9-28-15
Monday.

ch. 7
Review of fundamental concepts.
Antidrivatives.

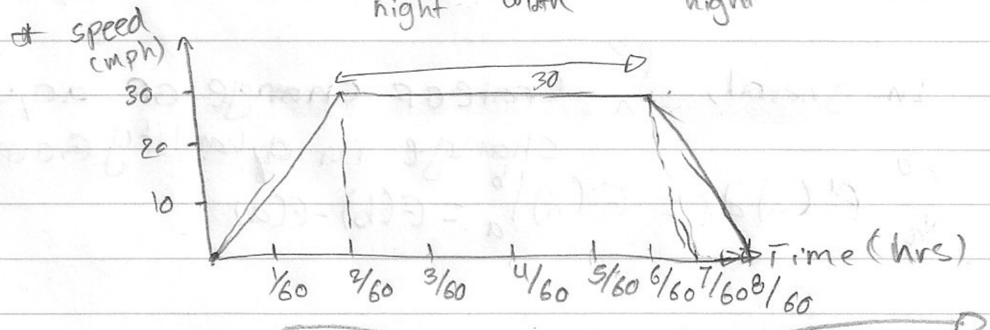
$\int x^2 dx = \frac{1}{3}x^3 + c$

Definite Integrals

$\int_1^3 x^2 dx = \frac{1}{3}x^3 \Big|_1^3 = \frac{1}{3}(3)^3 - \frac{1}{3}(1)^3 = 9 - \frac{1}{3} = 8\frac{2}{3}$

* sum of areas of rectangles =
$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x =$
$\sum_{i=1}^{4} f(x_i)\Delta x$

* area under the curve $y = f(x)$ on $[a,b]$
$\lim_{n \to \infty} \sum_{i=1}^{n} \underbrace{f(x_i)}_{\text{hight}} \underbrace{\Delta x}_{\text{width}} = \int_a^b \underbrace{f(x)}_{\text{hight}} \underbrace{dx}_{\text{width.}}$



$f(t) = \begin{cases} \boxed{900t} & 0 \le t < \frac{2}{60} \\ 30 & \frac{2}{60} \le t < \frac{6}{60} \\ -900t + 120 & \frac{6}{60} \le t \le \frac{8}{60} \end{cases}$

$y = mx + b$

$\frac{\Delta y}{\Delta x} = m = \frac{\text{rise}}{\text{run}} = \frac{30}{2/60}$

$\frac{30}{1/30} = 30 \cdot \frac{30}{1}$

$= 900$

$y - 30 = -900\left(t - \frac{6}{60}\right)$

$y - 30 = -900t + 90$

$y = -900t + 120$

$\frac{\Delta y}{\Delta t} = 900$ mph/hr

$$\int_{0}^{8/60} f(t)\,dt =$$

$$\int_{0}^{1/30} 900t\,dt + \int_{1/30}^{3/30} 30\,dt + \int_{3/30}^{4/30} ((-900t)+120)\,dt =$$

$$450t^2 \Big|_0^{1/30} + 30t \Big|_{1/30}^{3/30} + (-450t^2 + 120t)\Big|_{3/30}^{4/30} =$$

$$450(1/2)^2 + [30(3/30) - 30(1/30)] + -450(4/30)^2 + 120(4/30)$$

$$1/2 + 2 + 1/2 = 3 - (-450(3/30)^2 + 120(3/30)) =$$

sum of areas of rectangles =

$$f(1/60)\cdot\frac{1}{30} + f(3/60)\cdot 1/30 + f(5/60) + f(7/60)\cdot\frac{1}{30}$$

$$15\cdot 1/30 + 30\cdot\frac{1}{30} + 30\cdot\frac{1}{30} + 15\cdot\frac{1}{30}$$

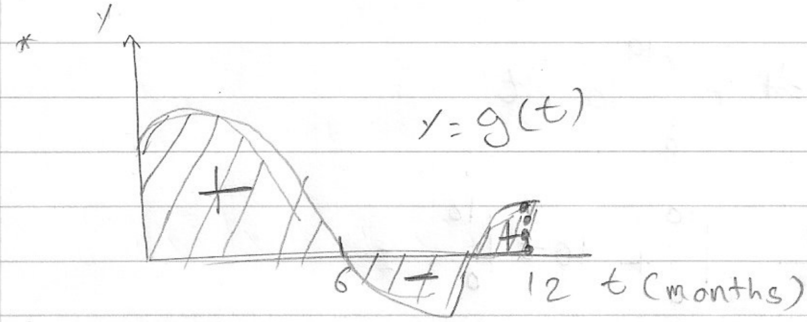
1/2 mile　　1 mile　　1 mile　　1/2 mile

In gneral, $\int_a^b$ (rate of change of a quantity) = change in quantity from a to b

$$\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b) - f(a)$$

2

*



suppose $y = g(t)$ reprsents the rate of change of profit for a company. Then $\int_0^{12} g(t)\,dt$ represents the net profit over

a 12 - month period.

