CHAPTER 13

ENGINEERING ECONOMICS

Engineering economics provides the framework for the preparation of economic feasibility studies. Accurate, meaningful analyses must be based on realistic and accurate engineering information, including detailed design of the proposed project or facility, and an accurate definition of all operating materials requirements, labor requirements, utility requirements, maintenance requirements, and support requirements. With all the necessary engineering information in-hand, the meaningful economic evaluation of a project can proceed. The latter requires that the engineer understand certain economic concepts.

TIME VALUE OF MONEY

The cost of borrowed money is called *interest*. Simple interest is defined as the interest rate times the principle for the specified period of time. Thus:

$$I = i \times n \times P$$

where: I = total interest earned

i = interest rate for the specified time period

P = amount of money for which the interest is calculated

n = number of unit time periods

Simple interest is rarely used alone in economic analyses. It is, however, used for short periods of time, for short-term borrowing and investment, and for the first step in developing the long-term interest costs of money. Note that dollars are used in the calculations and that the interest, expressed as a percent, must be converted to decimal form (by dividing by 100).

Compound interest means that the interest which has accrued during a unit time interest period is also subject to the interest rate for the next unit time period. Thus:

$$F_n = P \times (1 + i)^n$$

where: F_n = total interest

i = interest rate for the interest period

P = amount of money for which the interest is calculated

n = number of unit time periods over which the interest is calculated

Compounding can be calculated for any period such as a year, a quarter, semiannually, monthly, or daily. The effective interest rate for a longer period of time, when the interest is compounding at shorter intervals, is expressed as:

$$i_e = \left(1 + \frac{r}{k}\right)^k - 1$$

where: i_e = effective interest rate

r = interest rate for the unit time period

k = number of times of compounding during the unit time period

As defined above, the future worth F_n, of an amount of money P, is that amount of money times the compound interest factor $(1 + i)^n$. This relationship can be rearranged to determine the present value of money to be received or paid out at some future time, at a given interest rate. Thus:

$$P = \frac{F_n}{(1+i)^n}$$

For example, $1,469 to be received in five years with an available interest rate of 8% has a present value of $1,000. Put another way, $1,000 invested now would have to earn an additional $469 in five years, with an 8% available interest rate, to "break even."

PROBLEM 1:

What amount of money would have to be invested to have $4,000 at the end of three years at a 10% compound interest rate?

SOLUTION:

$$P = \frac{F}{(I+i)^n} = \frac{4,000}{(1+0.10)^3} = \$3,005$$

PROBLEM 2:

Five hundred thousand dollars is borrowed at a nominal rate of 8%, compounded quarterly. If no payments are made in the first three years, how much will be owed? How much would the amount be if interest was compounded annually? Daily?

SOLUTION:

For quarterly compounding, first find the effective interest rate:

$$\left(1 + \frac{0.08}{4}\right)^4 - 1 = 8.24\%$$

$$F = \$500,000\left(\frac{F}{P}, 8.24\%, 12\right)$$

$$= \$636,064$$

Compounded annually:

$$F = \$500,000\left(\frac{F}{P}, 8\%, 3\right)$$

$$= \$629,856$$

Compounded daily:

$$F = \$500,000\left(\frac{F}{P}, \frac{8}{365\%}, 1,095\right)$$

$$= \$635,608$$

INFLATION

Inflation is usually expressed as a percentage compounded annually. For a constant inflation rate, the cost of a commodity would increase in relation to its present cost by the following equation:

$$F_c = P_c \times (1 + f)^n$$

where: F_c = future cost of the commodity, in dollars
f = annual interest rate, %/100
P_c = present cost of the commodity, in dollars
n = number of unit time periods over which the interest is calculated

Also the future worth of money decreases due to inflation in relation to the present value. The resultant "devaluation" is given by the following equation for a constant inflation rate:

$$F = \frac{P}{(1+f)^n}$$

where: F = future worth of the money, in dollars
f = annual interest rate, %/100
P = present value of the money, in dollars
n = number of unit time periods over which the interest is calculated

However, if an amount P is invested at an annual interest rate of i, the future worth F at a constant annual inflation rate of f would be given by the following equation:

$$F = P \times \left[\frac{(1+i)}{(1+f)}\right]^n$$

Using the same example as earlier—for an inflation rate of f = 6%, when $1,000 is invested for a period of five years, at an 8% interest rate, the future worth is only $1,098 compared to the $1,469 returned when inflation is not considered.

