Chapter 16
Case studies in fast fracture and fatigue failure

Introduction

In this third set of Case Studies we examine three instances in which failure by crack-propagation was, or could have become, a problem. The first is the analysis of an ammonia tank that failed by fast fracture. The second concerns a common problem: the checking, for safety reasons, of cylinders designed to hold gas at high pressure. The last is a fatigue problem: the safe life of a reciprocating engine known to contain a large crack.

CASE STUDY 1: FAST FRACTURE OF AN AMMONIA TANK

Figure 16.1 shows part of a steel tank which came from a road tank vehicle. The tank consisted of a cylindrical shell about 6 m long. A hemispherical cap was welded to each end of the shell with a circumferential weld. The tank was used to transport liquid ammonia. In order to contain the liquid ammonia the pressure had to be equal to the saturation pressure (the pressure at which a mixture of liquid and vapour is in equilibrium). The saturation pressure increases rapidly with temperature: at 20°C the absolute pressure is 8.57 bar; at 50°C it is 20.33 bar. The *gauge* pressure at 50°C is 19.33 bar, or $1.9\,\text{MN}\,\text{m}^{-2}$. Because of this the tank had to function as a pressure vessel. The maximum operating pressure was $2.07\,\text{MN}\,\text{m}^{-2}$ gauge. This allowed the tank to be used safely to 50°C, above the maximum temperature expected in even a hot climate.

While liquid was being unloaded from the tank a fast fracture occurred in one of the circumferential welds and the cap was blown off the end of the shell. In order to decant

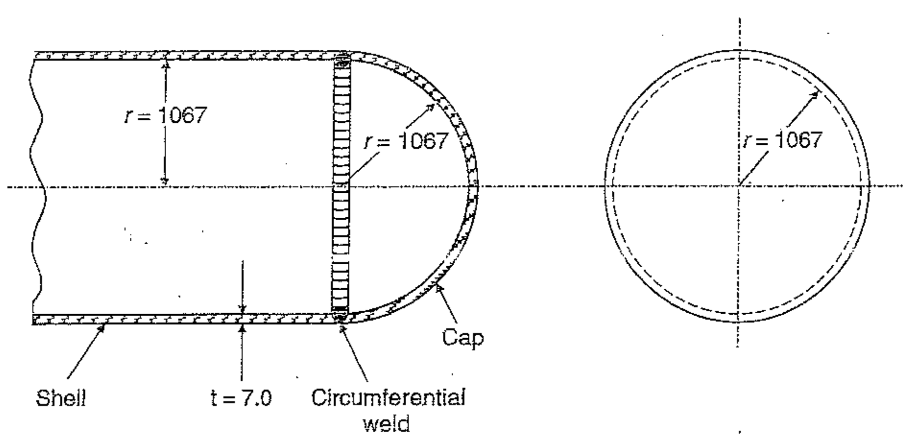

Fig. 16.1. The weld between the shell and the end cap of the pressure vessel. Dimensions in mm.

the liquid the space above the liquid had been pressurised with ammonia gas using a compressor. The normal operating pressure of the compressor was $1.83\,\mathrm{MN\,m^{-2}}$; the maximum pressure (set by a safety valve) was $2.07\,\mathrm{MN\,m^{-2}}$. One can imagine the effect on nearby people of this explosive discharge of a large volume of highly toxic vapour.

Details of the failure

The geometry of the failure is shown in Fig. 16.2. The initial crack, 2.5 mm deep, had formed in the heat-affected zone between the shell and the circumferential weld. The defect went some way around the circumference of the vessel. The cracking was intergranular, and had occurred by a process called stress corrosion cracking (see Chapter 23). The final fast fracture occurred by transgranular cleavage (see Chapter 14). This indicates that the heat-affected zone must have had a very low fracture toughness. In this case study we predict the critical crack size for fast fracture using the fast fracture equation.