Ex: It is estimate that a new oil field will produce oil ~~that~~ at the rate of $\frac{dR}{dt} = \frac{400t^2}{(t^3+20)^{20}} + 10$ thousand barrels per years.

How much oil can be expected from the field in the next 10 years.

$$\int_0^{10} \left[\frac{400t^2}{(t^3+20)^2} + 10 \right] dt = \int_0^{10} \frac{400t^2}{(t^3+20)^2}\,dt + \int_0^{10} 10\,dt$$

$$\int \frac{400t^2}{(t^3+20)^2} = \int 400t^2(t^3+20)^{-2}\,dt = 400\int (t^3+20)^{-2} \cdot t^2\,dt =$$

$u = t^3 + 20 \qquad du = 3t^2 du$

$$\boxed{\frac{1}{3}du = t^2\,dt}$$

$$400 \int u^{-2} \cdot \frac{1}{3}\,du =$$

$$\frac{400}{3} \int u^{-2}\,du$$

$$\frac{-400}{3} u^{-1} = \frac{-400}{3}$$

$$\frac{-400}{3} \cdot \frac{1}{t^3+20}$$

$$\int_0^{10} \frac{400t^2}{(t^3+20)^2}\,dt + \int_0^{10} 10\,dt$$

$$-\frac{400}{3} \cdot \frac{1}{t^3+20} \Big|_0^{10} + 10t \Big|_0^{10} =$$

$$-\frac{400}{3} \cdot \frac{1}{1020} + \frac{400}{3} \cdot \frac{1}{20} + 100 \approx 106.536$$

$\approx 106{,}536$ barrels are next 10 years.

* Methods of Integration

1. u - substitution

$$\int 4x^2(1-2x^3)^4\,dx = 4\int x^2(1-2x^3)^4\,dx =$$

$u = 1-2x^3 \qquad du = -6x^2\,dx$

$-\frac{1}{6}\,du = x^2\,dx$

$$4\int u^4 \cdot \left(-\tfrac{1}{6}\,du\right) =$$

$$-\frac{2}{3}\int u^4\,du = -\frac{2}{3} \cdot \frac{1}{5}u^5 + C =$$

$$-\frac{2}{15}(1-2x^3)^5 + C$$

check: $-\frac{2}{15} \cdot 5(1-2x^3)^4 \cdot -6x^2$

$$= -\frac{2}{3}(1-2x^3)^4 \cdot (-6x^2)$$

$$4(1-2x^3)^4 \cdot x^2$$

ex: $\displaystyle\int \frac{2-3x^2}{(2x-x^3)^2}\,dx = \int (2-3x^2)(2x-x^3)^{-2}\,dx =$

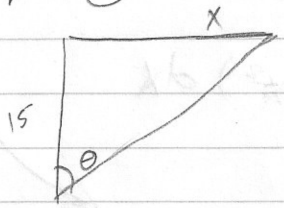
~~sol~~ $u = 2x - x^3 \qquad du = (2-3x^2)\,dx$

$$\int u^{-2}\,du = -u^{-1} + c = \frac{-1}{u} + c$$

$$= \frac{-1}{2x-x^3} + c$$

Test

qu. #6



$\tan\theta = \frac{x}{15}$

$\theta = \tan^{-1}\left(\frac{x}{15}\right)$

$\frac{d\theta}{dt} = \frac{d\theta}{dx} \cdot \frac{dx}{dt}$

$\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{x}{15}\right)^2} \cdot \frac{1}{15}$

After 1.5 s; $\frac{d\theta}{dt} = .75$, $x = 30$

$.75 = \frac{1}{1+\left(\frac{30}{15}\right)^2} \cdot \frac{1}{15} \cdot \frac{dx}{dt}$

