Stat 10x **Homework 3**

Q1) Suppose 20 patients arrive at a hospital on any given day. Assume that 10% of all the patients of this hospital are emergency cases.

a. Find the probability that exactly 5 of the 20 patients are emergency cases.

$\frac{20!}{5!\,15!} = \frac{20\times 19\times\cdots}{(5\times 4\times\cdots 1)(15\times\cdots)} = \frac{\cancel{20}\times 19\times \overset{3}{\cancel{18}}\times 17\times 16}{\cancel{20}\times\cancel{6}}$

$X \sim \text{binomial}(n=20,\ p=0.1)$

$P(X=5) = \binom{20}{5}(0.1)^5(0.9)^{15} = \frac{20!}{5!\,15!}(0.1)^5(0.9)^{15} \approx 0.0319$

In R: dbinom(5,20,0.1)

b. Find the probability that none of the 20 patients are emergency cases.

$P(X=0) = \binom{20}{0}(0.1)^0(0.9)^{20} = (0.9)^{20} = 0.1215767$

In R: dbinom(0, 20, 0.1)

c. Find the probability that all 20 patients are emergency cases.

$P(X=20) = \binom{20}{20}(0.1)^{20}(0.9)^0 = (0.1)^{20}$

d. Find the probability that at least 4 of the 20 patients are emergency cases.

$P(X\geq 4) = 1 - P(X\leq 3) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$= \ldots$

e. Find the probability that more than 4 of the 20 patients are emergency cases.

$P(X>4) = 1 - P(X\leq 4) = \ldots$

In R: pbinom(4, 20, 0.1, lower.tail= FALSE)

f. Find the probability that at most 3 of the 20 patients are emergency cases.

$P(X\leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \ldots$

g. Find the probability that less than 3 of the 20 patients are emergency cases.

$P(X<3) = P(X=0) + P(X=1) + P(X=2) = \ldots$

h. On average how many patients (from the 20) are expected to be emergency case?

$E(X) = np = 20\times 0.1 = 2$

Q2) The probability of receiving a one-pair poker hand is 0.42. What is the probability that a player will receive at least one one-pair hand in 10 games? it is binomial :

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0) = 1 - \binom{10}{0}(0.42)^0(0.58)^{10}$$
$$= 1 - 0.58^{10} = 0.995692$$

Q3) Compute the expected value and standard deviation of the maximum number when two dice are rolled. You will have to find the distribution of the maximum (X) first (i.e. complete the table below).

for Ex: $\max\{1,2\} = 2 \quad \max\{5,4\} = 5$

X	P(X)
1	1/36
2	3/36
3	5/36
4	7/36
5	9/36
6	11/36

$\sum P(x) = \frac{36}{36} = 1$

for $(X=1) \rightarrow P(X=1) = P(\{1,1\}) = 1/36$

for $(X=2) \rightarrow P(X=2) = P(\{1,2\} \text{ or } \{2,1\} \text{ or } \{2,2\}) = \frac{3}{36} = \frac{1}{12}$

for $(X=3) \rightarrow P(X=3) = \frac{5}{36}$

max 4 : (1,4) (4,1) (2,4) (4,2) (3,4) (4,3) (4,4)

$E(X) = \sum_x x\,P(x) = 1(\frac{1}{36}) + 2(\frac{3}{36}) + \dots + 6(\frac{11}{36}) = 4.472$

$E(X^2) = \sum_x x^2 P(x) = 1(\frac{1}{36}) + 2^2(\frac{3}{36}) + \dots + 6^2(\frac{11}{36}) = 21.97$

$Var(X) = 21.97 - (4.472)^2 = 1.97145$

$Sd(X) = \sqrt{1.97145}$

Q4) A random variable X has E(X) = 10 and SD(X) = 5. Find $E(2X^2 + X)$.

Note: $Var(X) = E(X^2) - E^2(X)$ and $E(aX + bY) = aE(X) + bE(Y)$

$E(2x^2 + x) = E(2x^2) + E(x) = 2E(x^2) + E(x)$

$Var(X) = E(x^2) - E^2(x) = 25$ (since $SD(X) = 5$)

$E(x^2) - 10^2 = 25 \Rightarrow E(x^2) = 125 \Rightarrow E(2x^2 + x) = 2 \times 125 + 10 = 260$

Q5) On a population of consumers, 60% prefer a certain brand of ice cream. If consumers are randomly selected,

a. what is the probability that exactly 3 people have to be interviewed to encounter the first consumer who prefers this brand of ice cream?

