Formulate and solve the following optimum design problem.

EcoCarnifex, a large chemical company, produces titanium dioxide from ilmenite. Titanium dioxide, which is bright white, is used as a pigment for paint and is also added to many food products to turn an otherwise yucky color into something more appealing (such as the filler for Twinkies). The reduction process to turn ilmenite ore into pure titanium dioxide uses big vats of hot chlorine gas. EcoCarnifex has two reduction facilities which each produce two grades of titanium dioxide (High grade and Low grade). Finished products utilizing the titanium dioxide are produced at EcoCarnifex's three fabrication plants. There are two finished products made at the plants: paint pigments and food additives. Due to new requirements from the Environmental Protection Agency, EcoCarnifex must minimize the amount of ore processed in its reduction plants while maintaining its production and demand constraints.

Production and demand constraints:

1. The total tonnage of ilmenite ore processed by both reduction facilities must equal the total tonnage processed into the two grades of titanium dioxide for shipment to the fabrication plants.
2. The total tonnage of ilmenite ore processed by each reduction facility cannot exceed its capacity.
3. The total tonnage of titanium dioxide manufactured into products at each fabrication plant must equal the tonnage of titanium dioxide shipped to it by the reduction facilities.
4. The total tonnage of titanium dioxide manufactured into products at each fabrication plant cannot exceed its available capacity.
5. The total tonnage of each product must equal its demand.

Nomenclature:

T(r,s) = Tonnage yield of titanium dioxide stock *s* (High or Low grade) from 1 ton of ilmenite ore processed at reduction plant *r*.

Y(s,f,p) = Total yield from 1 ton of titanium dioxide stock *s* shipped to fabricating plant *f* and manufactured into product *p*.

C(r) = Ilmenite ore processing capacity in tonnage at reduction plant *r*.

K(f) = Capacity of the fabrication plant *f* in tonnage for all stocks.

D(p) = Tonnage demand requirement for product *p*.

r = 1, 2 for reduction facilities 1 and 2.

s = 1 (High Grade), 2 (Low grade) titanium dioxide

f = 1, 2, 3 for fabrication plants 1, 2 and 3.

p = 1 (Pigments), 2 (Food Additives)

Constants for the problem

T(1,1) = 0.39	c(1) = 1,200,000	k(1) = 190,000	D(1) = 330,000
T(1,2) = 0.46	c(2) = 1,000,000	k(2) = 240,000	D(2) = 125,000
T(2,1) = 0.44		k(3) = 290,000	
T(2,2) = 0.48			
Y(1,1,1) = 0.79	Y(1,1,2)=0.84	Y(2,1,1) = 0.68	Y(2,1,2)=0.81
Y(1,2,1) = 0.73	Y(1,2,2)=0.85	Y(2,2,1) = 0.67	Y(2,2,2)=0.77
Y(1,3,1) = 0.74	Y(1,3,2)=0.72	Y(2,3,1) = 0.62	Y(2,3,2)=0.78