully examine the EFs of all immediate predecessors and choose the largest one.

LEARNING EXERCISE A new activity I, *EPA Approval*, takes 1 week. Its predecessor is activity H. What are I's ES and EF? [Answer: 15, 16]

RELATED PROBLEMS 3.11, 3.14c

EXCEL OM Data File **Ch03Ex4.xls** can be found at www.pearsonhighered.com/heizer.

Although the forward pass allows us to determine the earliest project completion time, it does not identify the critical path. To identify this path, we need to now conduct the backward pass to determine the LS and LF values for all activities.

Backward Pass

Just as the forward pass began with the first activity in the project, the **backward pass** begins with the last activity in the project. For each activity, we first determine its LF value, followed by its LS value. The following two rules are used in this process.

Backward pass

An activity that finds all the late start and late finish times.

Latest Finish Time Rule

This rule is again based on the fact that before an activity can start, all its immediate predecessors must be finished:

- ‣ If an activity is an immediate predecessor for just a single activity, its LF equals the LS of the activity that immediately follows it.
- ‣ If an activity is an immediate predecessor to more than one activity, its LF is the minimum of all LS values of all activities that immediately follow it. That is:

  LF = Min{LS of all immediate following activities} (3-3)

Latest Start Time Rule

The latest start time (LS) of an activity is the difference of its latest finish time (LF) and its activity time. That is:

LS = LF – Activity time (3-4)

Example 5 COMPUTING LATEST START AND FINISH TIMES FOR MILWAUKEE PAPER

TIMES FOR MILWAUKEE PAPER

Calculate the latest start and finish times for each activity in Milwaukee Paper's pollution project.

APPROACH Use Figure 3.10 as a beginning point. Overlay 1 of Figure 3.10 shows the complete project network for Milwaukee Paper, along with added LS and LF values for all activities. In what follows, we see how these values were calculated.

SOLUTION We begin by assigning an LF value of 15 weeks for activity H. That is, we specify that the latest finish time for the entire project is the same as its earliest finish time. Using the latest start time rule, the LS of activity H is equal to 13 (= 15 – 2).

Since activity H is the lone succeeding activity for both activities F and G, the LF for both F and G equals 13. This implies that the LS of G is 8 (= 13 – 5), and the LS of F is 10 (= 13 – 3).

Proceeding in this fashion, we see that the LF of E is 8 (= LS of G), and its LS is 4 (= 8 – 4). Likewise, the LF of D is 8 (= LS of G), and its LS is 4 (= 8 – 4).

We now consider activity C, which is an immediate predecessor to two activities: E and F. Using the latest finish time rule, we compute the LF of activity C as follows:

$$\text{LF of C} = \text{Min}(\text{LS of E}, \text{LS of F}) = \text{Min}(4, 10) = 4$$

The LS of C is computed as 2 (= 4 – 2). Next, we compute the LF of B as 4 (= LS of D) and its LS as 1 (= 4 – 3).

We now consider activity A. We compute its LF as 2 (= minimum of LS of C and LS of D). Hence, the LS of activity A is 0 (= 2 – 2). Finally, both the LF and LS of activity Start are equal to 0.

INSIGHT The LF of an activity that is the predecessor of only one activity is just the LS of that following activity. If the activity is the predecessor to more than one activity, its LF is the smallest LS value of all activities that follow immediately.

LEARNING EXERCISE A new activity I, *EPA Approval*, takes 1 week. Its predecessor is activity H. What are I's LS and LF? [Answer: 15, 16]

RELATED PROBLEMS 3.11, 3.14c.

Calculating Slack Time and Identifying the Critical Path(s)

After we have computed the earliest and latest times for all activities, it is a simple matter to find the amount of slack time that each activity has. Slack is the length of time an activity can be delayed without delaying the entire project. Mathematically:

Slack time

Free time for an activity. Also referred to as free float or free slack.

$$\text{Slack} = \text{LS} - \text{ES} \quad \text{or} \quad \text{Slack} = \text{LF} - \text{EF} \qquad (3\text{-}5)$$

Example 6 CALCULATING SLACK TIMES FOR

Example 6 CALCULATING SLACK TIMES FOR MILWAUKEE PAPER

Calculate the slack for the activities in the Milwaukee Paper project.

APPROACH Start with the data in Overlay 1 of Figure 3.10 in Example 5 and develop Table 3.3 one line at a time.

SOLUTION Table 3.3 summarizes the ES, EF, LS, LF, and slack time for all of the firm's activities. Activity B, for example, has 1 week of slack time since its LS is 1 and its ES is 0 (alternatively, its LF is 4 and its EF is 3). This means that activity B can be delayed by up to 1 week, and the whole project can still be finished in 15 weeks.

On the other hand, activities A, C, E, G, and H have *no* slack time. This means that none of them can be delayed without delaying the entire project. Conversely, if plant manager Julie Ann Williams wants to reduce the total project times, she will have to reduce the length of one of these activities.

Overlay 2 of Figure 3.10 shows the slack computed for each activity.

INSIGHT Slack may be computed from either early/late starts or early/late finishes. The key is to find which activities have zero slack.

