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Statistical Design I IENG 245

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Quiz 3

1. A fair die is tossed twice. Find the probability of getting at least 5 on the first toss and at most 3 on the second toss.

$P(A)$ = Prob of getting at least 5 on the first toss $= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$P(B)$ = at most 3 on the second toss $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B)$

$= \frac{1}{3} \cdot \frac{1}{2}$

$P(A \cap B) = \frac{1}{6}$ ✓ 10

2. How many distinct 6-letter words can be formed using the letters of the word *avatar* ?

a = 3
t = 1
v = 1
r = 1

No of Words can be form $= \frac{6!}{3!} = 6 \times 5 \times 4 = 120$ ✓

10

3. There are 5 bolts and 5 nuts in a box. An object is drawn at random from the box without replacement. What is the probability that after two drawings we would have a bolt and a nut?

$P(A)$ = Probability of Crossing a bolt
$P(B)$ = Probability of crossing a nut

$P(A \cap B) = \frac{5}{10} \cdot \frac{4}{9} + \frac{5}{10} \cdot \frac{4}{9} = \frac{20}{90} + \frac{20}{90} = \frac{4}{9}$ ✗

0

4. You have to organize a charitable Christmas dinner for 100 guests to benefit underprivileged children. To raise funds you plan a lottery for the guests. The prizes are as follows: first prize - \$1000, two second prizes \$500 each. How much should you charge for a ticket if you know that your guests' mathematical expectation should be −\$50.00 to ensure the sale of all 100 tickets?

20

5. In two Statistics classes taught at NYIT and NYU the average cumulative score is the same and is equal to 68. The range of 68 ± 6 corresponds to the grade of 'C'. Earning a grade other than that means that the cumulative score S_{cum} would be either greater than 74 or lower than 62, i.e. $|S_{\text{cum}} - 68| \geq 6$. The standard deviations however, differ for the two classes and are $\sigma_{\text{NYIT}} = 4$, $\sigma_{\text{NYU}} = 3$. In which class are there higher probability of 'non-C' students? Assuming that your estimate is optimal, compute the difference between the percentages of the 'non-C' students using the Chebyshev's inequality

$P(|S_{cum} - 68| \geq 6) \leq \frac{16}{36} = \frac{4}{9}$ ✓

$P(|S_{cum} - 68| \geq 6) \leq \frac{9}{36} = \frac{1}{4}$ ✓

+5

6. Let X and Y be random variables taking up values $\{0, 1, 2\}$. Their joint probability distribution function is $f(x, y) = \frac{3x+y}{36}$. Compute $E(2X - 5Y)$.

$E(2x - 5y) = 2E(x) - 5(Ey)$

$= 2\left(\frac{54}{36}\right) - 5\left(\frac{42}{36}\right)$

for calculation ← see opposite side

$= \frac{108}{36} - \frac{210}{36}$

$E(2x - 5y) = -\frac{102}{36}$ ✓

10

7. Exactly 10% of the bolts produced by a machine are defective. Calculate the probability that out of 6 randomly chosen bolts 3 are defective.

of the x bolts 10% are defective and (x − 10%) are not

$P = \frac{10\% \cdot C_3 \; (x - 10\%)\, C_3}{x\, C_6}$

X

0

8 Let X and Y be random variables taking up values $\{0, 1, 2\}$ Their joint probability distribution function is $f(x, y) = \frac{x(y+1)}{18}$ For these random variables compute $E(XY)$, $E(X)$, $E(Y)$. What is the relation (if any) between them?

$E(x) = \frac{30}{18}$ ✓ $E(y) = \frac{24}{18}$ ✓ $E(xy) = \frac{40}{18}$ ✓

$E(xy) = E(x) \cdot E(y)$

Find ~~for~~ calculation opposite page.

10

9 Let X and Y be random variables taking up values $\{0, 1, 2\}$ Their joint probability distribution function is $f(x, y) = \frac{x(y+1)}{18}$ Compute σ_{XY}

$\sigma_{xy} = E(xy) - E(x)E(y)$

$= \frac{40}{18} - \frac{30}{18} \times \frac{24}{18}$

$= 0$ ✓ 10

10 A family has six children Find the probability that there are more girls than boys Assume that the probabilities of girls and boys being born are equal

$P(\text{more girls than boys}) = \frac{21}{64}$

$P(\text{more girls than boys}) = \binom{6}{4} + \binom{6}{5} + \binom{6}{6}$

$= \frac{6!}{4!(6-4)!} + \frac{6!}{5!(6-5)!} + \frac{6!}{6!(6-6)!}$

$= \frac{6 \times 5 \times 4!}{4! \times 2} + \frac{6 \times 5!}{5!} + 0$

$= 15 + 6$

$= \frac{21}{64}$ ✓ 8