$.75 = \frac{1}{5} \cdot \frac{1}{15} \cdot \frac{dx}{dt}$

$75(.75) = \frac{dx}{dt}$

$56.25 \text{ m/s} = \frac{dx}{dt}$

9-30-15

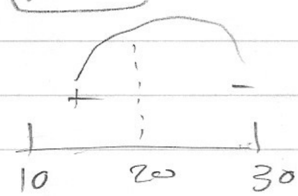
qu. #7

$\frac{ds}{dt} = 1800 \cdot \frac{1}{T+25} - 40 = 0$

$\frac{1800}{T+25} = 40$

$1800 = 40T + 1000$

$\boxed{T = 20}$



max at $T = 20$

u

Ex. $\int 2t\,(2t+1)^2\,dt$

$u = 2t + 1$ $\quad du = 2\,dt$

$2t = u - 1$ $\quad \frac{1}{2}du = dt$

$\int 2t(2t+1)^2\,dt =$

$\int (u-1)\,u^2 \cdot \frac{1}{2}\,du =$

$\frac{1}{2}\int (u-1)\cdot u^2\,du =$

$\frac{1}{2}\int (u^3 - u^2)\,du =$

$\frac{1}{2}\left(\frac{1}{4}u^4 - \frac{1}{3}u^3\right) + c =$

$\frac{1}{8}(2t+1)^4 - \frac{1}{6}(2t+1)^3 + c$

Ex. $\int x(x - 3x^4)\,dx$

$= \int (x^2 - 3x^5)\,dx$

$= \frac{1}{3}x^3 - \frac{1}{2}x^6 + c$

ex: $\int 3x\sqrt[3]{1+2x^2}$

$3\int x(1+2x)^{1/3}\,dx$

$u = 1+2x$ $\quad du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$3\int (1+2x^2)^{1/3} \cdot x\,dx =$

$3\int u^{1/3} \cdot \frac{1}{4}\,du =$

$\frac{3}{4}\int u^{1/3}\,du = \frac{3}{4}\cdot\frac{3}{4}u^{4/3} + c$

$= 9/16\,(1+2x^2)^{3/4} + c$

ex: $\int 4x^2(1-2x^3)^4 \, dx$

الحل النهائي: $\boxed{-\frac{2}{15}(1-2x^3)^5 + c}$

Antidervatives of Transcendental Function.

1. $\int \cos x \, dx = \sin x + c$

2. $\int \sin x \, dx = -\cos x + c$

3. $\int e^x \, dx = e^x + c$

4. $\int \frac{1}{x} \, dx = \ln|x| + c$

5. $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c$

6. $\int \frac{-1}{\sqrt{1-x^2}} \, dx = \cos^{-1} x + c$

7. $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$

لازم نستخدمهم في الاختبار #2

* 5a. $\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

* 7a. $\int \frac{1}{a^2+x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

* $\int \cos(kx) \, dx = \frac{1}{k} \sin(kx) + c$

* $\int \sin(kx) \, dx = -\frac{1}{k} \cos(kx) + c$

ex: $\int \frac{dx}{1-3x}$

$u = 1-3x \qquad du = -3dx$

$-\frac{1}{3}du = dx$

$\int \frac{1}{1-3x}dx = \int \frac{1}{u} \cdot \frac{-1}{3}du = \frac{-1}{3}\int \frac{1}{u}du = -\frac{1}{3}\ln|u| + c$

$\boxed{= \frac{-1}{3}\ln|1-3x| + c}$

Ex: $\int 8xe^{x^2}du = 8\int x \cdot e^{x^2}dx$

$u = x^2 \qquad du = 2x\,dx$

$\frac{1}{2} \cdot 8\int 2x\,e^{x^2}dx$

$4\int e^u du = 4e^u + c$

$\boxed{= 4e^{x^2} + c}$

ex: $\int \cos(5x)\,dx$

$u = 5x \qquad du = 5\,dx$

$\frac{1}{5}du = dx$

$\int \cos(5x)\,dx = \int \cos u \cdot \frac{1}{5}du$

$= \frac{1}{5}\int \cos u\,du$

$= \frac{1}{5}\sin u + c$

$= \frac{1}{5}\sin(5x) + c$

Ex. $\int \frac{2}{\sqrt{9-4x^2}}\,dx$

$= 2\int \frac{1}{\sqrt{3^3-(2x)^2}}\,dx = \int \frac{2}{\sqrt{3^2-(2x)^2}}\,dx$

$u = 2x \qquad du = 2\,dx$

$\int \frac{1}{\sqrt{3^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{3}\right)$