TABLE 3.3 Milwaukee Paper's Schedule and Slack Times

ACTIVITY	EARLIEST START ES	EARLIEST FINISH EF	LATEST START LS	LATEST FINISH LF	SLACK LS – ES	ON CRITICAL PATH
A	0	2	0	2	0	Yes
B	0	3	1	4	1	No
C	2	4	2	4	0	Yes
D	3	7	4	8	1	No
E	4	8	4	8	0	Yes
F	4	7	10	13	6	No
G	8	13	8	13	0	Yes
H	13	15	13	15	0	Yes

LEARNING EXERCISE A new activity I, *EPA Approval*, follows activity H and takes 1 week. Is it on the critical path? [Answer: Yes, it's LS – ES = 0]

RELATED PROBLEMS 3.6, 3.11, 3.27

ACTIVE MODEL 3.1 This example is further illustrated in Active Model 3.1 at www.pearsonhighered.com/heizer.

The activities with zero slack are called *critical activities* and are said to be on the critical path. The critical path is a continuous path through the project network that:

LO4 *Determine* a critical path

- ‣ Starts at the first activity in the project (Start in our example).
- ‣ Terminates at the last activity in the project (H in our example).
- ‣ Includes only critical activities (i.e., activities with no slack time).

Example 7 SHOWING CRITICAL PATH WITH BLUE ARROWS

Show Milwaukee Paper's critical path and find the project completion time.

APPROACH We use Table 3.3 and Overlay 3 of Figure 3.10. Overlay 3 of Figure 3.10 indicates that the total project completion time of 15 weeks corresponds to the longest path in the network. That path is Start-A-C-E-G-H in network form. It is shown with thick blue arrows.

INSIGHT The critical path follows the activities with slack = 0. This is considered the longest path through the network.

LEARNING EXERCISE Why are activities B, D, and F not on the path with the thick blue line? [Answer: They are not critical and have slack values of 1, 1, and 6 weeks, respectively.]

RELATED PROBLEMS 3.3, 3.4, 3.5, 3.6, 3.7, 3.12, 3.14b, 3.15, 3.17, 3.20a, 3.22a, 3.23, 3.26

Total Slack Time

Look again at the project network in Overlay 3 of Figure 3.10. Consider activities B and D, which have slack of 1 week each. Does it mean that we can delay *each* activity by 1 week, and still complete the project in 15 weeks? The answer is no.

Let's assume that activity B is delayed by 1 week. It has used up its slack of 1 week and now has an EF of 4. This implies that activity D now has an ES of 4 and an EF of 8. Note that these are also its LS and LF values, respectively. That is, activity D also has no slack time now. Essentially, the slack of 1 week that activities B and D had is, for that path, *shared* between them. Delaying either activity by 1 week causes not only that activity, but also the other activity, to lose its slack. This type of a slack time is referred to as *total slack*. Typically, when two or more noncritical activities appear successively in a path, they share total slack.

To plan, monitor, and control the huge number of details involved in sponsoring a rock festival attended by more than 100,000 fans, managers use Microsoft Project and the tools discussed in this chapter. The *Video Case Study* "Managing Hard Rock's Rockfest," at the end of the chapter, provides more details of the management task.

Variability in Activity Times

In identifying all earliest and latest times so far, and the associated critical path(s), we have adopted the CPM approach of assuming that all activity times are known and fixed constants. That is, there is no variability in activity times. However, in practice, it is likely that activity completion times vary depending on various factors.

For example, building internal components (activity A) for Milwaukee Paper Manufacturing is estimated to finish in 2 weeks. Clearly, supply-chain issues such as late arrival of materials, absence of key personnel, and so on could delay this activity. Suppose activity A actually ends up taking 3 weeks. Since A is on the critical path, the entire project will now be delayed by 1 week to 16 weeks. If we had anticipated completion of this project in 15 weeks, we would obviously miss our Earth Day deadline.

Although some activities may be relatively less prone to delays, others could be extremely susceptible to delays. For example, activity B (modify roof and floor) could be heavily dependent on weather conditions. A spell of bad weather could significantly affect its completion time.

This means that we cannot ignore the impact of variability in activity times when deciding the schedule for a project. PERT addresses this issue.

STUDENT TIP

PERT's ability to handle three time estimates for each activity enables us to compute the probability that we can complete the project by a target date.

Three Time Estimates in PERT

In PERT, we employ a probability distribution based on three time estimates for each activity, as follows:

Optimistic = time an activity will take if everything goes as planned. In estimating this value, there

time (*a*)		should be only a small probability (say, 1/100) that the activity time will be, a.
Pessimistic time (*b*)	=	time an activity will take assuming very unfavorable conditions. In estimating this value, there should also be only a small probability (also 1/100) that the activity time will be > *b*.
Most likely time (*m*)	=	most realistic estimate of the time required to complete an activity.

Optimistic time

The "best" activity completion time that could be obtained in a PERT network.

Pessimistic time

The "worst" activity time that could be expected in a PERT network.

Most likely time

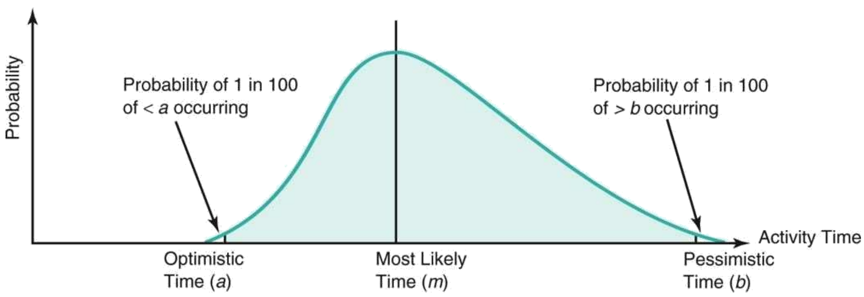
The most probable time to complete an activity in a PERT network.

