MATH 331
Final Exam
Spring 2015

Name_____

Part I: No calculators on this part. Also, don't worry about simplifying, just show me you know your calculus!

1. (4 pts each) Determine the exact value for each of the following limits:
   a. $\lim_{x\to 2}\left(\frac{x^2-2x}{x^2-x-2}\right)$

   b. $\lim_{x\to\infty}\left(\frac{2x^2}{3x^2-5x+3}\right)$

2. (4 pts each) Determine derivatives (with respect to x) for the following:
   a. $f(x)=4x^3+e^{x^2}$

   b. $h(x)=x^4\ln(x)$

   c. $g(x)=\left(5\sqrt{x}+3x\right)^4$

d. Determine $\frac{dy}{dx}$ for $y^3 + 3x^2 = xy$

e. Determine the partial derivative with respect to y $\left(f_y\right)$ for $f(x, y) = \ln\left(x^2 y + 3x + 4y^4\right)$

3. (4 pts each) Integrate the following:

a. $\int\left(x^5 + e^{3x} + 1\right)dx$

b. $\int x\left(4x^2 + 2\right)^3 dx$

c. $\int_0^3 \int_2^4 \left(x^2 y\right) dy dx$

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Part II: Calculators allowed on this part, but still make sure you show your work!

4. (6 pts) For the function $f(x) = x^2 - 4x$:
   a. Determine the average rate of change from $x = 1$ to $x = 4$

   b. Determine the instantaneous rate of change at $x = 1$.

5. (6 pts) Determine the equation of the tangent line at $x = 1$ for $f(x) = \dfrac{2\sqrt{x}}{4x^2 - 5}$.

6. (6 pts) For $f(x) = x^3 - 9x^2$, use calculus to determine the location of:
   a. Any maximum or minimum points (tell which are max and which are min)

   b. Any inflection points

7. (6 pts) A store owner currently sells an item for \$50 and can sell 100 items per week on average. However, market research says that for each \$3 decrease in price, the owner will be able to sell 25 more items per week. What price should the owner set so that the weekly maximum profit is realized?

8. (6 pts) Let $f(p)$ be a function which describes the relationship between the price charged p for membership to a health club, and the number of people f who buy memberships to the health club.
   a. Explain the meaning of the equation $f(100) = 50$ as it applies to the description above.

   b. Explain the meaning of the equation $f'(100) = -10$ as it applies to the description above.

9. (6 pts) Under certain conditions, the number of cancer cells $N(t)$ at time t increases at a rate of $N'(t) = 50e^{kt}$. Suppose that at 5 days, the number of cells is growing at a rate of 150 cells per day. Determine a number of cells after 12 days if there were 100 cells initially.

10. (12 points) The sales from a soda pop vending machine are given by $S(b,v)=.133bv^2$, where b is the number of businesses on the same floor of the building as the machine and v is the number of different kinds of soda in the machine (v for variety).
    a. Calculate $S(4,8)$ and explain it's meaning in the context of this problem.

    b. Calculate $S_v(4,8)$ and explain it's meaning in the context of this problem.

11. (6 pts) Determine the location of any maxima, minima, or saddle points for $f(x,y)=y^3-3y^2-x^2-9y+10x-60$ (tell what each one is)

12. (6 pts) Determine the area enclosed by the curves $f(x)=x^2-2x$ and $g(x)=4-x^2$