

- 1) Consider two random variables  $X$  and  $Y$  with following joint density function

$$f(x,y) = (xy)/9 \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 3.$$

- Find  $E[X|Y]$  and  $E[X^2|Y]$
  - Find  $\text{VAR}[X|Y]$
  - Find  $E[X^2|A]$  where  $A = \{ 0 \leq x \leq 0.5 \}$
  - Find covariance between  $X$  and  $Y$
- 2) The density function of two random variables  $X$  and  $Y$  is

$$f_{X,Y}(x,y) = u(x)u(y)16e^{-4(x+y)}$$

Find the mean value of the function

$$g(X,Y) = \begin{cases} 5 & 0 < X \leq 0.5 \text{ and } 0 < Y \leq 0.5 \\ -1 & 0.5 < X \text{ and/or } 0.5 < Y \\ 0 & \text{all other } X \text{ and } Y \end{cases}$$

- 3) Two random variables  $X$  and  $Y$  have means  $\bar{X} = 1$  and  $\bar{Y} = 2$ , variances  $\sigma_X^2 = 4$  and  $\sigma_Y^2 = 1$ , and correlation coefficient  $r_{XY} = 0.4$ . New random variables  $W$  and  $V$  are defined by

$$V = -X + 2Y \quad W = X + 3Y$$

Find:

- the means
- the variances
- the correlation and
- the correlation coefficient of  $V$  and  $W$ .

4) For two random variables X and Y:

$$f_{X,Y}(x,y) = 0.15\delta(x+1)\delta(y) + 0.1\delta(x)\delta(y) + 0.1\delta(x)\delta(y-2) \\ + 0.4\delta(x-1)\delta(y+2) + 0.2\delta(x-1)\delta(y-1) + 0.05\delta(x-1)\delta(y-3)$$

Find:

- (a) the correlation
- (b) the covariance
- (c) the correlation coefficient of X and Y.
- (d) Are X and Y either uncorrelated or orthogonal?

5) Three uncorrelated random variables  $X_1$ ,  $X_2$  and  $X_3$  have means  $\bar{X}_1 = 1$ ,  $\bar{X}_2 = -3$  and  $\bar{X}_3 = +1.5$  and second moments  $E[X_1^2] = 2.5$ ,  $E[X_2^2] = 11$ , and  $E[X_3^2] = 3.5$ . Let  $Y = X_1 - 2X_2 + 3X_3$  be a new random variable and find:

- (a) the mean value,
- (b) the variance of Y.

6) Let X and Y be statistically independent random variables with  $\bar{X} = 0.75$ ,  $\bar{X}^2 = 4$ ,  $\bar{Y} = 1$  and  $\bar{Y}^2 = 5$ . For a random variable  $W = X - 2Y + 1$  find:

- (a)  $R_{XY}$ , (b)  $R_{XW}$ , (c)  $R_{YW}$ , and (d)  $C_{XY}$ .
- (e) Are X and Y uncorrelated?

7) For random variables X and Y having means  $\bar{X} = 1$ ,  $\bar{Y} = 2$ ,

variances  $\sigma_X^2 = 6$ ,  $\sigma_Y^2 = 9$ , and correlation coefficient of  $\rho = -2/3$ , find:

- (a) the covariance of X and Y,
- (b) the correlation of X and Y,
- (c) the moments  $m_{20}$  and  $m_{02}$ .