When using PERT, we often assume that activity time estimates follow the beta probability distribution (see Figure 3.11). This continuous distribution is often appropriate for determining the expected value and variance for activity completion times.

To find the *expected activity time, t*, the beta distribution weights the three time estimates as follows:

$t = (a + 4m + b)/6$ (3-6)

Figure 3.11 Beta Probability Distribution with Three Time Estimates



That is, the most likely time (*m*) is given four times the weight as the optimistic time (*a*) and pessimistic time (*b*). The time estimate *t* computed using Equation (3-6) for each activity is used in the project network to compute all earliest and latest times.

To compute the *dispersion* or *variance of activity completion time*, we use the formula:[1]

$\text{Variance} = [(b - a)/6]^2$

(3-7)

Example 8 EXPECTED TIMES AND VARIANCES FOR MILWAUKEE PAPER

Julie Ann Williams and the project management team at Milwaukee Paper want an expected time and variance for Activity F (Installing the Pollution Control System) where:

$$a = 1 \text{ week}, m = 2 \text{ weeks}, b = 9 \text{ weeks}$$

APPROACH Use Equations (3–6) and (3–7) to compute the expected time and variance for F.

SOLUTION The expected time for Activity F is:

$$t = \frac{a + 4m + b}{6} = \frac{1 + 4(2) + 9}{6} = \frac{18}{6} = 3 \text{ weeks}$$

The variance for Activity F is:

$$\text{Variance} = \left[\frac{(b-a)}{6}\right]^2 = \left[\frac{(b-a)}{6}\right]^2 = \left(\frac{8}{6}\right)^2 = \frac{64}{36} = 1.78$$

LO5 *Calculate* the variance of activity times

INSIGHT Williams now has information that allows her to understand and manage Activity F. The expected time is, in fact, the activity time used in our earlier computation and identification of the critical path.

LEARNING EXERCISE Review the expected times and variances for all of the other activities in the project. These are shown in Table 3.4.

TABLE 3.4 Time Estimates (in weeks) for Milwaukee Paper's Project

ACTIVITY	OPTIMISTIC a	MOST LIKELY m	PESSIMISTIC b	EXPECTED TIME $t = (a + 4m + b)/6$	VARIANCE $[(b - a)/6]^2$
A	1	2	3	2	$[(3 - 1)/6]^2 = 4/36 = .11$
B	2	3	4	3	$[(4 - 2)/6]^2 = 4/36 = .11$
C	1	2	3	2	$[(3 - 1)/6]^2 =$

					4/36 = .11
D	2	4	6	4	$[(6 – 2)/6]^2$ = 16/36 = .44
E	1	4	7	4	$[(7 – 1)/6]^2$ = 36/36 = 1.00
F	1	2	9	3	$[(9 – 1)/6]^2$ = 64/36 = 1.78
G	3	4	11	5	$[(11 – 3)/6]^2$ = 64/36 = 1.78
H	1	2	3	2	$[(3 – 1)/6]^2$ = 4/36 = .11

STUDENT TIP

Can you see why the variance is higher in some activities than in others? Note the spread between the optimistic and pessimistic times.

RELATED PROBLEMS 3.13, 3.14a, 3.17a, b, 3.21a

EXCEL OM Data File **Ch03Ex8.xls** can be found at www.pearsonhighered.com/heizer.

[1] This formula is based on the statistical concept that from one end of the beta distribution to the other is 6 standard deviations (±3 standard deviations from the mean). Since $(b – a)$ is 6 standard deviations, the variance is $[(b – a)/6]^2$.

Paul Chesley/Getty Images

We see here a ship being built at the Hyundai shipyard, Asia's largest shipbuilder, in Korea. Managing this project uses the same techniques as managing the remodeling of a store, installing a new production line, or implementing a new computer system.

Probability of Project Completion

The critical path analysis helped us determine that Milwaukee Paper's expected project completion time is 15 weeks. Julie Ann Williams knows, however, that there is significant variation in the time estimates for several activities. Variation in activities that are on the critical path can affect the overall project completion time—possibly delaying it. This is one occurrence that worries the plant manager considerably.

PERT uses the variance of critical path activities to help determine the variance of the overall project. Project variance is computed by summing variances of *critical* activities:

$$\sigma_p^2 = \text{Project variance} = \sum (\text{variances of activities on critical path}) \quad (3\text{-}8)$$

Example 9 COMPUTING PROJECT VARIANCE AND STANDARD DEVIATION FOR MILWAUKEE PAPER

Milwaukee Paper's managers now wish to know the project's variance and standard deviation.

APPROACH Because the activities are independent, we can add the variances of the activities on the critical path and then take the square root to determine the project's standard deviation.

SOLUTION From Example 8 (Table 3.4), we have the variances of all of the activities on the critical path. Specifically, we know that the variance of activity A is 0.11, variance of activity C is 0.11, variance of activity E is 1.00, variance of activity G is 1.78, and variance of activity H is 0.11.

