EENG 3303 Fall 2014 Matlab 4,

See cover page at back.

Demonstrate capability of solving Laplace's Equation using numerical iteration with application based on the principle of mean value theorem of electrostatics. From voltage contours use program to predict directions for the electric fields

More Specific Objective
Write a computer program which will solve Laplace's Equation within domain of a square. The sides of the square form the boundary of the domain. The voltages for each side can by controlled via selection (terminal input or input statement) at start of the program (V_1, V_2, V_3, V_4). Thru user input via selection of positive integer 'N". the $N \times N$ grid of points which defines the full domain (boundary and interior points) is set. The solution construction is based on the mean value rule for 2 dimensions discussed in class

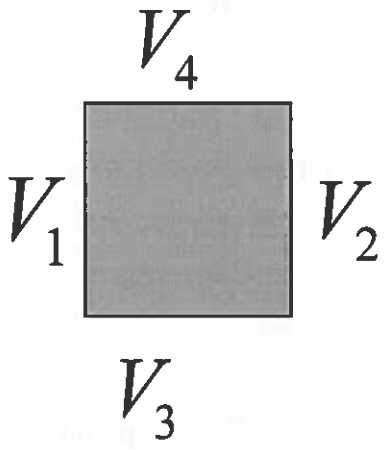
$$V(i,j) = \frac{V(i+1,j)+V(i-1,j)+V(i,j+1),V(i,j-1)}{4}$$

The natural mathematical structure to represent values on a two dimension square grid is a matrix. A minimum of two –three matrices are needed to be used to complete task.

$$M_{new}(i,j) = \frac{M(i+1,j)+M(i-1,j)+M(i,j+1),M(i,j-1)}{4}$$

Order of statement recommended

$$M(1:N,\ 1) = V1$$
$$M(1:N,\ N) = V2$$
$$M(1:N,\ N) = V3$$
$$M(N:1,\ N) = V4$$



Each updated $M_{new}(i,j)$ (new) version of the matrix can be compared with the current $M(i,j)$ and a difference matrix formed.

$$M_{diff} = M_{new} - M$$

At east step in an iterative process the maximum of the absolute values of the elements M_{diff} is identified (call it max-value). When this value is less than a program input "eps" (for epsilon) the iterative process can end, i.e.

$$\max\_value \le eps \quad (\text{computer test})$$

and then outputs such as a matlab surf plot can be generated . The mean value theorem can be used to check agreement with the expected value at center with the computer prediction at the approximate center is

$$Center \approx M(N/2, N/2).$$

Using Matlab plotting utility "contour" a contour plot can be generated which will show constant value contours. Discrete approximations for the electric field an be predicted using Matlab operator known as "gradient" and results plotted with contour plot (combined plot) with Matlab plot tool called "quiver".

Matlab has a discrete version of differential operator the Laplacian known as "del2". Apply the differential operator del2 to starting conditions matrix M (before iteration) and final matrix M . Let program predict the max of the absolute value of elements and compare. Use logical program labels "start_error", "end_error".

$$error = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\left|\nabla^2 M_{i,j}\right|}{N^2}$$ concept applies start, end errors in Laplacian over domain

Program inputs

```
N=20;%
V0=5
V1=V0;
V2=0;
V3=0;
V4=V0;
maxvalue=V0;( initialize to  start check)
eps=10^(-5);

hint,  helpful matlab commands I used
surf,  colorbar,  gradient,  contour,
clabel, del2,  quiver
```

Take it or leave it: should be helpful, from my code details on plotting,

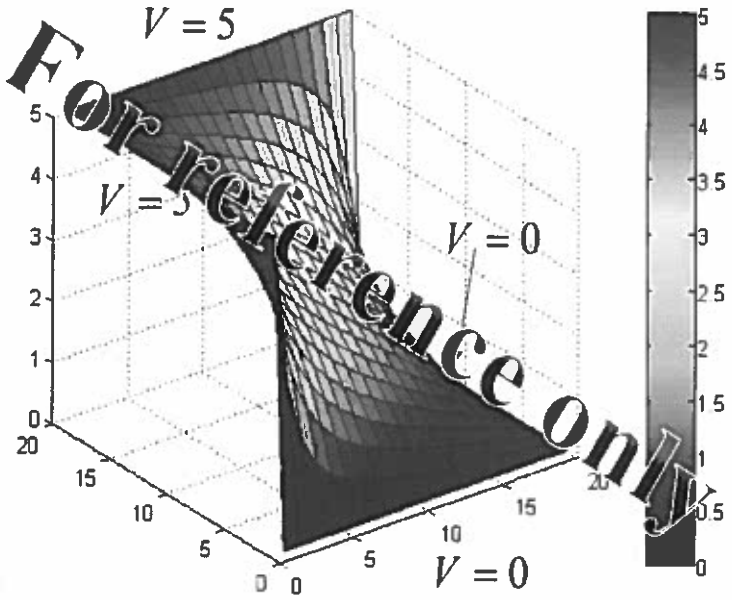
```
z=M;
x=1:N;
y=1:N;
%mesh(x,y,z);
figure( 1)
surf(x,y,z);
colorbar( 'vert' );
```

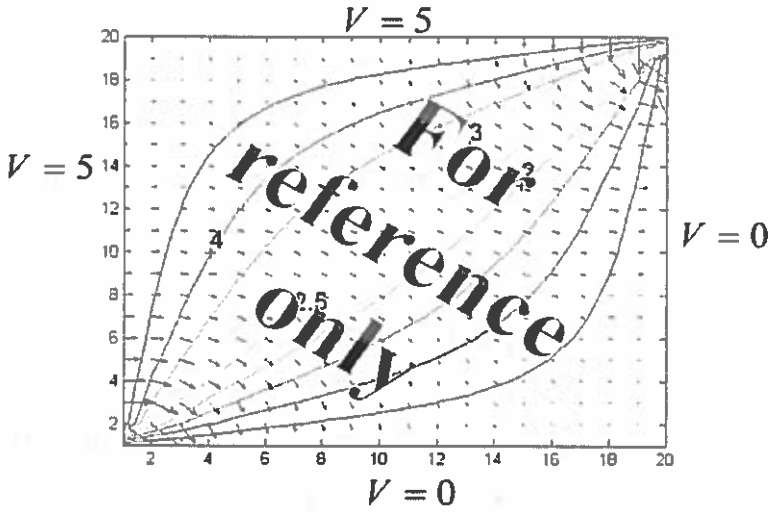
Also from my code

```
figure (2)
% find efield
v=1:N;
[px,py]=gradient(z,1,1);
C=contour(v,v,z);
V=[1 2 2.5 3 4]
clabel(C,V,'fontsize',15)
hold on;
quiver(v,v,-px,-py,3);
hold off
```

error estimation

```
start_error= sum(sum(abs(del2(M))))/N^2;
maxvalue=max(max(Mdiff));
```


V = 5
V = 5
V = 0
V = 0
For reference only


V = 5
V = 5
V = 0
V = 0
For reference only

Matlab 4 EENG 3303 Cover page,

Name _____

Numerical predictions

Computer program prediction for center (of square) value after iterations.

Center estimate=_____

Laplacian operator start_error = _____

Laplacian operator end_error=_____

Graphical output
Attach two figures (1) showing the surf with color bar (2) contour plots for V showing contour values and the efield directions on the contour map.

Deliverables, this page, two color figures, and attach code.