PROBLEM 3:

If the inflation rate is 8%, and you invest $20,000 at an 11% simple interest rate, will you have retained your buying power at the end of (a) 5 years? (b) 10 years?

SOLUTION:

(a) The principal will grow to:

$$F = \$20{,}000[1 + 5(0.11)] = \$31{,}000$$

To see if the buying power is the same, the future amount must be discounted for inflation, so that $1.00 in the future can be converted to an equivalent amount today.

$$F_I = \frac{P_I}{(1+r)^n}$$

where F_I is the future worth, measured in today's dollars P_I, and where r is equal to the rate of inflation. Notice that discounting for inflation is the reverse of compound interest problems.

$$F_I = \frac{31{,}000}{(1+0.08)^5}$$

$$F_I = \$21{,}098$$

Buying power is retained, plus a little extra.

(b) $$F = \$20{,}000[1 + (10 \times 0.11)]$$

$$= \$42{,}000$$

$$F_I = \frac{42{,}000}{(1+0.08)^{10}}$$

$$F_I = \$19{,}454 < \$20{,}000$$

With an increase in time, buying power diminishes because of the compounding nature of inflation.

TAXES

The taxes paid on any interest earned during a tax period must be subtracted from that earned interest before compounding. Consequently, the future worth F after one year accounting for taxes is:

$$F = P + (1 - t) \times i \times P, \text{ or } F = P \times [1 + (1 - t) \times i]$$

where: t = tax rate.

If the inflation proceeds at a rate f during that same year, the above equation can be written:

$$F = P \times \frac{[1 + (1 - t) \times i]}{[1 + f]}$$

Again, using the same earlier example, for a tax rate of 20% and an inflation rate of f = 6%, when $1,000 is invested for a period of five years at an 8% interest rate, the future worth is only $1,019 compared to the $1,469 when neither inflation nor taxes are considered.

CASH FLOW

The cash flow for a period of time, such as a month, quarter, or year, is the difference between all of the funds received and all of the funds disbursed. A cash flow projection will show what is expected to take place over the life of a project. Such projections are often presented as a diagram, with the horizontal axis representing the time periods and the vertical axis representing the annual amount of cash flow or the cumulative cash flow. For a typical project, the projection will show a negative cash flow during the earlier years, rising to a positive cash flow in later years. The *break-even point* is defined as the years when the cumulative cash flow equals zero. The *break-even capacity* of the project is the production rate at which all the costs, excluding depreciation, are equal to the sales realized. A break-even analysis is one of the tools often used in analyzing the viability of a project.

INTEREST FACTORS

There are several interest factors, or parameters, that are routinely used in economic engineering calculations. These include present value, P, future worth, F, and other parameters. The formulas defining these factors and parameters require a constant interest rate i, over a series of uniform

time intervals or periods (n). The values for many of these factors and parameters have been tabulated and are available in appropriate financial handbooks.

Some important factors include:

***Single-Payment, Compound-Amount Factor,* F/P:**

$$F/P = (1 + i)^n$$

This factor can be multiplied by the present value, P, to determine the future worth, F. There is a standard notation that is sometimes used to denote this factor: (F/P, i%, n).

***Single-Payment, Present-Worth Factor,* P/F:**

$$P/F = (1 + i)^{-n}$$

This factor is the reciprocal of the previous factor. This parameter is also called a *discount factor*. It can be multiplied by the future worth, F, to determine the present value, P.