Compute the total project variance and project standard deviation:

$$\text{Project variance}\left(\sigma_p^2\right) = 0.11 + 0.11 + 1.00 + 1.78 + 0.11 = 3.11$$

which implies:

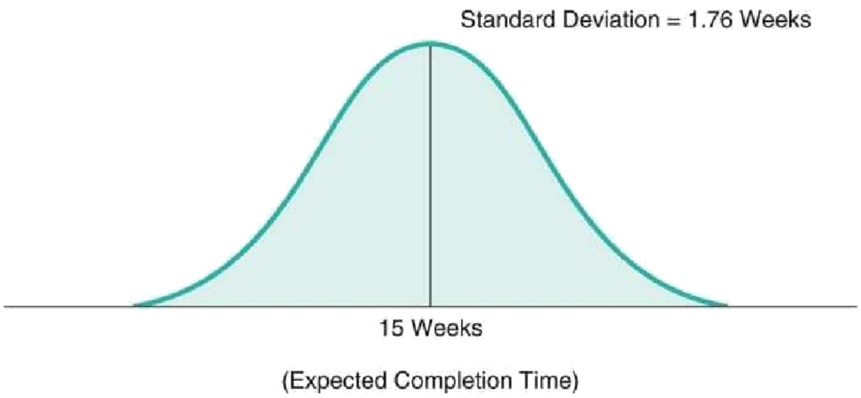
$$\text{Project standard deviation}\left(\sigma_p\right) = \sqrt{\text{Project variance}} = \sqrt{3.11} = 1.76 \text{ weeks}$$

INSIGHT Management now has an estimate not only of expected completion time for the project but also of the standard deviation of that estimate.

LEARNING EXERCISE If the variance for activity A is actually 0.30 (instead of 0.11), what is the new project standard deviation? [Answer: 1.817.]

RELATED PROBLEM 3.17e

Figure 3.12 Probability Distribution for Project Completion Times at Milwaukee Paper



How can this information be used to help answer questions regarding the probability of finishing the project on time? PERT makes two more assumptions: (1) total project completion times follow a normal probability distribution, and (2) activity times are statistically independent. With these assumptions, the bell-shaped normal curve shown in Figure 3.12 can be used to represent project completion dates. This normal curve implies that there is a 50% chance that the manufacturer's project completion time will be less than 15 weeks and a 50% chance that it will exceed 15 weeks.

Example 10 PROBABILITY OF COMPLETING A PROJECT ON TIME

Julie Ann Williams would like to find the probability that her project will be finished on or before the 16-week Earth Day deadline.

APPROACH To do so, she needs to determine the appropriate area under the normal curve. This is the area to the left of the 16th week.

SOLUTION The standard normal equation can be applied as follows:

$$Z = (\text{Due date} - \text{Expected date of completion}) / \sigma_p \quad (3\text{-}9)$$

$$= (16 \text{ weeks} - 15 \text{ weeks}) / 1.76 \text{ weeks} = 0.57$$

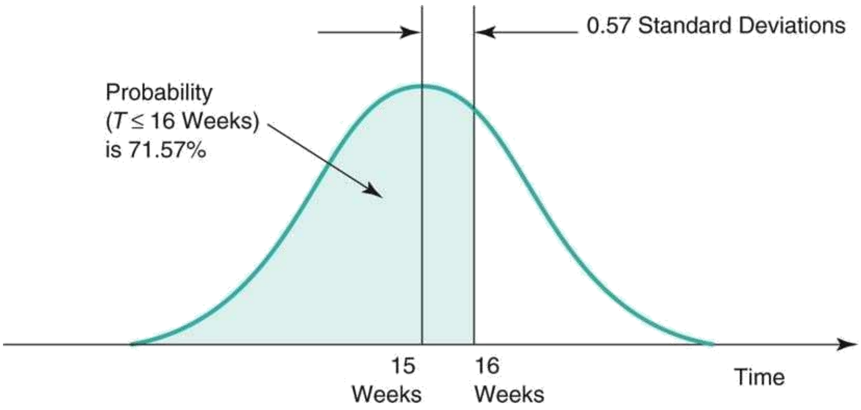
STUDENT TIP

Here is a chance to review your statistical skills and use of a normal distribution table (Appendix I).

where Z is the number of standard deviations the due date or target date lies from the mean or expected date.

Referring to the Normal Table in Appendix I, we find a Z value of 0.57 to the right of the mean indicates a probability of 0.7157. Thus, there is a 71.57% chance that the pollution control equipment can be put in place in 16 weeks or less. This is shown in Figure 3.13.

Figure 3.13 Probability That Milwaukee Paper Will Meet the 16-Week Deadline



INSIGHT The shaded area to the left of the 16th week (71.57%) represents the probability that the project will be completed in less than 16 weeks.

LEARNING EXERCISE What is the probability that the project will be completed on or before the 17th week? [Answer: About 87.2%.]

RELATED PROBLEMS 3.14d, 3.17f, 3.21d, e, 3.22b, 3.24

Determining Project Completion Time for a Given Confidence Level

Let's say Julie Ann Williams is worried that there is only a 71.57% chance that the pollution control equipment can be put in place in 16 weeks or less. She thinks that it may be possible to plead with the board of directors for more time. However, before she approaches the board, she wants to arm herself with sufficient information about the project. Specifically, she wants to find the deadline by which she has a 99% chance of completing the project. She hopes to use her analysis to convince the board to agree to this extended deadline, even though she is aware of the public relations damage the delay will cause.