$= \sin^{-1}\left(\frac{2x}{3}\right) + c$

Ex: $\int \cos^3 x \cdot \sin x\,dx$

$u = \cos x \qquad du = -\sin x\,dx$

$\qquad\qquad -du = \sin x\,dx$

$\int (\cos)^3 \cdot \sin x\,dx = \int u^3(-du)$

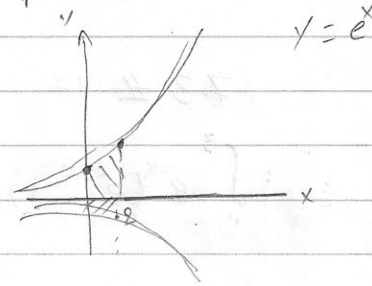
$= -\frac{1}{4}u^4 + c$

$\boxed{= -\frac{1}{4}(\cos)^4 + c}$

H.W 7.1 3, 5, 11, 13, 17, 19, 21, 31, 33, 41

6

7.1 # 33 10-5-2015



$$V = \int_a^b \pi [f(x)]^2 dx$$

$$\int_0^2 \pi e^{2x} dx$$

$$= \int_0^2 \pi e^{2x} dx$$

$$= \pi \int_0^2 e^{2x} dx$$

$$= \pi (\tfrac{1}{2} e^{2x} \Big|_0^2$$

$$= \frac{\pi}{2} e^4 - \frac{\pi}{2} e^0$$

$$\boxed{= \frac{\pi}{2} e^4 - \pi/2}$$

7.2: 3, 11, 21, 27

7.3: 5, 11, 17, 27

7.4: 4, 13, 18, 28

7.6: 7, 9, 33, 25

Gp: 2

7.2 27#

$$\int \frac{x+2}{x^2}\,dx$$

$u = x+2 \quad du = dx$

$$= \int \frac{u}{(u-2)^2}$$

$$= \int \frac{u}{u^2-4u+4}$$

$$= \int \frac{2u-4}{2(u^2-4u+4)} + \frac{2}{u^2-4u+4}$$

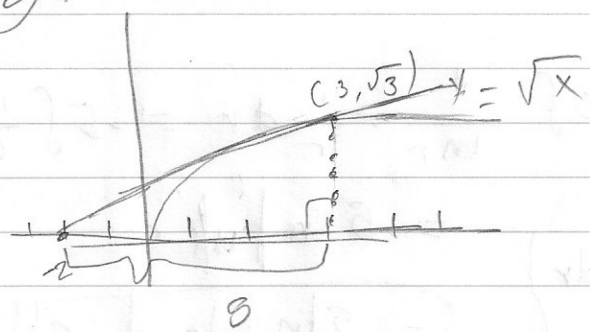
7.3 # 17

$$\int_1^3 3e^{2x}(e^{-2x}-1)\,dx$$

7

(2) # from the h.w.

10-7-15



$\tan\theta = \frac{\sqrt{3}}{6}$

θ

7.2# 3

$\int \frac{dx}{1+4x} = \int \frac{1}{u}\cdot\frac{du}{4} = \frac{1}{4}\int \frac{1}{u}du = \frac{1}{4}\ln|u| + c$

$u = 1+4x$

$du = 4\,dx$

$dx = \frac{du}{4}$

$= \frac{1}{4}\ln|1+4x| + c$

7.2# 27 $\int \frac{x+2}{x^2}\,dx$

$\int \frac{x}{x^2} + \frac{2}{x^2}\,dx = \int \frac{1}{x}dx + \int 2x^{-2}dx$

$\ln|x| + (-2x^{-1}) + c$

$\ln|x| - \frac{2}{x} + c$

7.2# 11

$\int \frac{\cos x}{1+\sin x}\,dx$

$u = 1+\sin x$

$du = \cos x\,dx$

$= \int \frac{1}{u}\,du = \ln|u| + c$

$= \ln|1+\sin x| + c$

$\int_0^{\pi/2} \frac{\cos x}{1+\sin}\,dx$

$\ln|1+\sin x|\Big|_0^{\pi/2} = \ln|2| - \ln|1| = \boxed{\ln 2}$

7.2 #2

$.5\int \frac{1}{r \ln r} dr = .5\int \frac{1}{\ln r} \cdot \frac{1}{r} dr = .5\int \frac{1}{u} du$