***Uniform-Series, Compound-Amount Factor,* F/A:**

$$F/A = \frac{\left[(1+i)^n - 1\right]}{i}$$

This factor gives the future worth F of a series of uniform annual payments or receipts A that are made of a period of n years at an interest rate of i.

***Uniform-Series, Sinking-Fund Factor,* A/F:**

$$A/F = \frac{i}{\left[(1+i)^n - 1\right]}$$

This factor is the reciprocal of the previous factor. This parameter provides a means to calculate the uniform payment A required to have a total amount of F accumulated at the end of n years at an interest rate of i.

***Uniform-Series, Capital-Recovery Factor,* A/P:**

$$A/P = \frac{i}{\left[1 - (1+i)^{-n}\right]}$$

This factor gives the uniform amount of money A that depletes an amount of total present value P dollars over a period of n years at an interest rate of i.

***Uniform-Series, Present-Worth Factor,* P/A:**

$$P/A = \frac{\left[1-(1+i)^{-n}\right]}{i}$$

This factor is the reciprocal of the previous parameter, and it is used to compute the principal needed to assure a uniform series of payments A, over a period of n years at an interest rate of i.

***Gradient Series Factor,* A/G:**

$$A/G = \left(\frac{1}{i}\right) - \left\{\frac{n}{\left[(1+i)^n - 1\right]}\right\}$$

This factor is used to convert a series of increasing amounts into a series of uniform payments (A). The increasing series consists of an initial series amount of A_o, with each succeeding year increased by an amount, G. After n years the series amount would be: $[A_o + (n - 1) \times G]$, and the total amount accumulated would be F.

METHODS FOR PROJECT ANALYSIS

The present worth and annual worth methods are two commonly used methods for the evaluation of project viability. These methods assume that all engineering and technical information has been already generated, and economic parameters, such as initial investment, cash flow, and minimum acceptable rate of return (*MARR*), have been set.

Present-Worth Method

The present-worth method converts project cash flow into an equivalent present value for a given minimum acceptable rate of return. The present worth (PW) is defined by the formula:

$$PW = \sum_{j=1}^{n} CF_j \left(\frac{P}{F}\right)$$

where: PW = present worth
CF = annual cash flow
n = total number of years
(P/F) = single-payment, present-worth factor evaluated for the interest rate i, and the year j, for each cash flow

PROBLEM 4:

Consider the following series of payments and profits from ABC Pipes Inc.'s plan to institute a new product line of pipe fittings.

\$150,000	initial cost for new equipment
\$22,000	yearly after-tax cash flow
\$40,000	maintenance of equipment in year 10
\$30,000	salvage value of equipment in 20 years

SOLUTION:

To find the present worth of this project at a 10% *MARR*, the equation gives:

$$PW = \$150{,}000 + \$22{,}000\left(\frac{P}{A}, 10\%, 20\right) - \$40{,}000\left(\frac{P}{F}, 10\%, 10\right)$$
$$+ \$30{,}000\left(\frac{P}{F}, 10\%, 20\right)$$
$$= \$9{,}238$$

The present worth is greater than zero, and this project would be acceptable to a corporation requiring a 10% rate of return.

Annual-Worth Method

The annual-worth method converts uneven project cash flow into uniform project cash flow. The results are similar to that of the previously described present-worth method. The annual worth (also referred to as a Equivalent Uniform Annual Series, *EUAS*) is determined by first determining the present worth (PW), and then converting the PW to a uniform annual cash flow using the following formula:

$EUAS = PW\left(\frac{A}{P}\right)$, where $\left(\frac{A}{P}\right)$ is the uniform-series, capital-recovery factor defined earlier.