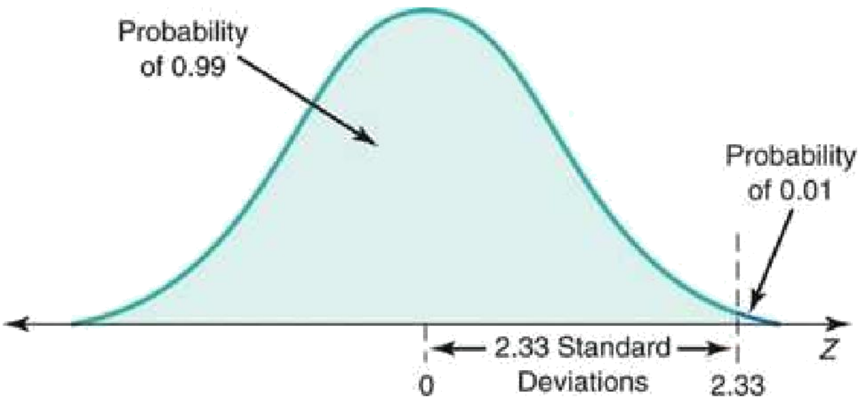
Clearly, this due date would be greater than 16 weeks. However, what is the exact value of this new due date? To answer this question, we again use the assumption that Milwaukee Paper's project completion time follows a normal probability distribution with a mean of 15 weeks and a standard deviation of 1.76 weeks.

Example 11 COMPUTING PROBABILITY FOR ANY COMPLETION DATE

Julie Ann Williams wants to find the due date that gives her company's project a 99% chance of *on-time* completion.

APPROACH She first needs to compute the *Z*-value corresponding to 99%, as shown in Figure 3.14. Mathematically, this is similar to Example 10, except the unknown is now the due date rather than *Z*.

Figure 3.14 *Z*-Value for 99% Probability of Project Completion at Milwaukee Paper



SOLUTION Referring again to the Normal Table in Appendix I, we identify a *Z*-value of 2.33 as being closest to the probability of 0.99. That is, Julie Ann Williams's due date should be 2.33 standard deviations above the mean project completion time. Starting with the standard normal equation [see Equation (3-9)], we can solve for the due date and rewrite the equation as:

$$\begin{aligned} \text{Due date} &= \text{Expected completion time} + \left(Z \times \sigma_p \right) \\ &= 15 + (2.33 \times 1.76) = 19.1 \text{ weeks} \end{aligned} \qquad (3\text{-}10)$$

INSIGHT If Williams can get the board to agree to give her a new deadline of 19.1 weeks (or more), she can be 99% sure of finishing the project by that new target date.

LEARNING EXERCISE What due date gives the project a 95% chance of on-time completion? [Answer: About 17.9 weeks.]

RELATED PROBLEMS 3.22c, 3.24e

Variability in Completion Time of Noncritical Paths

In our discussion so far, we have focused exclusively on the variability in the completion times of activities on the critical path. This seems logical since these activities are, by definition, the more important activities in a project network. However, when there is variability in activity times, it is important that we also investigate the variability in the completion times of activities on *noncritical* paths.

Consider, for example, activity D in Milwaukee Paper's project. Recall from Overlay 3 in Figure 3.10 (in Example 7) that this is a noncritical activity, with a slack time of 1 week. We have therefore not considered the variability in D's time in computing the probabilities of project completion times. We observe, however, that D has a variance of 0.44 (see Table 3.4 in Example 8). In fact, the pessimistic completion time for D is 6 weeks. This means that if D ends up taking its pessimistic time to finish, the project will not finish in 15 weeks, even though D is not a critical activity.

For this reason, when we find probabilities of project completion times, it may be necessary for us to not focus only on the critical path(s). Indeed, some research has suggested that expending project resources to reduce the variability of activities not on the critical path can be an effective element in project management. We may need also to compute these probabilities for noncritical paths, especially those that have relatively large variances. It is possible for a noncritical path to have a smaller probability of completion within a due date, when compared with the critical path. Determining the variance and probability of completion for a noncritical path is done in the same manner as Examples 9 and 10.

What Project Management Has Provided So Far

Project management techniques have thus far been able to provide Julie Ann Williams with several valuable pieces of management information:

- **1.** The project's expected completion date is 15 weeks.
- **2.** There is a 71.57% chance that the equipment will be in place within the 16-week deadline. PERT analysis can easily find the probability of finishing by any date Williams is interested in.
- **3.** Five activities (A, C, E, G, and H) are on the critical path. If any one of these is delayed for any reason, the entire project will be delayed.
- **4.** Three activities (B, D, F) are not critical and have some slack time built in. This means that Williams can borrow from their resources, and, if necessary, she may be able to speed up the whole project.
- **5.** A detailed schedule of activity starting and ending dates, slack, and critical path activities has been made available (see Table 3.3 in Example 6).

Cost–Time Trade-Offs and Project Crashing

While managing a project, it is not uncommon for a project manager to be faced with either (or both) of the following situations: (1) the project is behind schedule, and (2) the scheduled project completion time has been moved forward. In either situation, some or all of the remaining activities need to be speeded up (usually by adding resources) to finish the project by the desired due date. The process by which we shorten the duration of a project in the cheapest manner possible is called project **crashing**.

Crashing

Shortening activity time in a network to reduce time on the critical path so total completion time is reduced.

CPM is a technique in which each activity has a *normal* or *standard* time that we use in our computations. Associated with this normal time is the *normal* cost of the activity. However, another time in project management is the *crash time*, which is defined as the shortest duration required to

complete an activity. Associated with this crash time is the *crash cost* of the activity. Usually, we can shorten an activity by adding extra resources (e.g., equipment, people) to it. Hence, it is logical for the crash cost of an activity to be higher than its normal cost.