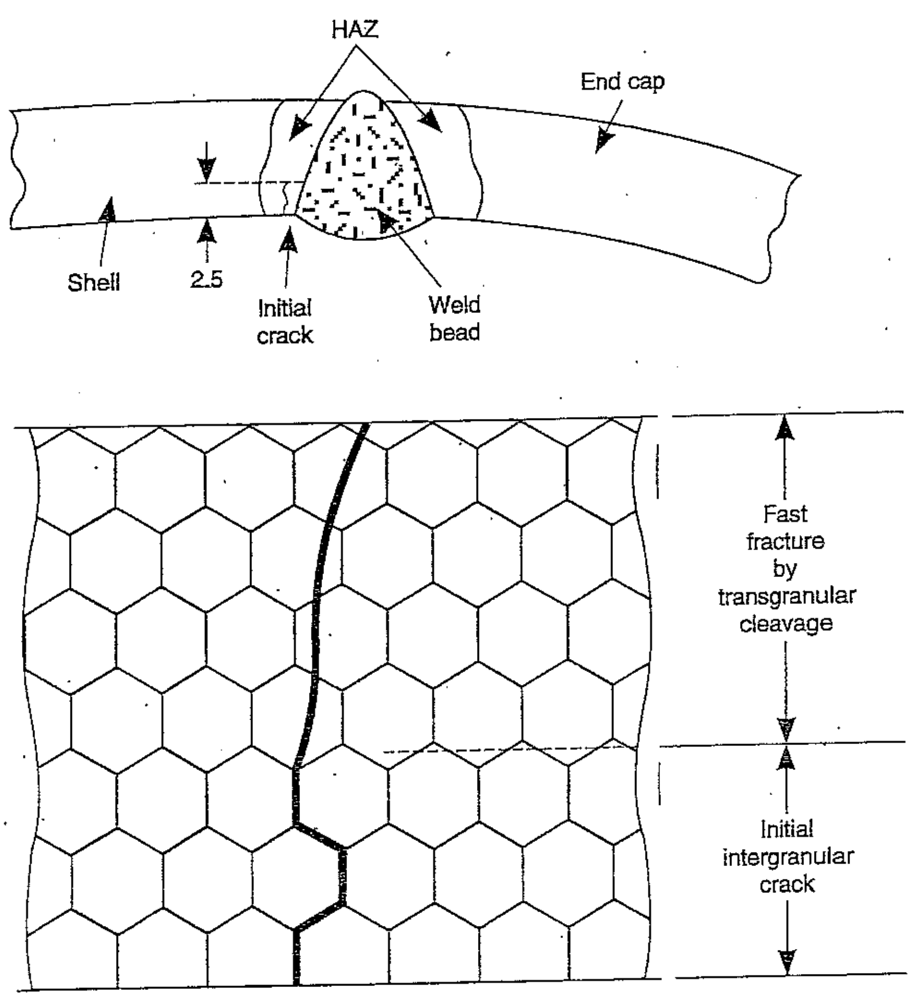

Fig. 16.2. The geometry of the failure. Dimensions in mm.

Material properties

The tank was made from high-strength low-alloy steel with a yeild strength of $712\,\mathrm{MN\,m^{-2}}$ and a fracture toughness of $80\,\mathrm{MN\,m^{-3/2}}$. The heat from the welding process had altered the structure of the steel in the heat-affected zone to give a much greater yield strength ($940\,\mathrm{MN\,m^{-2}}$) but a much lower fracture toughness ($39\,\mathrm{MN\,m^{-3/2}}$).

Calculation of critical stress for fast fracture

The longitudinal stress σ in the wall of a cylindrical pressure vessel containing gas at pressure p is given by

$$\sigma = \frac{pr}{2t},$$

provided that the wall is thin ($t \ll r$). $p = 1.83\,\mathrm{MN\,m^{-2}}$, $r = 1067\,\mathrm{mm}$ and $t = 7\,\mathrm{mm}$, so $\sigma = 140\,\mathrm{MN\,m^{-2}}$. The fast fracture equation is

$$Y\sigma\sqrt{\pi a} = K_c.$$

Because the crack penetrates a long way into the wall of the vessel, it is necessary to take into account the correction factor Y (see Chapter 13). Figure 16.3 shows that $Y = 1.92$ for our crack. The critical stress for fast fracture is given by

$$\sigma = \frac{K_c}{Y\sqrt{\pi a}} = \frac{39}{1.92\sqrt{\pi .0.0025}} = 229\,\mathrm{MN\,m^{-2}}.$$

