joint costs. Cost accountants use accounting records to track costs of individual products. Fixed and variable resources used by each product are recorded. These product costs, calculated by the cost accountants, typically are used to estimate short-run and long-run average and marginal costs.

Despite these estimation problems, cost curves play an important role in managerial decision making. Nonetheless, it is important that managers maintain a healthy skepticism when using these estimates. For instance, in making major decisions, it generally is instructive for managers to examine whether a proposed decision is still attractive with reasonable variation in the estimated parameters of the cost function—that is, to conduct *sensitivity analysis.*

Summary

A *production function* is a descriptive relation that connects inputs with outputs. It specifies the *maximum* possible output that can be produced for given amounts of inputs. *Returns to scale* refers to the relation between output and a proportional variation in *all inputs* taken together. A production function displays *constant returns to scale* when a 1 percent change in all inputs results in a 1 percent change in output. With *increasing returns to scale,* a 1 percent change in all inputs results in a greater than 1 percent change in output. Finally, with *decreasing returns to scale,* a 1 percent change in all inputs results in a less than 1 percent change in output. *Returns to a factor* refers to the relation between output and the variation in only one input, *holding other inputs fixed.* Returns to a factor can be expressed as total, marginal, or average quantities. The *law of diminishing returns* states that the marginal product of a variable factor will eventually decline as the use of the input is increased.

Most production functions allow some substitution of inputs. An *isoquant* displays all combinations of inputs that produce the same quantity of output. The optimal input mix to produce any given output depends on the costs of the inputs. An *isocost line* displays all combinations of inputs that cost the same. *Cost minimization* for a given output occurs where the isoquant is tangent to the isocost line. Changes in input prices change the slope of the isocost line and the point of tangency. When the price of an input increases, the firm will reduce its use of this input and increase its use of other inputs (*substitution effect*).

Cost curves can be derived from the isoquant/isocost analysis. The *total cost curve* depicts the relation between total costs and output. *Marginal cost* is the change in total cost associated with a one-unit change in output. *Average cost* is total cost divided by total output. Average cost falls when marginal cost is below average cost; average cost rises when marginal cost is above average cost. Average and marginal costs are equal when average cost is at a minimum. There is a direct link between the production function and cost curves. Holding input prices constant, the slopes of cost curves are determined by the underlying production technology.

Opportunity cost is the value of a resource in its next best alternative use. Current market prices more closely reflect the opportunity costs of inputs than *historical costs.* The *relevant costs* for managerial decision making are the opportunity costs.

Cost curves can be depicted for both the *short run* and the *long run.* The short run is the operating period during which at least one input (typically capital) is fixed in supply. During this period, *fixed costs* can be incurred even if the firm produces no output. In the long run, there are no fixed costs—all inputs and costs are *variable.* Short-run cost curves are sometimes called *operating curves* because they are used in making near-term production and pricing decisions. Fixed costs are irrelevant for these decisions. Long-run cost curves are referred to as *planning curves,* since they play a key role in longer-run planning decisions relating to plant size and equipment acquisitions.

The *minimum efficient scale* is defined as that plant size at which long-run average cost is first minimized. The minimum efficient scale affects both the optimal plant size and the level of potential competition. Industries where the average cost declines over a broad range of output are characterized as having *economies of scale.*

A *learning curve* displays the relation between average cost and the cumulative volume of production. For some firms, the long-run average cost for producing a given level of output declines as the firm gains experience from producing the output (that is, there are significant learning effects).

Economies of scope exist when the cost of producing a joint set of products in one firm is less than the cost of producing the products separately across independent firms. Economies of scope help explain why firms often produce multiple products.

The profit-maximizing output level occurs at the point where *marginal revenue equals marginal cost.* At this point, the marginal benefits of increasing output are offset exactly by the marginal costs.

The *marginal revenue product* of input i (MRP_i) equals the marginal product of the input times marginal revenue. Profit-maximizing firms use an input up to the point where the MRP of the input equals the input price. At this point, the marginal benefit of employing more of the input is offset exactly by its marginal cost.

Managers often use estimates of cost curves in decision making. A common statistical tool for estimating these curves is regression analysis. One common problem in statistical estimation is the difficulty of obtaining good information on the opportunity costs of resources. Another problem with estimating cost curves involves allocating fixed costs in a multiproduct plant. Cost accountants track the costs and estimate product costs.

Appendix

The Factor-Balance Equation[20]

This appendix derives the factor-balance equation—Equation (5.9) in the text:

$$MP_i/P_i = MP_j/P_j \tag{5.15}$$

This condition must hold if the firm is producing output in a manner that minimizes costs (assuming an interior solution).

Recall that at the cost-minimizing method of production, the isoquant curve and isocost line are tangent. Thus, they must have equal slopes. The factor-balance equation is found by setting the slope of the isoquant equal to the slope of the isocost line and rearranging the expression. In the text, we showed that the slope of the isocost line is $-P_j/P_i$. We now derive the slope of an isoquant.

Slope of an Isoquant

The production function in the two-input case takes the following general form:

$$Q = f(x_i, x_j) \tag{5.16}$$

To find the slope of an isoquant, we totally differentiate Equation (5.16). We set this differential equal to zero, since quantity does not change along an isoquant:

$$dQ = [\partial Q/\partial x_i dx_i] + [\partial Q/\partial x_j dx_j] = 0 \tag{5.17}$$