The amount by which an activity can be shortened (i.e., the difference between its normal time and crash time) depends on the activity in question. We may not be able to shorten some activities at all. For example, if a casting needs to be heat-treated in the furnace for 48 hours, adding more resources does not help shorten the time. In contrast, we may be able to shorten some activities significantly (e.g., frame a house in 3 days instead of 10 days by using three times as many workers).

Likewise, the cost of crashing (or shortening) an activity depends on the nature of the activity. Managers are usually interested in speeding up a project at the least additional cost. Hence, when choosing which activities to crash, and by how much, we need to ensure the following:

- ‣ The amount by which an activity is crashed is, in fact, permissible
- ‣ Taken together, the shortened activity durations will enable us to finish the project by the due date
- ‣ The total cost of crashing is as small as possible

LO6 *Crash* a project

Crashing a project involves four steps:

- **STEP 1:** Compute the crash cost per week (or other time period) for each activity in the network. If crash costs are linear over time, the following formula can be used:

$$\text{Crash cost per period} = \frac{(\text{Crash cost} - \text{Normal cost})}{(\text{Normal time} - \text{Crash time})} \quad (3\text{-}11)$$

- **STEP 2:** Using the current activity times, find the critical path(s) in the project network. Identify the critical activities.
- **STEP 3:** If there is only one critical path, then select the activity on this critical path that (a) can still be crashed and (b) has the smallest crash cost per period. Crash this activity by one period. If there is more than one critical path, then select one activity from each critical path such that (a) each selected activity can still be crashed and (b) the total crash cost per period of *all* selected activities is the smallest. Crash each activity by one period. Note that the same activity may be common to more than one critical path.
- **STEP 4:** Update all activity times. If the desired due date has been reached, stop. If not, return to Step 2.

We illustrate project crashing in Example 12.

Example 12 PROJECT CRASHING TO MEET A DEADLINE AT MILWAUKEE PAPER

Suppose the plant manager at Milwaukee Paper Manufacturing has been given only 13 weeks (instead of 16 weeks) to install the new pollution control equipment. As you recall, the length of Julie Ann Williams's critical path was 15 weeks, but she must now complete the project in 13 weeks.

APPROACH Williams needs to determine which activities to crash, and by how much, to meet this 13-week due date. Naturally, Williams is interested in speeding up the project by 2 weeks, at the least additional cost.

SOLUTION The company's normal and crash times, and normal and crash costs, are shown in Table

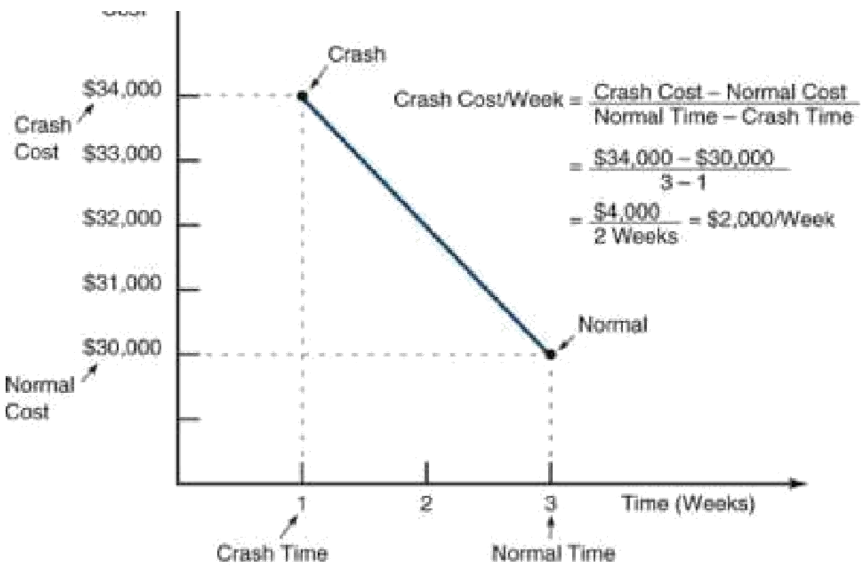
3.5. Note, for example, that activity B's normal time is 3 weeks (the estimate used in computing the critical path), and its crash time is 1 week. This means that activity B can be shortened by up to 2 weeks if extra resources are provided. The cost of these additional resources is $4,000 (= difference between the crash cost of $34,000 and the normal cost of $30,000). If we assume that the crashing cost is linear over time (i.e., the cost is the same each week), activity B's crash cost per week is $2,000 (= $4,000/2).

TABLE 3.5 Normal and Crash Data for Milwaukee Paper Manufacturing

ACTIVITY	TIME (WEEKS)		COST ($)		CRASH COST PER WEEK ($)	CRITICAL PATH?
	NORMAL	CRASH	NORMAL	CRASH		
A	2	1	22,000	22,750	750	Yes
B	3	1	30,000	34,000	2,000	No
C	2	1	26,000	27,000	1,000	Yes
D	4	3	48,000	49,000	1,000	No
E	4	2	56,000	58,000	1,000	Yes
F	3	2	30,000	30,500	500	No
G	5	2	80,000	84,500	1,500	Yes
H	2	1	16,000	19,000	3,000	Yes

