G13 f_x =G11*(9/12)

	A	B	C	D	E	F	G	H
1	**Concrete Retaining Wall**							
2								
3		X (ft)	Y_U (ft)	Y_L (ft)	A_U (ft^2)	A_L (ft^2)		
4		0.00	3.00	0.50	2.92	0.60		
5		1.00	2.84	0.70	2.60	0.92		
6		2.00	2.35	1.13	2.10	1.33		
7		3.00	1.85	1.53	2.65	2.70		
8		4.63	1.40	1.78	2.98	3.42		
9		6.88	1.25	1.26				
10					A_U (ft^2)	A_L (ft^2)	A_{FORM} (ft^2)	
11				Totals:	13.25	8.96	22.21	
12								
13						Volume (ft^3):	16.7	
14								

Figure 16.20
Determining the required volume of concrete.

16.4.3 Integration by Using Smooth Curves (Simpson's Rule)

Simpson's rule requires

1. An odd number of data points
2. Evenly spaced data (i.e., uniform Δx)

The procedure for integration using Simpson's rule is quite similar to that used in the preceding cases, but you must be careful to distinguish between the *data interval* between adjacent x values and the *integration step* which, for Simpson's rule, incorporates two data intervals. It is because of this that Simpson's rule is restricted to an even number of intervals (an odd number of data points).

Procedure for Simpson's Rule

1. Enter or import the data.
2. Compute h, the distance between any two adjacent x values (because the x values are uniformly spaced).
3. Enter the area formula for one integration step (not data interval).
4. Copy the formula over all integration regions.
5. Sum the areas of all integration regions.

Step 1. Enter or Import the Data. No change from previous cases.

Step 2. Compute h, the Distance between Adjacent x Values. The value of h can be computed by using any two adjacent x values, because Simpson's rule requires uniformly spaced x values. The first two x values in cells A5 and A6 have been used in Figure 16.21, and h is computed with the formula `=B7-B6` and stored in cell C3.

Figure 16.21
Calculating the h value for Simpson's Rule integration.

C3 *fx* =B7-B6

	A	B	C	D	E	F
1	**Simpson's Rule Integration**					
2						
3		**h:**	0.1571			
4						
5		**X**	**Y**	**Int. Step**		
6		0.0000	1.0000	1		
7		0.1571	0.9877	1		
8		0.3142	0.9511	2		
9		0.4712	0.8910	2		
10		0.6283	0.8090	3		
11		0.7854	0.7071	3		
12		0.9425	0.5878	4		
13		1.0996	0.4540	4		
14		1.2566	0.3090	5		
15		1.4137	0.1564	5		
16		1.5708	0.0000			
17						

Note: h is the width of the interval between any two data points, not the entire integration step.

$$h = \Delta x. \tag{16.7}$$

Step 3. Enter the Formula for the Area of One Integration Step. The formula for Simpson's rule is

$$\boxed{A_s = \frac{h}{3}(y_L + 4y_M + y_R),} \tag{16.8}$$

where A_s is the area of an integration step for Simpson's rule, which covers two data intervals. The subscripts L, M, and R stand for left, middle, and right sides of the integration step, respectively. (The integration steps are numbered on the worksheet in Figure 16.21 as a reminder.)
The first area formula is entered into cell E6 as `=($C$3/3)*(C6+4*C7+C8)`, as shown in Figure 16.22.

Note: The dollar signs on the reference to cell $C$3 indicate that the reference to h should not be changed when the formula in cell E6 is copied.

Step 4. Copy the Area Formula to Each Integration Step. The formula in cell E6 must be copied to each integration step (i.e., every other row). A quick way to do this is to:

1. Copy the formula to the Windows clipboard [Ctrl-c].
2. Select the destination cells by first clicking on cell E8 and holding down the [Ctrl] key while clicking on cells E10, E12, and E14. (Excel uses the [Ctrl] key to select multiple, noncontiguous regions.)
3. Paste the formula into the cells from the clipboard [Ctrl-v].