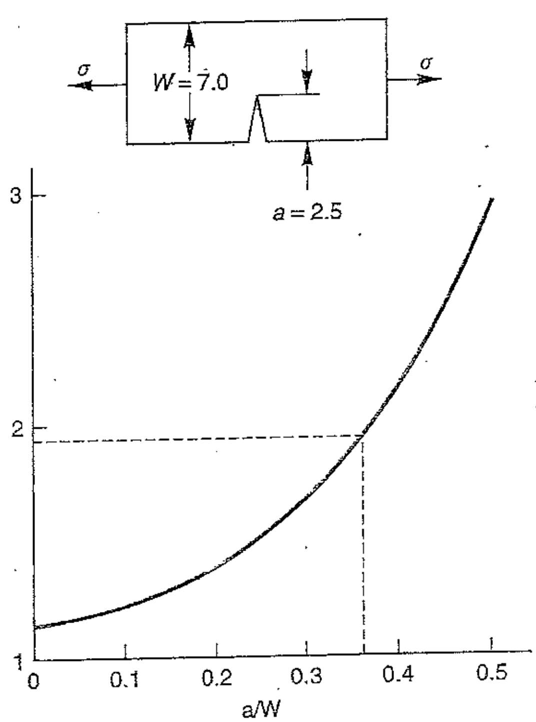

Fig. 16.3. Y value for the crack. Dimensions in mm.

The critical stress is 64% greater than the longitudinal stress. However, the change in section from a cylinder to a sphere produces something akin to a stress concentration; when this is taken into account the failure is accurately predicted.

Conclusions and recommendations

This case study provides a good example of the consequences of having an inadequate fracture toughness. However, even if the heat-affected zone had a high toughness, the crack would have continued to grow through the wall of the tank by stress-corrosion cracking until fast fracture occurred. The critical crack size would have been greater, but failure would have occurred eventually. The only way of avoiding failures of this type is to prevent stress corrosion cracking in the first place.

Case study 2: Compressed air tanks for a supersonic wind tunnel

The supersonic wind tunnels in the Aerodynamic Laboratory at Cambridge University are powered by a bank of twenty large cylindrical pressure vessels. Each time the tunnels are used, the vessels are slowly charged by compressors, and then quickly discharged through a tunnel. How should we go about designing and checking pressure vessels of this type to make sure they are safe?

Criteria for design of safe pressure vessels

First, the pressure vessel must be safe from plastic collapse: that is, the stresses must everywhere be below general yield. Second, it must not fail by fast fracture: if the largest cracks it could contain have length $2a$ (Fig. 16.4), then the stress intensity $K \approx \sigma\sqrt{\pi a}$ must everywhere be less than K_c. Finally, it must not fail by fatigue: the slow growth of a crack to the critical size at which it runs.

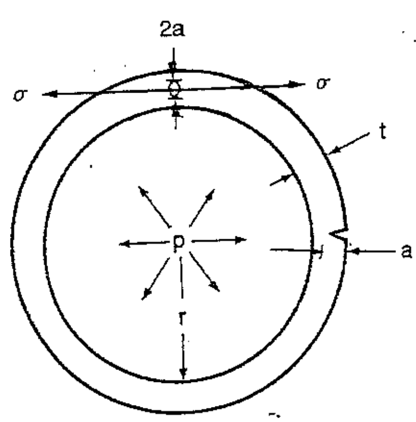

Fig. 16.4. Cracks in the wall of a pressure vessel.

The hoop stress σ in the wall of a cylindrical pressure vessel containing gas at pressure p is given by

$$\sigma = \frac{pr}{t},$$

provided that the wall is thin ($t \ll r$).