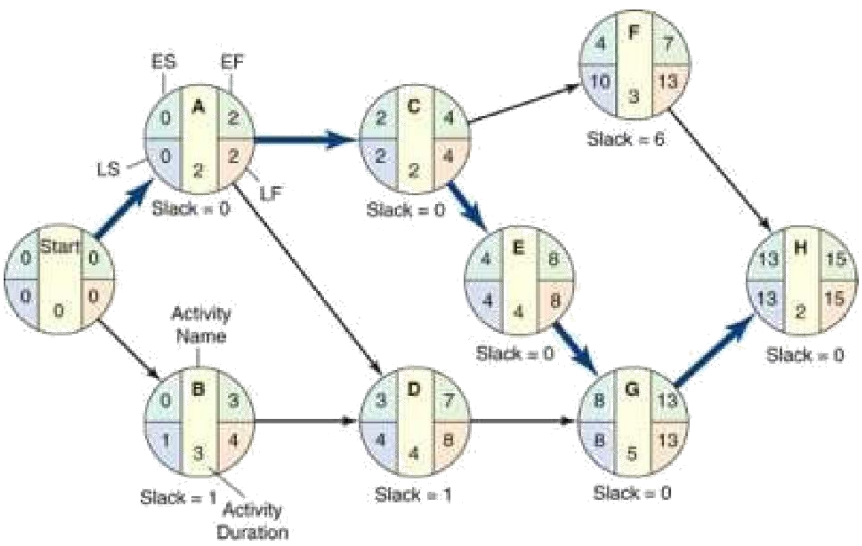
This calculation for Activity B is shown in Figure 3.15. Crash costs for all other activities can be computed in a similar fashion.

Figure 3.15 Crash and Normal Times and Costs for Activity B



Steps 2, 3, and 4 can now be applied to reduce Milwaukee Paper's project completion time at a minimum cost. We show the project network for Milwaukee Paper again in Figure 3.16.

Figure 3.16 Critical Path and Slack Times for Milwaukee Paper



The current critical path (using normal times) is Start–A–C–E–G–H, in which Start is just a dummy starting activity. Of these critical activities, activity A has the lowest crash cost per week of $750. Julie Ann Williams should therefore crash activity A by 1 week to reduce the project completion time to 14 weeks. The cost is an additional $750. Note that activity A cannot be crashed any further, since it has reached its crash limit of 1 week.

At this stage, the original path Start–A–C–E–G–H remains critical with a completion time of 14 weeks. However, a new path Start–B–D–G–H is also critical now, with a completion time of 14 weeks. Hence, any further crashing must be done to both critical paths.

On each of these critical paths, we need to identify one activity that can still be crashed. We also want the total cost of crashing an activity on each path to be the smallest. We might be tempted to simply pick the activities with the smallest crash cost per period in each path. If we did this, we would select activity C from the first path and activity D from the second path. The total crash cost would then be $2,000 (= $1,000 + $1,000).

But we spot that activity G is common to both paths. That is, by crashing activity G, we will simultaneously reduce the completion time of both paths. Even though the $1,500 crash cost for activity G is higher than that for activities C and D, we would still prefer crashing G, since the total crashing cost will now be only $1,500 (compared with the $2,000 if we crash C and D).

INSIGHT To crash the project down to 13 weeks, Williams should crash activity A by 1 week and activity G by 1 week. The total additional cost will be $2,250 (= $750 + $1,500). This is important because many contracts for projects include bonuses or penalties for early or late finishes.

LEARNING EXERCISE Say the crash cost for activity B is $31,000 instead of $34,000. How does this change the answer? [Answer: no change.]

RELATED PROBLEMS 3.16, 3.18, 3.19, 3.20, 3.25

EXCEL OM Data File **Ch3Ex12.xls** can be found at www.pearsonhighered.com/heizer.

A Critique of PERT and CPM

As a critique of our discussions of PERT, here are some of its features about which operations managers need to be aware:

Advantages

- **1.** Especially useful when scheduling and controlling large projects.
- **2.** Straightforward concept and not mathematically complex.
- **3.** Graphical networks help highlight relationships among project activities.
- **4.** Critical path and slack time analyses help pinpoint activities that need to be closely watched.
- **5.** Project documentation and graphs point out who is responsible for various activities.
- **6.** Applicable to a wide variety of projects.
- **7.** Useful in monitoring not only schedules but costs as well.

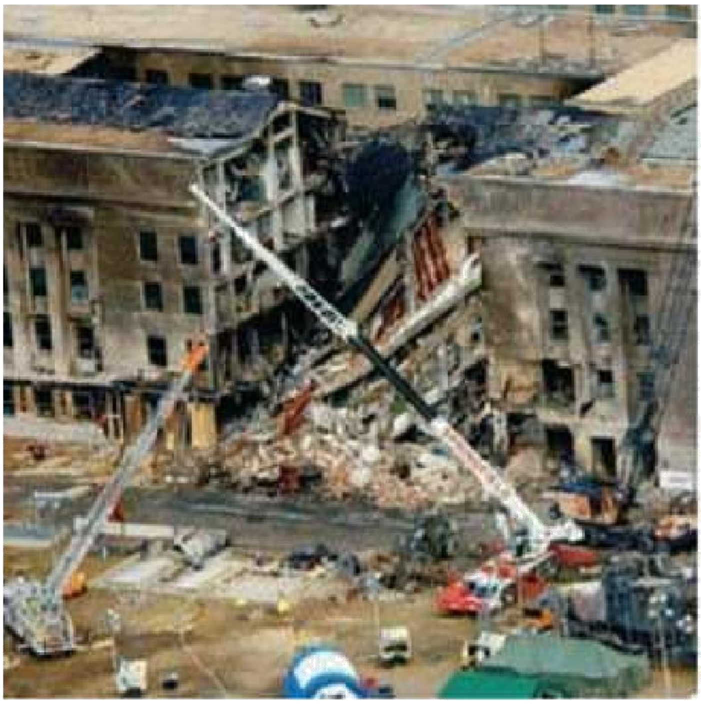
OM in Action: Rebuilding the Pentagon After 9/11