Geometric distribution : $P(X=3) = (1-0.6)^{3-1}\,0.6^1 = 0.4^2(0.6)$

b. what is the probability that at least 3 people have to be interviewed to encounter the first consumer who prefers this brand of ice cream?

$q = 0.4$
$P = 0.6$

$$P(X \geq 3) = P(X > 2) = 1 - P(X \leq 2)$$
$$= 1 - (1 - (0.4)^2) = (0.4)^2 = 0.16$$

at least 3 people have to b interviewed is the same as first two do not prefer this Ice Cream.

Q6) Suppose that 30% of the applicants for a certain industrial job have advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool. Find the probability that the first applicant having advanced training in computer programming is found on the fifth interview.

Geometric dist.

$$P(X=5) = (0.7)^4 (0.3)^1$$

Q7) At a survey poll before the elections candidate A receives the support of 650 voters in a sample of 1200 voters.

a. Construct a 95% confidence interval for the population proportion p that supports candidate A.

$$\hat{P} = \frac{650}{1200} \qquad \hat{p} \pm \sqrt{\frac{\hat{P}(1-\hat{P})}{1200}} \times \underset{Z_{0.025}}{1.96}$$

b. Find the sample size needed so that the margin of error will be _0:01 with confidence level 95%.

$$Z_{\alpha/2} = 1.96$$

$$n = \frac{Z^2_{\alpha/2}\, p(1-p)}{E^2} = \frac{(1.96)^2 \frac{1}{4}}{(0.01)^2} = \frac{1.96}{4(0.01)^2} = 4900$$

Q8) The UCLA housing office wants to estimate the mean monthly rent for studios around the campus. A random sample of size n = 36 studios is taken from the area around UCLA. The sample mean is found to be $\overline{X}$ = $900. Assume that the population standard deviation is σ = $150.

a. Construct a 95% confidence interval for the mean monthly rent of studios in the area around UCLA.

$n > 30$

$$900 \pm 1.96 \frac{150}{\sqrt{36}}$$

b. Construct a 99% confidence interval for the mean monthly rent of studios in the area around UCLA.

$$Z_{0.005} = 2.575 \qquad 900 \pm 2.575 \frac{150}{\sqrt{36}}$$

c. What sample size is needed so that the length of the interval is $60 with 95% confidence?

$$E = \frac{60}{2} = 30 \qquad n = \frac{(1.96)^2 (150)^2}{(30)^2} =$$

Q9) A chemist has prepared a product designed to kill 60% of a particular type of insect. What sample size should be used if he desires to be 95% confident that he is within 0.02 of the true fraction of insects killed?

$E = 0.02$ $\qquad n = \frac{(1.96)^2 \times 0.6 \times (0.4)}{(0.02)^2}$

Q10) In question one, Suppose 140 patients arrive at a hospital on any given week. Assume that 10% of all the patients of this hospital are emergency cases.

a. Find the probability that at least 20 patients are emergency cases. $0.1 \times 140 = 14$

$P(X \geq 20) = P\left(Z \geq \frac{19.5 - 14}{\sqrt{0.1(0.9)140}}\right)$

or $P\left(\hat{P} \geq \frac{20}{140}\right)$

b. Find the probability that the number of patients that are emergency cases are less than 10.

$P(X < 10) = P\left(Z < \frac{10 - 14}{\sqrt{140(0.1)(0.9)}}\right)$

c. Find the probability that between 10 and 30 patients are emergency cases.

Zscore for 10 $\Rightarrow \frac{10 - 14}{\sqrt{140(0.1)(0.9)}}$ $\qquad$ for 30 $\frac{30 - 14}{\sqrt{140(0.1)(0.9)}}$

d. Find numbers of patients with emergency cases in one week that are extremely unusual to observe. First you need to define what unusual means to you.

Let say unusual happens less than 5% of the time. Can be low or very high.

$Z_{0.025} = 1.96$ $\qquad 1.96 = \frac{X - 14}{\sqrt{140(0.1)(0.9)}} \Rightarrow X > 14 + 1.96\sqrt{140(0.1)(0.9)}$

and $X < 14 -$

Q11) A manufacturer claims that 20% of the public preferred her product. A sample of 100 persons is taken to check her claim. It is found that 8 of these 100 persons preferred her product.

$H_0: P = 0.2$

a. Find the p-value of the test (use a two-tailed test).

$H_a: P \neq 0.2$ $\qquad 2 * P\left(Z < \frac{.08 - .2}{\sqrt{\frac{0.2(1-.2)}{100}}}\right) = 2 * 0.0013 = 0.0026$

b. Using the 0:05 level of significance test her claim.

