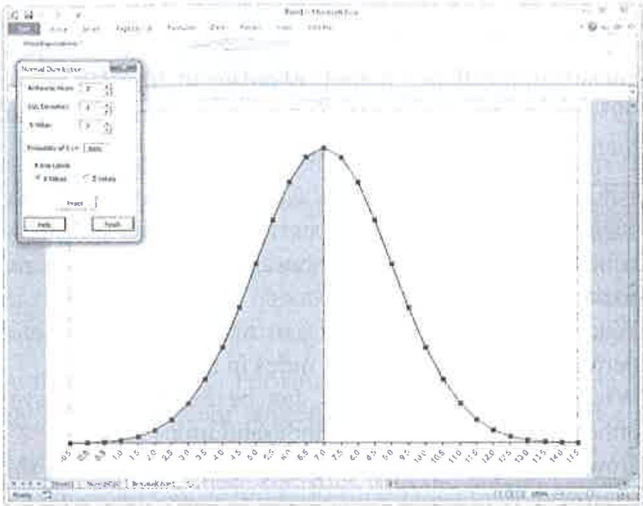
VISUAL EXPLORATIONS Exploring the Normal Distribution

Use the Visual Explorations Normal Distribution procedure to see the effects of changes in the mean and standard deviation on the area under a normal distribution curve. Open the **Visual Explorations add-in workbook** (see Appendix Section D.4). Select **Add-ins ➔ VisualExplorations ➔ Normal Distribution.**

The add-in displays a normal curve for the OurCampus! download example and a floating control panel (see illustration at right). Use the control panel spinner buttons to change the values for the mean, standard deviation, and *X* value and note the effects of these changes on the probability of *X* < value and the corresponding shaded area under the curve (see illustration at right). If you prefer to see the normal curve labeled with *Z* values, click **Z Values.**

Click the **Reset** button to reset the control panel values or click **Help** for additional information about the problem. Click **Finish** when you are done exploring.



Problems for Section 6.2

LEARNING THE BASICS

6.1 Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

a. *Z* is less than 1.57?
b. *Z* is greater than 1.84?
c. *Z* is between 1.57 and 1.84?
d. *Z* is less than 1.57 or greater than 1.84?

6.2 Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

a. *Z* is between −1.57 and 1.84?
b. *Z* is less than −1.57 or greater than 1.84?
c. What is the value of *Z* if only 2.5% of all possible *Z* values are larger?
d. Between what two values of *Z* (symmetrically distributed around the mean) will 68.26% of all possible *Z* values be contained?

6.3 Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

a. *Z* is less than 1.08?
b. *Z* is greater than −0.21?
c. *Z* is less than −0.21 or greater than the mean?
d. *Z* is less than −0.21 or greater than 1.08?

6.4 Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), determine the following probabilities:

a. $P(Z > 1.08)$
b. $P(Z < -0.21)$
c. $P(-1.96 < Z < -0.21)$
d. What is the value of *Z* if only 15.87% of all possible *Z* values are larger?

6.5 Given a normal distribution with $\mu = 100$ and $\sigma = 10$, what is the probability that

a. $X > 75$?
b. $X < 70$?
c. $X < 80$ or $X > 110$?
d. Between what two *X* values (symmetrically distributed around the mean) are 80% of the values?

6.6 Given a normal distribution with $\mu = 50$ and $\sigma = 4$, what is the probability that

a. $X > 43$?
b. $X < 42$?
c. 5% of the values are less than what *X* value?
d. Between what two *X* values (symmetrically distributed around the mean) are 60% of the values?

APPLYING THE CONCEPTS

6.7 In 2008, the per capita consumption of coffee in the United States was reported to be 4.2 kg, or 9.24 pounds (data extracted from **en.wikipedia.org/wiki/List_of_countries_by_coffee_consumption_per_capita**). Assume that the per capita consumption of coffee in the United States is approximately distributed as a normal random variable, with a mean of 9.24 pounds and a standard deviation of 3 pounds.

a. What is the probability that someone in the United States consumed more than 10 pounds of coffee in 2008?
b. What is the probability that someone in the United States consumed between 3 and 5 pounds of coffee in 2008?
c. What is the probability that someone in the United States consumed less than 5 pounds of coffee in 2008?
d. 99% of the people in the United States consumed less than how many pounds of coffee?

SELF Test **6.8** Toby's Trucking Company determined that the distance traveled per truck per year is normally distributed, with a mean of 50 thousand miles and a standard deviation of 12 thousand miles.
a. What proportion of trucks can be expected to travel between 34 and 50 thousand miles in a year?
b. What percentage of trucks can be expected to travel either below 30 or above 60 thousand miles in a year?
c. How many miles will be traveled by at least 80% of the trucks?
d. What are your answers to (a) through (c) if the standard deviation is 10 thousand miles?

6.9 Consumers spend an average of $21 per week in cash without being aware of where it goes (data extracted from "Snapshots: A Hole in Our Pockets," *USA Today*, January 18, 2010, p. 1A). Assume that the amount of cash spent without being aware of where it goes is normally distributed and that the standard deviation is $5.
a. What is the probability that a randomly selected person will spend more than $25?
b. What is the probability that a randomly selected person will spend between $10 and $20?
c. Between what two values will the middle 95% of the amounts of cash spent fall?