PROBLEM 5:

A company needs additional warehouse space for product as a result of plant expansion. The options include constructing a prefabricated steel building, a tilt-up concrete building, or renting space. The steel building has a cost of \$150,000 and a service life of 25 years with annual maintenance and property taxes of \$6,000 per year. The concrete building has a cost of \$200,000 and a service life of 50 years with annual maintenance and property taxes of \$4,000. Both buildings have no realizable salvage value, and the company uses a 15% minimum attractive rate of return. The company can rent suitable space for \$32,000 per year. Basing the decision for additional warehouse space on the Equivalent Uniform Annual Cost, should the company construct the steel building, construct the concrete building, or rent warehouse space?

SOLUTION:

The Equivalent Uniform Annual Cost (*EUAC*) for the building is given by:

$$EUAC = -P\left(\frac{A}{P}, i\%, n\right) - \text{[maintenance and taxes]}$$

where $\left(\frac{A}{P}, i\%, n\right)$ is the uniform series capital-recovery factor and P is the cost of the building.

Steel Building:

$$EUAC = -150{,}000\left(\frac{A}{P}, 15\%, 25\right) - 6{,}000$$

$$EUAC = -150{,}000\ (0.15470) - 6{,}000 = -\$29{,}205$$

Concrete Building:

$$EUAC = -200{,}000\left(\frac{A}{P}, 15\%, 50\right) - 4{,}000$$

$$EUAC = -200{,}000(0.15014) - 4{,}000 = -\$34{,}028$$

Comparing the above annual rates with renting at \$32,000 per year, the best decision is to build the steel building for \$29,205 Equivalent Uniform Annual Cost.

PROFITABILITY ANALYSIS

The two standard methods used by private companies to measure the profitability of a project are the net present value (*NPV*) and the rate of return (*ROR*). The net present value is the sum of all the cash flows discounted to the present value. The rate of return is the interest rate used in the present value calculation which will give a net present value of zero. The rate of return is sometimes called the discounted cash flow rate of return (*DCFRR*) or the internal rate of return (*IRR*).

Net Present Value

The net present value (*NPV*) is evaluated using the net annual cash flows CF_j in the following formula:

$$NPV = -CF_0 + \sum_{j=1}^{n} CF_j(1+i)^{-j}$$

where: CF_0 = initial capital investment

i = interest rate specified for the project

n = number of years specified for the project

PROBLEM 6:

A straight-run fuel oil stream in a refinery can be converted to a high octane fuel for blending into premium gasoline using hydrocracking. A proposal has been made to add a 15,000 bbl/day unit at a capital cost of \$71.0 million. The annual net profit in million dollars is given below for the estimated life of the hydrocracking unit. The net present value is to be evaluated for interest rates of 15% and 25%, and the profitability compared. These results are shown in the following table.

SOLUTION:

Computing the net present value gives:

$$NPV(15\%) = -71.0 + 80.84 = 9.64$$

$$NPV(25\%) = -71.0 + 66.66 = -4.34$$

The investment is marginally attractive with a positive net present value if funds are available at 15%, but the project is not considered with a negative net present value for funds available at 25%.

End of Year n	Annual Net Profit, F	$\frac{P}{F}(15\%)(1.15)^{-n}$	Present Value	$\frac{P}{F}(25\%)(1.25)^{-n}$	Present Value
1	32.0	0.8695	27.83	0.8000	25.60
2	28.0	0.7561	21.17	0.6400	17.92
3	22.0	0.6575	14.47	0.5120	11.26
4	17.0	0.5718	9.72	0.4091	6.96
5	15.0	0.4972	7.46	0.3277	4.92
Total	114.0		80.65		66.66

Table 1. Annual Net Profit for Estimated Life of Hydrocracking Unit

Rate of Return

The rate of return (ROR) or internal rate of return (IRR) is the interest rate or discount rate at which the net present value is equal to zero. Thus:

$$CF_0 = +\sum_{j=1}^{n} CF_j(1+i)^{-j}$$

where: CF_0 = initial capital investment
i = interest rate specified for the project
n = number of years specified for the project

PROBLEM 7:

A division of a company has been allocated \$100,000 to invest at the start of the next fiscal year in cost-reduction projects. Three projects are under consideration and are summarized below.