Since $0.0026 < 0.05$

We can reject H_0.

Q12) An experimenter has prepared a drug dosage level that he claims will induce sleep for at least 80% of those people suffering from insomnia. After examining the dosage, we feel that his claims regarding the effectiveness of the dosage are inflated. In an attempt to disprove his claim, we administer his prescribed dosage to 10 insomniacs, and we observe X, the number having sleep induced by the drug dose. We wish to test the hypothesis H0 : p = 0.8 against the alternative Ha : p < 0.8. Assume the rejection region X ≤ 5 is used.

a. Find the type I error α. $\alpha = P(X \leqslant 5 \mid P = 0.8) = 0.0327935$

$X \sim bin(10, 0.8)$

b. Find the type II error β if the true p = 0.6.

$\beta = P(X > 5 \mid P = 0.6) = P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

c. Find the type II error β if the true p = 0.4.

$\beta = P(X > 5 \mid P = 0.4)$

Q13) For a certain candidate's political poll n = 15 voters are sampled. Assume that this sample is taken from an infinite population of voters. We wish to test H0: p = 0.5 against the alternative Ha: p < 0.5. The test statistic is X, which is the number of voters among the 15 sampled favoring this candidate.

a. Calculate the probability of a type I error if we select the rejection region to be RR ={ x ≤ 2}.

$\alpha = P(X \leqslant 2 \mid P = 0.5) = P(X=0) + P(X=1) + P(X=2)$
$= 0.00369$

b. Is our test good in protecting us from concluding that this candidate is a winner if, in fact, he will lose? Suppose that he really will win 30% of the vote (p = 0.30). What is the probability of a type II error that the sample will erroneously lead us to conclude that H0 is true?

α is very small therefore the test will protect us from a large type one error.
P (Type II error) might be large.
$P(X > 2 \mid P = 0.3) = 1 - P(X \leqslant 2 \mid P = 0.3) = 0.873$
so the test wont protect us from type II error.

Q14)

The researchers believe the free range eggs have more contaminants because they are found in the environments where free range hens roam. Studies have found the chemicals in feedstuffs, soil, plants, worms and insects. In one study researchers in Taiwan collected 60 free range eggs from farms in southern Taiwan and compared them with 120 eggs from caged hens that were selected randomly throughout the country. They found that about 10% of free range eggs and 7% of the eggs from caged hens exceeded the certain limit of contaminants. Set a hypothesis test for this problem and find the p_value for the test. Using significant level equal to 0.05 what is the result of the test?

$\hat{P}_1 = 0.1 \quad \hat{P}_2 = 0.07$

$H_0: P_1 = P_2$

$H_a: P_1 > P_2$

$P(\hat{P}_1 - \hat{P}_2 \geq 0.1 - 0.07) = P(Z \geq Z_{score})$

Q15) A hypothesis test results in a rejection of the null hypothesis at the α = .05 level. If the α-level is changed to .01, then, using the same data:(select the correct answer)

P-value < 0.05 we don't know P-value < 0.01

maybe P-value = 0.02

A. We will still reject the null hypothesis.
B. We will fail to reject the null hypothesis.
C. We cannot determine whether or not we will reject the null hypothesis.

C is correct.

Q16) Assume the weight of Angus steer is normally distributed with mean 2000 pounds and standard deviation 100 pounds.

a. Find the 95th percentile of the above distribution.

$P(Z \cancel{\geq} 1.645) = 0.95$

$X = 2000 + 1.645(100) = 2164.5$

b. Find probability that a randomly selected Angus Steer to weight more than 2050 lbs.

$P(X \geq 2050) \quad P^* = P\left(Z \geq \frac{2050-2000}{100}\right) = P(Z > 0.5)$

c. ~~If we select 5 Angus Steer randomly find probability that majority of them weight more than 2050.~~ $P(Y \geq 3)$ when $Y \sim bin(n=5, P = \text{Part (b)})$

d. We want to select Angus Steer until we get one with weight more than 2050. What is the probability that at most we select 5 to find our 2050 plus Angus Steer.

$(1-P^*)^4 P^*$

e. What is the probability that average of 10 randomly selected Steers weight more than 2050?

$P(\bar{X} > 2050) \qquad \bar{X} \sim N\left(2000, \frac{100}{\sqrt{10}}\right)$

$= P\left(\cancel{Z} > \frac{2050-2000}{100}\right)$