$u = r \cdot \ln r$
$du = (\ln r + r \cdot \frac{1}{r}) dr$
$du = (\ln r + 1)$

$= .5 \ln |u| + c$

$\boxed{= .5 \ln |\ln r| + c}$

7.3 #5

$\int 4e^{2x+5} dx$

$u = 2x + 5$
$du = 2dx$
$dx = \frac{1}{2} du$

$\frac{4}{2} \int e^{2x+5} = 2e^{2x+5} + c$

$4\int e^{u} (\frac{1}{2} du) = \frac{4}{2} \int e^{u} du = 2e^{u} + c = \boxed{2e^{2x+5} + c}$

7.3 #17 $\int 3e^{2x}(e^{-2x} - 1) dx = 3\int e^{2x}(e^{-2x} - 1) dx$

$u = e^{2x}$ $du = e^{2x} \cdot 2dx$
$u = 2x$ $du = 2dx$

$= 3\int (1 - e^{2x}) dx = \ln |u|$

$= 3\int 1 dx - \int e^{2x} dx$

$= 3[x - \frac{1}{2} e^{2x} + c]$

$\int_1^3 3e^{2x}(e^{-2x} - 1) dx = 3x - \frac{3}{2} e^{2x} \Big|_1^3 =$

$\boxed{(9 - \frac{3}{2} e^{6}) - (3 - \frac{3}{2} e^{2})}$

8

قوانين حق الاختبار

$\sin^2 x + \cos^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

7.4 # 4

$\int 4\sin(2-x)\,dx$

$= 4\int \sin(2-x)\,dx$

$u = (2-x) \quad du = -1\,dx \qquad -du = dx$

$4\int \sin(u)\,(du) = +\cos(u) + C$

$\boxed{4\cos(2-x) + C}$

7.4 # 8

$\int \frac{\sin 2x}{2\cos^2 x}\,dx$

$= \int \frac{2\sin x\cos x}{2\cos^2 x}\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{1}{u}\,du$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= -\ln|u| + c$

$\boxed{= -\ln|\cos x| + C}$

7.6 # 7

$\int \frac{8x}{\sqrt{1-16x^4}}\,dx$

$u = 4x^2$

$du = 8x\,dx$

$\int \frac{du}{\sqrt{1-u^2}}$

$= \sin^{-1}(u) + C$

$= \sin^{-1}(4x^2) + c$

* 7.6 # 33 may be the same in exame.

$\int \frac{2}{\sqrt{4-9x^2}}\,dx$

$2\int \frac{1}{\sqrt{4-9x^2}}\,dx =$

$a = 2$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}\,du = dx$

$2\int \frac{1}{\sqrt{2^2-u^2}}\left(\frac{1}{3}du\right)$

$\frac{2}{3}\int \frac{1}{\sqrt{2^2-u^2}}\,du$

$= \frac{2}{3}\sin^{-1}\left(\frac{u}{2}\right) + c$

$= \frac{2}{3}\sin^{-1}\left(\frac{3x}{2}\right) + c$

31# from 7.6

$$\int \frac{2x}{\sqrt{4-9x^2}}\,dx$$

$u = 4-9x^2$

$du \; -18x\,dx$

$-\frac{1}{18}\,du = x\,dx$

$$2\int \frac{1}{\sqrt{4-9x^2}}\,(x\,dx)$$

$$= 2\int \frac{1}{\sqrt{u}}\cdot\left(\frac{-1}{18}\,du\right)$$

$$= -\frac{1}{9}\int u^{-1/2}\,du$$

$$= -\frac{1}{9}\cdot\frac{1}{\frac{1}{2}}\,u^{1/2}$$

$$= -\frac{1}{9}\cdot 2u^{1/2} + c$$

$$= -\frac{2}{9}(4-9x^2)^{1/2} + c$$

$$\int \frac{2}{(4-9x)\,dx} = 2\int \frac{1}{(4-9x)\,dx}$$

$u = 4-9x \quad du = -9\,dx$

$dx = -\frac{1}{9}\,du$

$$2\int \frac{1}{4-9x}\,dx$$

$$= 2\int \frac{1}{u}\left(-\frac{1}{9}\,du\right)$$

$$= -\frac{2}{9}\int \frac{1}{u}\,du$$

$$= -\frac{2}{9}\ln|u| + c$$

7.2: 5, 7, 13, 15, 25, 25

7.3: 3, 7, 9 15, 19, 21, 25

7.4 3, 12, 17, 30, 35

7.6 3, 9, 13, 15, 17, 23, 27