6.10 A set of final examination grades in an introductory statistics course is normally distributed, with a mean of 73 and a standard deviation of 8.
a. What is the probability that a student scored below 91 on this exam?
b. What is the probability that a student scored between 65 and 89?
c. The probability is 5% that a student taking the test scores higher than what grade?
d. If the professor grades on a curve (i.e., gives A's to the top 10% of the class, regardless of the score), are you better off with a grade of 81 on this exam or a grade of 68 on a different exam, where the mean is 62 and the standard deviation is 3? Show your answer statistically and explain.

6.11 A statistical analysis of 1,000 long-distance telephone calls made from the headquarters of the Bricks and Clicks Computer Corporation indicates that the length of these calls is normally distributed, with μ = 240 seconds and σ = 40 seconds.
a. What is the probability that a call lasted less than 180 seconds?
b. What is the probability that a call lasted between 180 and 300 seconds?
c. What is the probability that a call lasted between 110 and 180 seconds?
d. 1% of all calls will last less than how many seconds?

6.12 In 2008, the per capita consumption of coffee in Sweden was reported to be 8.2 kg, or 18.04 pounds (data extracted from **en.wikipedia.org/wiki/List_of_countries_by_coffee_consumption_per_capita**). Assume that the per capita consumption of coffee in Sweden is approximately distributed as a normal random variable, with a mean of 18.04 pounds and a standard deviation of 5 pounds.
a. What is the probability that someone in Sweden consumed more than 10 pounds of coffee in 2008?
b. What is the probability that someone in Sweden consumed between 3 and 5 pounds of coffee in 2008?
c. What is the probability that someone in Sweden consumed less than 5 pounds of coffee in 2008?
d. 99% of the people in Sweden consumed less than how many pounds of coffee?

6.13 Many manufacturing problems involve the matching of machine parts, such as shafts that fit into a valve hole. A particular design requires a shaft with a diameter of 22.000 mm, but shafts with diameters between 21.990 mm and 22.010 mm are acceptable. Suppose that the manufacturing process yields shafts with diameters normally distributed, with a mean of 22.002 mm and a standard deviation of 0.005 mm. For this process, what is
a. the proportion of shafts with a diameter between 21.99 mm and 22.00 mm?
b. the probability that a shaft is acceptable?
c. the diameter that will be exceeded by only 2% of the shafts?
d. What would be your answers in (a) through (c) if the standard deviation of the shaft diameters were 0.004 mm?

6.3 Evaluating Normality

As discussed in Section 6.2, many continuous variables used in business closely follow a normal distribution. To determine whether a set of data can be approximated by the normal distribution, you either compare the characteristics of the data with the theoretical properties of the normal distribution or construct a normal probability plot.

SOLUTION The download time is uniformly distributed from 4.5 to 9.5 seconds. The area between 9 and 9.5 seconds is equal to 0.5 seconds, and the total area in the distribution is 9.5 – 4.5 = 5 seconds. Therefore, the probability of a download time between 9 and 9.5 seconds is the portion of the area greater than 9, which is equal to 0.5/5.0 = 0.10. Because 9.5 is the maximum value in this distribution, the probability of a download time above 9 seconds is 0.10. In comparison, if the download time is normally distributed with a mean of 7 seconds and a standard deviation of 2 seconds (see Example 6.1 on page 253), the probability of a download time above 9 seconds is 0.1587.

Problems for Section 6.4

LEARNING THE BASICS

6.23 Suppose you select one value from a uniform distribution with $a = 0$ and $b = 10$. What is the probability that the value will be

a. between 5 and 7?
b. between 2 and 3?
c. What is the mean?
d. What is the standard deviation?

APPLYING THE CONCEPTS

SELF Test **6.24** The time between arrivals of customers at a bank during the noon-to-1 P.M. hour has a uniform distribution between 0 to 120 seconds. What is the probability that the time between the arrival of two customers will be

a. less than 20 seconds?
b. between 10 and 30 seconds?
c. more than 35 seconds?
d. What are the mean and standard deviation of the time between arrivals?

6.25 A study of the time spent shopping in a supermarket for a market basket of 20 specific items showed an approximately uniform distribution between 20 minutes and 40 minutes. What is the probability that the shopping time will be

a. between 25 and 30 minutes?
b. less than 35 minutes?
c. What are the mean and standard deviation of the shopping time?

6.26 How long does it take you to download a game for your iPod? According to Apple's technical support site, **www.apple.com/support/itunes**, downloading an iPod game using a broadband connection should take 3 to 6 minutes. Assume that the download times are uniformly distributed between 3 and 6 minutes. If you download a game, what is the probability that the download time will be

a. less than 3.3 minutes?
b. less than 4 minutes?
c. between 4 and 5 minutes?
d. What are the mean and standard deviation of the download times?

6.27 The scheduled commuting time on the Long Island Railroad from Glen Cove to New York City is 65 minutes. Suppose that the actual commuting time is uniformly distributed between 64 and 74 minutes. What is the probability that the commuting time will be

a. less than 70 minutes?
b. between 65 and 70 minutes?
c. greater than 65 minutes?
d. What are the mean and standard deviation of the commuting time?

6.5 The Exponential Distribution

The **exponential distribution** is a continuous distribution that is right-skewed and ranges from zero to positive infinity (see Panel C of Figure 6.1 on page 248). The exponential distribution is widely used in waiting-line (i.e., queuing) theory to model the length of time between arrivals in processes such as customers arriving at a bank's ATM, patients entering a hospital emergency room, and hits on a website.

The exponential distribution is defined by a single parameter, λ, the mean number of arrivals per unit of time. The probability density function for the length of time between arrivals is given by Equation (6.7).