On September 11, 2001, American Airlines Flight 77 slammed into the Pentagon. The world was shocked by this and the other terrorist attacks on the Twin Towers in New York City. One hundred and twenty-five people died when a large portion of the Pentagon was severely damaged. Among the first to react were construction workers renovating another portion of the Pentagon. Their heroism saved lives and eased suffering. Within hours of the disaster, heavy equipment began arriving on the site, accompanied by hundreds of volunteer construction workers driven by patriotism and pride.

Just four days after the attack, Walker Evey, named program manager for “Project Phoenix,” promised to rebuild the damaged portions of the Pentagon “faster than anyone has a right to expect … and to have people back in the damaged portion of the building, right where the plane hit, by September 11, 2002.”

Preliminary construction reports estimated it would take 3 to 4 years and $750 million to rebuild. By directing the project with teamwork, handshake contracts, creativity, and ingenuity—not to mention emotional 20-hour days 6 to 7 days a week—Evey’s Project Phoenix met its psychological and physical goal. In less than 11 months, and for only $501 million, workers demolished and rebuilt the damaged sections—400,000 square feet of structure, 2 million square feet of offices, 50,000 tons of debris—using 1,000 construction workers from 80 companies. By September 9, 2002, more than 600 military and civilian personnel were sitting at their desks in rebuilt Pentagon offices.

Outside, the blackened gash is long gone. Instead, some 4,000 pieces of limestone—mined from the same Indiana vein that the Pentagon’s original stone came from 65 years ago—have been placed on the building’s façade. For this impressive accomplishment, the Pentagon and Walker Evey were nominated for the Project Management Institute's Project of the Year Award.



Mai/Mai/Getty Images/Time Life Pictures

Sources: MIT Sloan Management Review (March 23, 2011); *Knight-Ridder Tribune Business News* (February 1, 2004); *U.S. News & World Report* (September 16, 2002).

Limitations

- **1.** Project activities have to be clearly defined, independent, and stable in their relationships.
- **2.** Precedence relationships must be specified and networked together.
- **3.** Time estimates tend to be subjective and are subject to fudging by managers who fear the dangers of being overly optimistic or not pessimistic enough.
- **4.** There is the inherent danger of placing too much emphasis on the longest, or critical, path. Near-critical paths need to be monitored closely as well.

Using Microsoft Project to Manage Projects

The approaches discussed so far are effective for managing small projects. However, for large or complex projects, specialized project management software is much preferred. In this section, we provide a brief introduction to the most popular example of such specialized software, Microsoft Project. A time-limited version of Microsoft Project may be requested with this text.

Microsoft Project is extremely useful in drawing project networks, identifying the project schedule, and managing project costs and other resources.

Milwaukee Paper Co. Activities

ACTIVITY	TIME (WKS)	PREDECESSORS
A	2	—
B	3	—

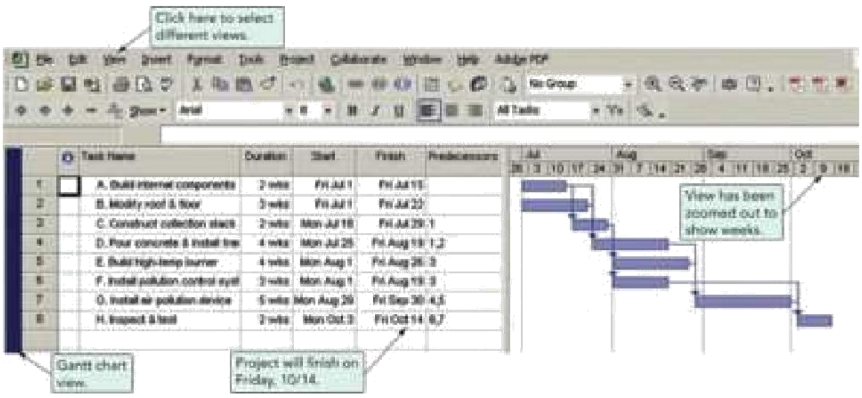
C	2	A
D	4	A, B
E	4	C
F	3	C
G	5	D, E
H	2	F, G

Entering Data

Let us again consider the Milwaukee Paper Manufacturing project. Recall that this project has eight activities (repeated in the margin). The first step is to define the activities and their precedence relationships. To do so, we select **File|New** to open a blank project. We type the project start date (as July 1), then enter all activity information (see Program 3.1). For each activity (or task, as Microsoft Project calls it), we fill in the name and duration. The description of the activity is also placed in the Task Name column in Program 3.1. As we enter activities and durations, the software automatically inserts start and finish dates.

The next step is to define precedence relationships between these activities. To do so, we enter the relevant activity numbers (e.g., 1, 2) in the Predecessors column.

Program 3.1 Gantt Chart in Microsoft Project for Milwaukee Paper Manufacturing



Viewing the Project Schedule

When all links have been defined, the complete project schedule can be viewed as a Gantt chart. We can also select **View|Network Diagram** to view the schedule as a project network (shown in Program

3.2). The critical path is shown in red on the