For general yielding,

$$\sigma = \sigma_y.$$

For fast fracture,

$$\sigma\sqrt{\pi a} = K_c.$$

Failure by general yield or fast fracture

Figure 16.5 shows the loci of general yielding and fast fracture plotted against crack size. The yield locus is obviously independent of crack size, and is simply given by $\sigma = \sigma_y$. The locus of fast fracture can be written as

$$\sigma = \frac{K_c}{\sqrt{\pi}}\left(\frac{1}{\sqrt{a}}\right),$$

which gives a curved relationship between σ and a. If we pressurise our vessel at point A on the graph, the material will yield *before* fast fracture; this yielding can be detected by strain gauges and disaster avoided. If we pressurise at point B, fast fracture will occur at a stress less than σ_y, without warning and with catastrophic consequences; the point where the two curves cross defines a critical flaw size at which fracture by general yield and by fast fracture coincide. Obviously, if we know that the size of the largest flaw in our vessel is less than this critical value, our vessel will be safe (although we

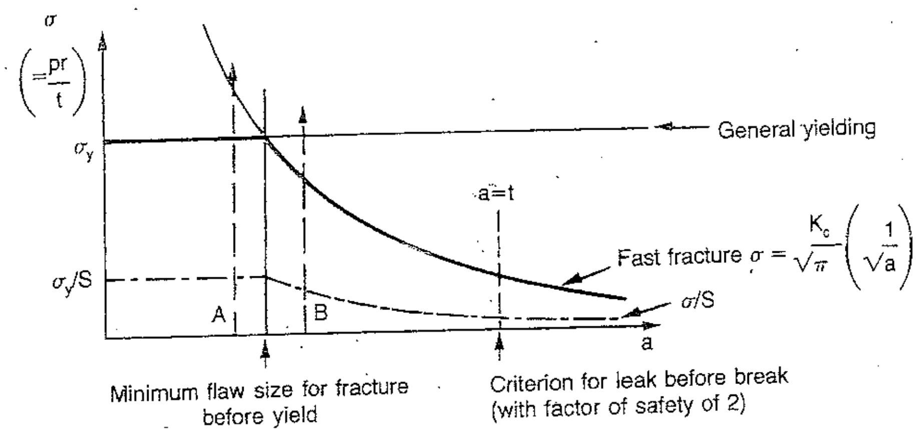

Fig. 16.5. Fracture modes for a cylindrical pressure vessel.

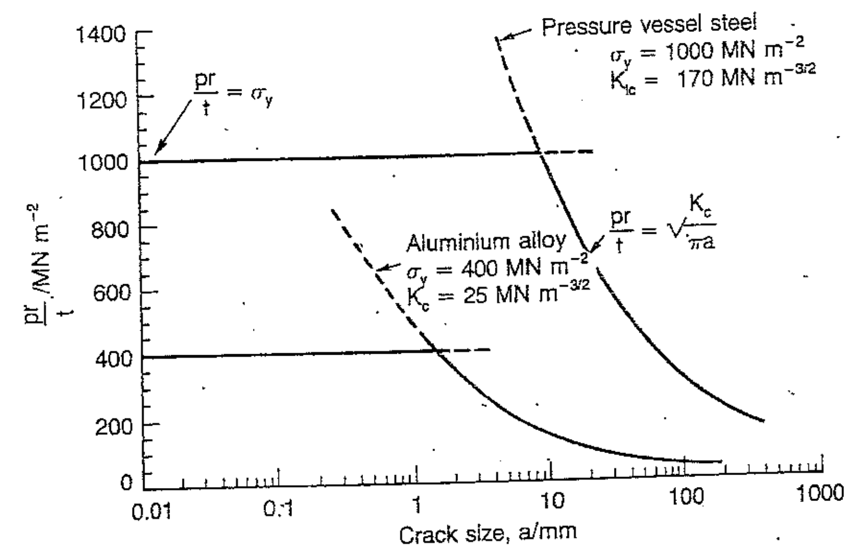

Fig. 16.6. Design against yield and fast fracture for a cylindrical pressure vessel.

should also, of course, build in an appropriate safety factor S as well – as shown by the dash – dot line on Fig. 16.5).