The minimum attractive rate of return for the company is 20% for projects with this economic life. Would the recommendation based on the rate of return for these projects be (A) invest in A only, (B) invest in B only, (C) invest in A and B, (D) invest in C only, or (E) seek other alternatives?

SOLUTION:

Alternatives are evaluated for investing \$100,000 by comparing the rate of return for the projects with the minimum attractive rate of return of

Project	Investment Required	Estimated Economic Life (years)	Net Annual Cash Flow
A	\$50,000	9	\$16,600
B	\$50,000	8	\$15,000
C	\$100,000	6	\$30,000

Table 2. Sample Cost = Reduction Projects

20%. The rate of return (i) is the interest rate where the net present value is zero. For a uniform net annual cash flow (A), the equation for the net present value (NPV) is:

$$NPV = CF_0 = A\left(\frac{P}{A}, i\%, n\right)$$

where CF_0 is the capital investment and $\left(\frac{P}{A}, i\%, n\right) = \frac{\left[1-(1+i)^{-n}\right]}{i}$ is the uniform series capital recovery factor.

For Project A:

$$0 = -50{,}000 + 16{,}600\left(\frac{P}{A}, i\%, 9\right) \text{ or } \left(\frac{P}{A}, i\%, 9\right) = 3.012$$

In addition to using tabulations of $\left(\frac{P}{A}, i\%, 9\right)$ factors in standard texts gives $i = 30.0\%$.

For Project B:

$$0 = -50{,}000 + 15{,}000\left(\frac{P}{A}, i\%, 8\right) \text{ or } \left(\frac{P}{A}, i\%, 8\right) = 3.3333$$

Using standard tables of compound interest factors, gives $i = 25.0\%$.

For Project C:

$$0 = -100{,}000 + 30{,}000\left(\frac{P}{A}, i\%, 6\right) \text{ or } \left(\frac{P}{A}, i\%, 6\right) = 3.33$$

Using standard tables of compound interest factors, gives $i = 20.0\%$.

Summary:

Project	Rate of Return
A	30.0%
B	25.0%
C	20.0%

The investment decision is to select Projects *A* and *B* because their rate of return is greater than the minimum attractive rate of return; and all of the available capital is used.

It should be noted that the rate of return method is best when comparing independent alternatives. It is probably one of the most popular tools used in capital budgeting. Net present value is widely used to choose among dependent alternatives. A thorough economic analysis will use more than one method, and it will try to include as much information as can be made available.

Equivalence

The concept of "equivalence" means that the present value of two or more projects are equal or "equivalent." The projects in question may have very different initial capital costs and very different cash flows, but with equal present value the projects are considered equivalent. As examples:

a) A uniform annual payment of $655.56 for 20 years at an interest rate of 8% is equivalent to, that is, has the same present value of, a single payment of $30,000 in 20 years.

b) A single payment today of $30,000 would be equivalent to an annual payment of $3,055.57 over 20 years at an 8% interest (discount) rate, that is, has the same present value as $30,000 today.

PROBLEM 8:

Manufacturers of motors for small household equipment wonder if they should close one of their two plants (Plant *A*) and expand operations at Plant *B*. It seems there is not enough business to keep both plants running at capacity, yet expansion of Plant *B* would have to occur for it to handle double the usual load. Net income has been a steady $800,000 at Plant *A*, and $840,000 at Plant *B*. Due to savings in overhead costs, it is thought that, with the same level of business, net income from an upgraded Plant *B* alone would equal $1,780,000. Salvage value of Plant *A* and sale of the land is estimated at $120,000. The investment to expand Plant *B* would cost $1.1 million, and it would take three years before the level of production could be increased.

(a) If the company has a required *MARR* of 11%, and operations are thought to be steady for the next 15 years, how desirable would this course of action be, if disassembly of Plant *A* were to begin immediately and the total cost of upgrading was incurred at time zero?