The result is shown in Figure 16.23.

Figure 16.22
Calculating the area of the first integration region.

E6 f_x =($C$3/3)*(C6+4*C7+C8)

	A	B	C	D	E	F
1	**Simpson's Rule Integration**					
2						
3		**h:**	0.1571			
4						
5		**X**	**Y**	**Int. Step**	**Area**	
6		0.0000	1.0000	1	0.3090	
7		0.1571	0.9877	1		
8		0.3142	0.9511	2		
9		0.4712	0.8910	2		
10		0.6283	0.8090	3		
11		0.7854	0.7071	3		
12		0.9425	0.5878	4		
13		1.0996	0.4540	4		
14		1.2566	0.3090	5		
15		1.4137	0.1564	5		
16		1.5708	0.0000			
17						

Figure 16.23
Calculating the area of each integration region.

E14 f_x =($C$3/3)*(C14+4*C15+C16)

	A	B	C	D	E	F
1	**Simpson's Rule Integration**					
2						
3		**h:**	0.1571			
4						
5		**X**	**Y**	**Int. Step**	**Area**	
6		0.0000	1.0000	1	0.3090	
7		0.1571	0.9877	1		
8		0.3142	0.9511	2	0.2788	
9		0.4712	0.8910	2		
10		0.6283	0.8090	3	0.2212	
11		0.7854	0.7071	3		
12		0.9425	0.5878	4	0.1420	
13		1.0996	0.4540	4		
14		1.2566	0.3090	5	0.0489	
15		1.4137	0.1564	5		
16		1.5708	0.0000			
17						

Step 5. Sum the Areas for Each Region. You can use `=SUM(E6:E14)` to compute this sum because empty cells are ignored during the summing process. This is illustrated in Figure 16.24.

The resulting estimate for the area under the curve is 1.0 (actually 1.000003), which is virtually equivalent to the true value, 1:

Figure 16.24
The area under the cosine curve, determined with Simpson's Rule integration.

E18 fx =SUM(E6:E14)

	A	B	C	D	E	F
1	Simpson's Rule Integration					
2						
3		h:	0.1571			
4						
5		X	Y	Int. Step	Area	
6		0.0000	1.0000	1	0.3090	
7		0.1571	0.9877	1		
8		0.3142	0.9511	2	0.2788	
9		0.4712	0.8910	2		
10		0.6283	0.8090	3	0.2212	
11		0.7854	0.7071	3		
12		0.9425	0.5878	4	0.1420	
13		1.0996	0.4540	4		
14		1.2566	0.3090	5	0.0489	
15		1.4137	0.1564	5		
16		1.5708	0.0000			
17						
18				Total Area:	1.0000	
19						

Simpson's Rule: Summary

Simpson's rule is pretty straightforward to apply and does a good job of fitting most functions, but there are two important limitations on Simpson's rule. You must have the following:

1. An odd number of data points (even number of intervals)
2. Uniformly spaced x values

The trapezoidal rule is slightly less accurate, but it is very simple to use and does not have the restrictions just listed. Sometimes Simpson's method is used with an even number of data points (with uniformly spaced x values) by using the Simpson's rule formula for all intervals except the last one. A trapezoid is used to compute the area of the final interval (half-Simpson integration step).

16.5 USING REGRESSION EQUATIONS FOR INTEGRATION

Another general approach to integrating a data set is to:

1. Use *regression* to fit a curve to the data, then
2. Analytically integrate the *regression equation*.

Excel provides a couple of alternatives for linear regression: *trendlines* with limited regression forms and regression by using the Data Analysis package.

For the cosine data used here ($y = \cos(x)$, $0 \le x \le \pi/2$), the obvious fitting function is a cosine. To give the regression analysis something to work with, significant random noise has been added to the data values shown in Figure 16.25.