Figure 16.6 shows the general yield and fast fracture loci for a pressure-vessel steel and an aluminium alloy. The critical flaw size in the steel is ≈9 mm; that in the aluminium alloy is ≈1 mm. It is easy to detect flaws of size 9 mm by ultrasonic testing, and pressure-vessel steels can thus be accurately tested non-destructively for safety – vessels with cracks larger than 9 mm would not be passed for service. Flaws of 1 mm size cannot be measured so easily or accurately, and thus aluminium is less safe to use.

Failure by fatigue

In the case of a pressure vessel subjected to *cyclic loading* (as here) cracks can grow by fatigue and a vessel initially passed as safe may subsequently become unsafe due to this crack growth. The probable extent of crack growth can be determined by making fatigue tests on pre-cracked pieces of steel of the same type as that used in the pressure vessel, and the safe vessel lifetime can be estimated by the method illustrated in Case Study 3.

Extra safety: leak before break

It is worrying that a vessel which is safe when it enters service may become unsafe by slow crack growth – either by fatigue or by stress corrosion. If the consequences of catastrophic failure are very serious, then additional safety can be gained by designing the vessel so that it will *leak before it breaks* (like the partly inflated balloon of Chapter 13). Leaks are easy to detect, and a leaking vessel can be taken out of service and repaired. How do we formulate this leak-before-break condition?

If the critical flaw size for fast fracture is less than the wall thickness (t) of the vessel, then fast fracture can occur with no warning. But suppose the critical size ($2a_{crit}$) is greater than t – then gas will leak out through the crack before the crack is big enough to run. To be on the safe side we shall take

$$2a_{\text{crit}} = 2t.$$

The stress is defined by

$$\sigma\sqrt{\pi a_{\text{crit}}} = K_c$$

so that the permissible stress is

$$\sigma = \frac{K_c}{\sqrt{\pi t}}$$

as illustrated on Fig. 16.5.

There is, of course, a penalty to be paid for this extra safety: either the pressure must be lowered, or the section of the pressure vessel increased – often substantially.

Pressure testing

In many applications a pressure vessel may be tested for safety simply by hydraulic testing to a pressure that is higher – typically 1.5 to 2 times higher – than the normal operating pressure. Steam boilers (Fig. 16.7) are tested in this way, usually once a year. If failure does not occur at twice the working pressure, then the normal operating stress is at most one-half that required to produce fast fracture. If failure *does* occur under