(b) If disassembling Plant *A* was to occur at the end of year three, when the expansion is completed?

(c) If the disbursement for upgrading was spread evenly over the first three years of construction?

SOLUTION:

(a) The first task is to separate all factors unique to the proposed course of action. If disassembling the plant were to occur immediately, that is, at time zero, a cost of $980,000 would be incurred, which is the expenditure for upgrading minus the income from sale and salvage of Plant *A*. The first three years of the project would incur a loss of $800,000 from Plant *A* with no changes in income from Plant *B*. The next 15 years would see a positive cash flow of $140,000, which is the change in income caused by the project.

The internal rate of return is a negative 4.65%, and the return on investment is –2,198,866, clearly unacceptable.

(b) If disassembling is delayed until the third year, the cost in year zero is $1.1 million, no income change occurs in years one and two, with a positive cash flow in year three from sale of land and Plant *A*. Income of $140,000 is then constant for 15 years. Delaying the plant disassembly is clearly the more practical choice, yet return on investment at 7.4%, still does not meet the requirement of the company. Net present value is –$276,150.

(c) If the disbursements for upgrading were spread evenly over the three years, instead of a lump sum at time zero, then at the close of years one and two, –$366,667 is incurred, –$246,667 at year three ($366,667 – $120,000), and $140,000 for the 15 years thereafter. This makes the investment more attractive at a return on investment of 9.6%, but this is still not sufficient to meet an *MARR* of 11%. The new present value is –$72,180.

TAXES AND DEPRECIATION

It is important to distinguish between *before tax* cash flows and *after tax* flows (BTCF and ATCF). Although project decisions are best made using after tax cash flows, before tax cash flows are often used in preliminary stages of project analysis. Tax analysis requires an understanding of the concept of depreciation, which, in turn, requires an understanding of the terms *fixed capital, working capital*, and *capital goods.*

Capital goods are those accumulated in order to produce other goods. Fixed capital is that capital which cannot be readily converted into another type of asset (such as the buildings and machinery used to produce the goods). Working capital is the investment that puts a project, or plant, into production. Total capital is the sum of the fixed and working capital, but does *not* include operating expenses. Depreciation for any given year depends upon the *fixed* capital only.

Though the depreciable base of a project consists of fixed capital only, the value of land is not included. The IRS has guidelines for the write-off life of assets, and has three methods of computing depreciation: straight line (steady linear loss over a period of time), accelerated (more depreciation in earlier years), and decelerated (more depreciation in the latter years).

Straight-Line Method

With straight-line depreciation, the annual depreciation of fixed capital is constant over the depreciation period. The depreciation charge for any year r can be computed by the following formula:

$$D_r = \frac{(P-S)}{n}$$

where: D_r = depreciation in year, r

P = original cost

S = salvage value

n = total number of years over which the depreciation is taken

For this case the book value (BV) at the end of the rth year would be:

$$BV_r = P - r \times D_r$$

For example, the original cost of a piece of equipment is $20,000, the depreciation period is 12 years, and the salvage value is equal to zero. The

depreciation for year 1 is: $D_1 = (20{,}000 - 0)/12 = \$1{,}667$; the book value is: $BV_1 = 20{,}000 - (1 \times 1{,}667) = \$18{,}333$.

Sum-of-Years-Digits Method

The sum-of-years (SY) depreciation is an accelerated depreciation method whereby about 75% of the cost is in the first half of the project life. The depreciation charge for any year r can be computed by the following formulas:

- Calculate the sum-of-years, n: $SY = n \times (n+1)/2$
- Calculate a "unit of depreciation," C: $C = (P - S)/SY$
- Calculate the depreciation D for any year, r: $D_r = (n + 1 - r) \times C$

Using the same information as in the straight-line depreciation method:

- $SY = 12 \times (12+1)/2 = 78$ (sum of the 12 years)
- $C = (20{,}000 - 0)/78 = \$256$ (the "unit depreciation")

Now the depreciation for each year can be calculated:

- $D_1 = (12 + 1 - 1) \times 256 = 12 \times 256 = \$3{,}072$ for year 1
- $D_2 = (12 + 1 - 2) \times 256 = 11 \times 256 = \$2{,}816$ for year 2

The book value for the project at any year, can be calculated by substituting the value of the sum-of-the-digits depreciation for that year into the same book value formula as for straight-line depreciation.

