3) (25 pts) A continuous stir tank reactor (CSTR) is used to produce a product **P** from chemicals **A** and **B**. The reaction is **A** + **B** → **P**. **A** is in excess and the rate of decomposition of **B** is given by:

$$r_b = \frac{k_1 x_2}{(1+k_2 x_2)^2}$$

where k_1 and k_2 are constants and x_2 is the product concentration.
The equations describing the system are given by:

$$\frac{dx_1}{dt} = u_1 + u_2 - 0.2\sqrt{x_1}$$

$$\frac{dx_2}{dt} = (C_{b1} - x_2)\frac{u_1}{x_1} + (C_{b2} - x_2)\frac{u_2}{x_1} - \frac{k_1 x_2}{(1+k_2 x_2)^2}$$

The parameters are: $C_{b1} = 24.9$, $C_{b2} = 0.1$, $k_1 = k_2 = 1$, and $u_1 = u_2 = 1$.
The initial conditions are: $x_1(0) = 10$ and $x_2(0) = 0$.

a) First, simulate the equation for x_1 only since it does not depend on x_2. Note the steady-state value that you find for x_1.

b) In the model for x_2, initialize the integrator for x_1 to the steady-state value you found in part (a) and initialize $x_2(0) = 0$. Run the simulation for 1000 seconds to determine $x_2(t)$ and the steady-state value of x_2.

c) Repeat the simulation for x_2 using an initial value $x_2(0) = 10$. Note the new steady-state value you find for x_2.

d) Both of the previous cases are stable. There is another steady state corresponding to everything else being the same and the steady states for x_2 and x_1 being $x_2 = 2.793$ and $x_1 = 100$. This is an unstable steady state. Demonstrate this by setting the initial value of the integrator for x_2 to $x_2(0) = 2.80$ and show that the simulation goes to the upper steady-state value. Repeat the simulation with $x_2(0) = 2.79$ and show that it goes to the lower steady-state value. This means that any small fluctuation will cause the system to fall to steady state of part (b) or rise to the steady state of part (c).

e) Demonstrate that the steady state with $x_2 = 2.793$ is unstable by linearizing the equation for x_2 about the steady-state values and showing that the linear system that results is unstable. You can do this by either solving the differential equation or by simulating the system with a small initial Δx_2.