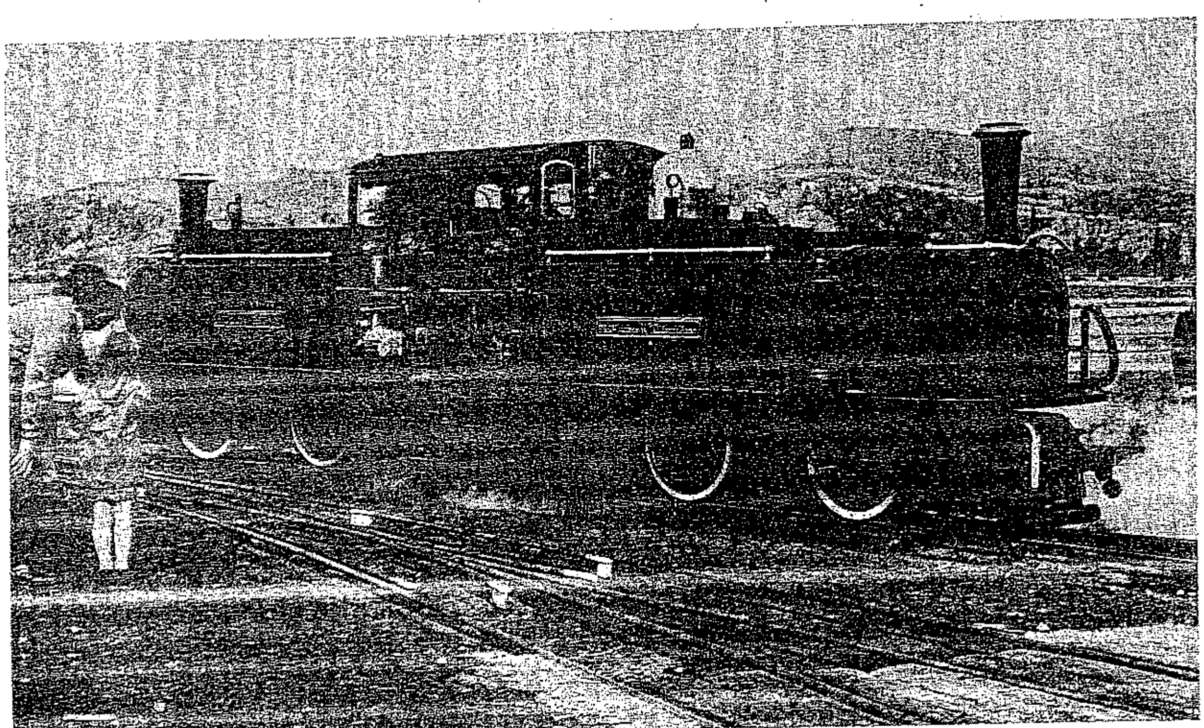

Fig. 16.7. A pressure vessel in action – the boiler of the articulated steam locomotive *Merddin Emrys*, built in 1879 and still hauling passengers on the Festiniog narrow-gauge railway in North Wales.

hydraulic test nobody will get hurt because the stored energy in compressed water is small. Periodic testing is vital because cracks in a steam boiler will grow by fatigue, corrosion, stress corrosion and so on.

Case study 3: the safety of the Stretham engine

The Stretham steam pumping engine (Fig. 16.8) was built in 1831 as part of an extensive project to drain the Fens for agricultural use. In its day it was one of the largest beam engines in the Fens, having a maximum power of 105 horsepower at 15 rpm (it could