Double-Rate Declining Balance Method

The double-rate declining balance depreciation is an accelerated depreciation method at double the straight-line depreciation rate for the remaining depreciable balance. The depreciation charge for any year, r, can be computed by the following formulas:

- Calculate the double-declining factor, f: $f = 2/n$
- Then calculate the book value, BV_r: $BV_r = P \times (1 - f)^r$
- Calculate the depreciation, D, for year, r: $D_r = (BV_{r-1} - BV_r)$; or $D_r = f \times (BV_{r-1})$

Using the same information as in the straight-line depreciation method:

- $f = 2.0/12 = 0.167$
- $BV_1 = 20{,}000 \times (1 - 0.167)^1 = \$16{,}660$ for year 1

- $D_1 = (20{,}000 - 16{,}660) = \$3{,}340$ for year 1
- $BV_2 = 20{,}000 \times (1 - 0.167)^2 = \$13{,}889$ for year 2
- $D_2 = (16{,}000 - 13{,}889) = \$3{,}111$ for year 2

Other Methods

The *Sinking Fund* method depreciates equipment with an imaginary sinking fund that is equivalent to the company making a series of equal annual payments to a fund which will equal the cost of replacing the equipment at the end of its useful life. This method is used only with equipment that has to be replaced with equipment which costs at least as much as the original. This is a decelerated depreciation method with larger annual depreciation values taken in the later years.

Often it is more convenient to group assets bought in the same year for depreciation purposes into a *group account*. A *classified account* groups items according to use, and a *composite account* includes items which have diverse uses and lives. All of the methods discussed work. They are selected for use by consideration of a company's overall economic situation.

SENSITIVITY AND RISK ANALYSIS

A "sensitive" project is one whose desirability is highly affected by the small change in one or more certain variables. A project is "insensitive" to certain variables if wide variations in the value of that variable does not alter the conclusions of the project evaluation. The variables which are sources of risk and which could lead to project sensitivity, include everything that is estimated such as: disbursements, receipts, project service time, salvage value, tax rate, cost of raw materials, cost of energy, and revenues from products.

Risk analysis is linked to sensitivity analysis and attempts to quantify the risks, uncertainties, and variability associated with a project. A simple risk assessment for economic decision analysis involves the following steps:

- Determine the range of possible outcomes that would effect the profitability of the project (e.g., the range in product selling price).
- Evaluate the profit over the range of possible outcomes.

- Estimate the probability of occurrence of each of the possible outcomes (e.g., the probability that the price will be at the lowest value when the plant is constructed and begins operation).
- Evaluate the weighted average of the profit by computing the sum of the profits times the probabilities.

This weighted average profit is an estimate of the expected value of the profit from the project.

As an example, if it was determined that the supply of seafood is directly related to weather conditions, the probability that a seafood processing plant will be used to capacity in any one year could be predicted using weather data from past years and what the effect of seafood harvests will be. With this information, for this example it is predicted that the probability of the plant being used at 75% capacity in a given year is 30%, the probability for a full capacity year is 45%, and the probability of usage at 125% of capacity is 25%. The following table shows the calculations that give the expected value of the annual worth:

% Capacity	Annual Worth	Probability	Expected Value
75%	$22,314	0.30	$6,694.20
100%	$124,045	0.45	$55,820.25
125%	$212,151	0.25	$53,037.75
		Expected Value =	$115,552.75

Table 3. Calculation of Expected Worth

Note that the sum of the probabilities is equal to one. This must always be the case.