

Unit 3 Exam
Math 331 VA
Spring 2015

Name _____

DIRECTIONS: Show as much work as possible within each question as I grade on both the process and the final answer. TI-89's are wonderful calculators, but they don't show me if you know anything about calculus! You may have 2 hours maximum to complete the exam. Show all work on the exam itself, you should not use any outside paper, notes, etc.

1. (6 pts each) Determine the following antiderivatives:

a. $\int (e^{3x} + x^3 + \sqrt[3]{x}) dx$

b. $\int x^3 \sqrt{x^4 + 3} dx$

c. $\int \left(\frac{3}{x} + \frac{5}{x^{5/2}} \right) dx$

2. (6 pts each) Calculate the value of each definite integral (Show work Don't use calculators!):

a. $\int_0^3 \left(\frac{x^2}{x^3 + 9} \right) dx$

b. $\int_2^{\infty} \frac{1}{(1+x)^{2.5}} dx$

c. $\int_{-\infty}^{-9} \frac{3}{x^4} dx$

3. (16 pts) a. Approximate the area under the curve $f(x) = \frac{1}{\sqrt{x}}$ and above the x-axis by splitting the region from $x = 1$ to $x = 9$ into 8 equal subintervals (rectangles) and using the left endpoints of the subintervals as the heights.

- b. Use the Simpson's Rule with $n = 8$ to estimate area under the curve $f(x) = \frac{1}{\sqrt{x}}$

- c. Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}} dx$, and compare with the answer obtained from part (a), and (b). Which method is more accurate, part A or part B?

4. (7 pts) Determine the area between the curves $f(x) = x^2$ and $g(x) = x^3$

5. (6 pts) Use **geometry** to determine the value of the following definite integral: $\int_0^1 \sqrt{1-x^2} dx$

6. (6 pts) A Pizza company receives 425 cases of tomato sauce every 30 days. The number of cases of sauce on inventory t days after the shipment arrives is $N(t) = 425 - 20\sqrt{30t}$. Find the *average* daily inventory.

7. (6 pts each) Evaluate the following:

a. $\int_1^3 \int_0^2 (y + x^2) dx dy$

b. $\int_0^5 \int_0^{2y} (xy^2 - x^2) dx dy$

8. (8 pts) Calculate the volume of the solid of revolution formed by rotating the region bounded by $f(x) = 2x + 3$, $y = 0$, $x = 1$, and $x = 4$ around the x-axis. Provide the diagram.

9. The function $f(x) = 240$ represents the rate of flow of money in dollars per year. Assume a 8-year period for t and a rate r of 10% compounded continuously and determine the following:

a. (6 pts) The present value $\left(P = \int_0^t f(x)e^{-rx} dx \right)$

b. (3 pts) The accumulated amount $\left(A = e^{rt} \int_0^t f(x)e^{-rx} dx \right)$