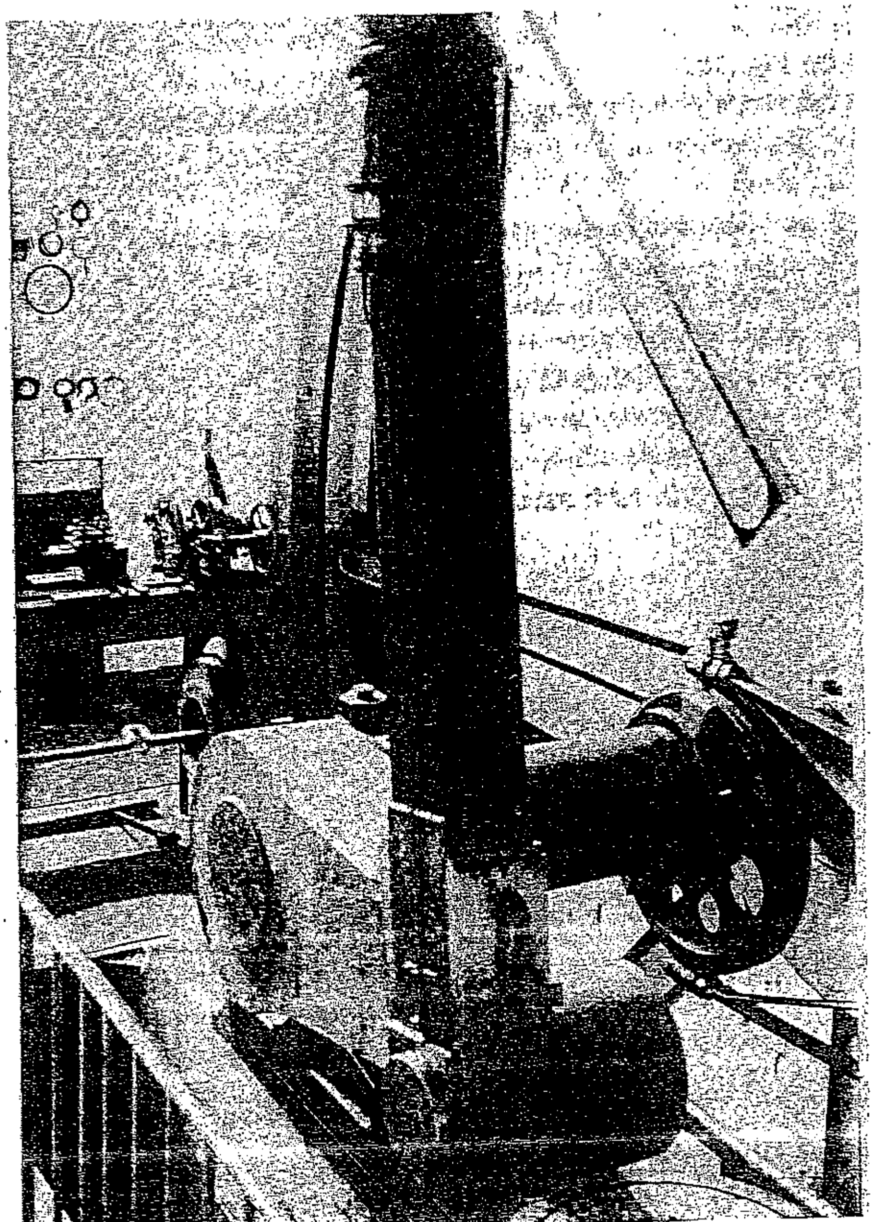

Fig. 16.8. Part of the Stretham steam pumping engine. In the foreground are the crank and the lower end of the connecting rod. Also visible are the flywheel (with separate spokes and rim segments, all pegged together), the eccentric drive to the valve-gear and, in the background, an early treadle-driven lathe for on-the-spot repairs.

lift 30 tons of water per revolution, or 450 tons per minute); it is now the sole surviving steam pump of its type in East Anglia.*

The engine could still be run for demonstration purposes. Suppose that you are called in to assess its safety. We will suppose that a crack 2 cm deep has been found in the connecting rod – a cast-iron rod, 21 feet long, with a section of 0.04 m^2. Will the crack grow under the cyclic loads to which the connecting rod is subjected? And what is the likely life of the structure?

Mechanics

The stress in the crank shaft is calculated approximately from the power and speed as follows. Bear in mind that approximate calculations of this sort may be in error by up to a factor of 2 – but this makes no difference to the conclusions reached below. Referring to Fig. 16.9:

$$\begin{aligned} \text{Power} &= 105 \text{ horsepower} \\ &= 7.8 \times 10^4 \,\text{J s}^{-1}, \\ \text{Speed} &= 15 \,\text{rpm} = 0.25 \,\text{rev s}^{-1}, \\ \text{Stroke} &= 8 \text{ feet} = 2.44 \,\text{m}, \end{aligned}$$

$$\text{Force} \times 2 \times \text{stroke} \times \text{speed} \approx \text{power},$$

$$\therefore \text{Force} \approx \frac{7.8 \times 10^4}{2 \times 2.44 \times 0.25} \approx 6.4 \times 10^4 \,\text{N}.$$

Nominal stress in the connecting rod $= F/A = 6.4 \times 10^4/0.04 = 1.6 \,\text{MN m}^{-2}$ approximately.

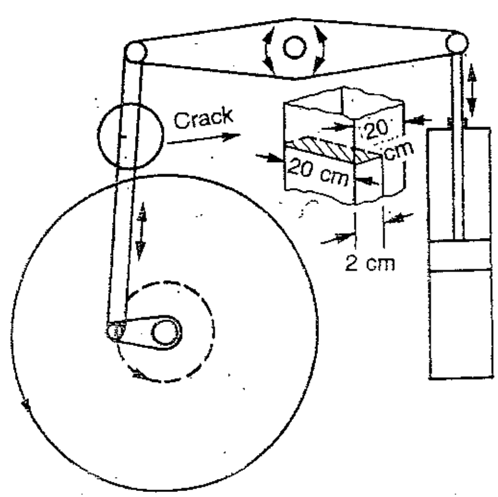

Fig. 16.9. Schematic of the Stretham engine.

*Until a couple of centuries ago much of the eastern part of England which is now called East Anglia was a vast area of desolate marshes, or fens, which stretched from the North Sea as far inland as Cambridge.

Failure by fast fracture

For cast iron, $K_c = 18\,\mathrm{MN\,m^{-3/2}}$.

First, could the rod fail by fast fracture? The stress intensity is:

$$K = \sigma\sqrt{\pi a} = 1.6\sqrt{\pi.0.02}\,\mathrm{MN\,m^{-3/2}} = 0.40\,\mathrm{MN\,m^{-3/2}}.$$

It is so much less than K_c that there is no risk of fast fracture, even at peak load.

Failure by fatigue

The growth of a fatigue crack is described by

$$\frac{da}{dN} = A(\Delta K)^m. \qquad (16.1)$$

For cast iron,

$$A = 4.3 \times 10^{-8}\,\mathrm{m}\,(\mathrm{MN\,m^{-3/2}})^{-4},$$
$$m = 4.$$

We have that

$$\Delta K = \Delta\sigma\sqrt{\pi a}$$

where $\Delta\sigma$ is the range of the tensile stress (Fig. 16.10). Although $\Delta\sigma$ is constant (at constant power and speed), ΔK increases as the crack grows. Substituting in eqn. (16.1) gives

$$\frac{da}{dN} = A\Delta\sigma^4\pi^2a^2$$

and

$$dN = \frac{1}{(A\Delta\sigma^4\pi^2)}\,\frac{da}{a^2}.$$

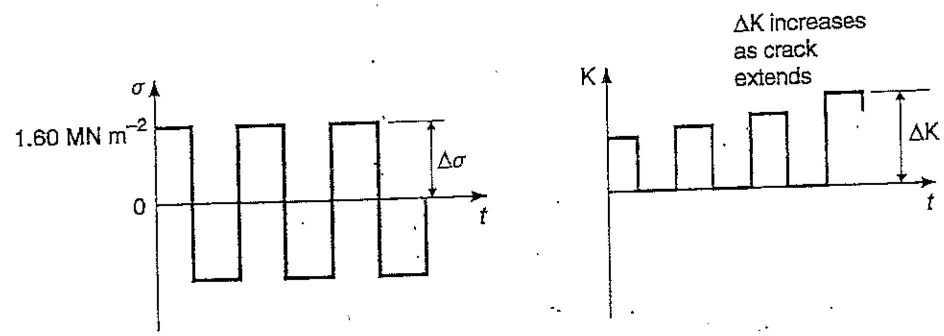

Fig. 16.10. Crack growth by fatigue in the Stretham engine.

Integration gives the number of cycles to grow the crack from a_1 to a_2:

$$N = \frac{1}{(A\Delta\sigma^4 \pi^2)} \left\{\frac{1}{a_1} - \frac{1}{a_2}\right\}$$

for a range of a small enough that the crack geometry does not change appreciably. Let us work out how long it would take our crack to grow from 2 cm to 3 cm. Then

$$N = \frac{1}{4.3 \times 10^{-8} (1.6)^4 \pi^2} \left\{\frac{1}{0.02} - \frac{1}{0.03}\right\}$$

$$= 5.9 \times 10^6 \text{ cycles.}$$

This is sufficient for the engine to run for 8 hours on each of 832 open days for demonstration purposes, i.e. to give 8 hours of demonstration each weekend for 16 years. A crack of 3 cm length is still far too small to go critical, and thus the engine will be perfectly safe after the 5.9×10^6 cycles. Under demonstration the power delivered will be far less than the full 105 horsepower, and because of the $\Delta\sigma^4$ dependence of N, the number of cycles required to make the crack grow to 3 cm might be as much as 30 times the one we have calculated.

The estimation of the total lifetime of the structure is more complex – substantial crack growth will make the crack geometry change significantly; this will have to be allowed for in the calculations by incorporating a correction factor, Y.

Conclusion and recommendation

A simple analysis shows that the engine is likely to be safe for limited demonstration use for a considerable period. After this period, continued use can only be sanctioned by regular inspection of the growing crack, or by using a more sophisticated analysis.

Further reading

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