If you're not comfortable working with symbols, the upcoming sections on truth-functional arguments and deductions might look intimidating. But they are not as forbidding as they may appear. We presume that the whole matter of a symbolic system is unfamiliar to you, so we'll start from absolute scratch. Keep in mind, though, that everything builds on what goes before. It's important to master each concept as it's explained and not fall behind. Catching up can be very difficult. If you find yourself having difficulty with a section or a concept, put in a bit of extra effort to master it before moving ahead. It will be worth it in the end.

TRUTH TABLES AND THE TRUTH-FUNCTIONAL SYMBOLS

Our "logical vocabulary" will consist of claim variables and truth-functional symbols. Before we consider the real heart of the subject, truth tables and the symbols that represent them, let's first clarify the use of letters of the alphabet to symbolize terms and claims.

Claim Variables

In Chapter 8, we used uppercase letters to stand for terms in categorical claims. Here, we use uppercase letters to stand for claims. Our main interest is now in the way that words such as "not," "and," "or," and so on affect claims and link them together to produce compound claims out of simpler ones. So, don't confuse the Ps and Qs, called **claim variables,** that appear in this chapter with the variables used for terms in Chapter 8.*

Truth Tables

Let's now consider truth tables and symbols. In truth-functional logic, any given claim, P, is either true or false. The following little table, called a **truth table,** displays both possible truth values for P:

P
T
F

Whichever truth value the claim P might have, its negation or contradictory, which we'll symbolize ~P, will have the other. Here, then, is the truth table for ***negation:***

P	~P
T	F
F	T

*It is customary to use one kind of symbol, usually lowercase letters or Greek letters, as *claim variables* and plain or italicized uppercase letters for *specific claims.* Although this use has some technical advantages and makes possible a certain theoretical neatness, beginning students can find it confusing. Therefore, we'll use uppercase letters for both variables and specific claims and simply make it clear which way we're using the letters.

The word "and," when used in questions, can produce some interesting and amusing results. In this case, Brutus means to ask, "How many of them are boys, and how many of them are girls?" But Jack thinks he asks, "How many of them are girls or boys?" There's even a third version: "How many of them are *both* girls and boys?" Presumably, none.

The left-hand column of this table sets out both possible truth values for P, and the right-hand column sets out the truth values for ~P based on P's values. This is a way of defining the negation sign, ~, in front of the P. The symbol means "change the truth value from T to F or from F to T, depending on P's values." Because it's handy to have a name for negations that you can say aloud, we read ~P as "not-P." So, if P were "Parker is at home," then ~P would be "It is not the case that Parker is at home," or, more simply, "Parker is not at home." In a moment we'll define other symbols by means of truth tables, so make sure you understand how this one works.

Because any given claim is either true or false, two claims, P and Q, must both be true, both be false, or have opposite truth values, for a total of four possible combinations. Here are the possibilities in truth-table form:

P	Q
T	T
T	F
F	T
F	F

A **conjunction** is a compound claim made from two simpler claims, called conjuncts. *A conjunction is true if and only if both of the simpler claims that make it up (its conjuncts) are true.* An example of a conjunction is the claim "Parker is at home and Moore is at work." We'll express the conjunction of P and Q by connecting them with an ampersand (&). The truth table for conjunctions looks like this:

P	Q	P & Q
T	T	T
T	F	F
F	T	F
F	F	F

P & Q is true in the first row only, where both P and Q are true. Notice that the "truth conditions" in this row match those required in the italicized statement above the truth table.*

Here's another way to remember how conjunctions work: If either part of a conjunction is false, the conjunction itself is false. Notice finally that, although the word "and" is the closest representative in English to our ampersand symbol, there are other words that are correctly symbolized by the ampersand: "but" and "while," for instance, as well as such phrases as "even though." So, if we let P stand for "Parsons is in class" and let Q stand for "Quincy is absent," then we should represent "Parsons is in class even though Quincy is absent" by P & Q. The reason is that the compound claim is true only in one case: where both parts are true. And that's all it takes to require an ampersand to represent the connecting word or phrase.

A **disjunction** is another compound claim made up of two simpler claims, called *disjuncts. A disjunction is false if and only if both of its disjuncts are false.* Here's an example of a disjunction: "Either Parker is at home, or Moore is at work." We'll use the symbol ∨ ("wedge") to represent disjunction when we symbolize claims—as indicated in the example, the closest word in English to this symbol is "or." The truth table for disjunctions is this:

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

Notice here that a disjunction is false only in the last row, where both of its disjuncts are false. In all other cases, a disjunction is true.

The third kind of compound claim made from two simpler claims is the **conditional claim.** In ordinary English, the most common way of stating conditionals is by means of the words "if . . . then . . . ," as in the example "If Parker is at home, then Moore is at work."

We'll use an arrow to symbolize conditionals: P → Q. The first claim in a conditional, the P in the symbolization, is the **antecedent,** and the second—Q in this case—is the **consequent.** *A conditional claim is false if and only if its antecedent is true and its consequent is false.* The truth table for conditionals looks like this:

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

*Some of the words that have truth-functional meaning have other kinds of meanings as well. For example, "and" can signify not only that two things happened but that one happened earlier than the other. An example: "Melinda got on the train and bought her ticket" is quite different from "Melinda bought her ticket and got on the train." In this case, "and" operates as if it were "and then."

Only in the second row, where the antecedent P is true and the consequent Q is false, does the conditional turn out to be false. In all other cases, it is true.*

Of the four types of truth-functional claims—negation, conjunction, disjunction, and conditional—the conditional typically gives students the most trouble. Let's have a closer look at it by considering an example that may shed light on how and why conditionals work. Let's say that Moore promises you that, if his paycheck arrives this morning, he'll buy lunch. So, now we can consider the conditional

If Moore's paycheck arrives this morning, then Moore will buy lunch.

We can symbolize this using P (for the claim about the paycheck) and L (for the claim about lunch): P → L. Now let's try to see why the truth table above fits this claim.

The easiest way to see this is by asking yourself what it would take for Moore to break his promise. A moment's thought should make this clear: Two things have to happen before we can say that Moore has fibbed to you. The first is that his paycheck must arrive this morning. (After all, he didn't say what he was going to do if his paycheck *didn't* arrive, did he?) Then, it being true that his paycheck arrives, he must then *not* buy you lunch. Together, these two items make it clear that Moore's original promise was false. Notice: Under no other circumstances would we say that Moore broke his promise. And *that* is why the truth table has a conditional false in one and only one case, namely, where the antecedent is true and the consequent is false. Basic information about all four symbols is summarized in Figure 1.

Our truth-functional symbols can work in combination. Consider, for example, the claim "If Paula doesn't go to work, then Quincy will have to work a double shift." We'll represent the two simple claims in the obvious way, as follows:

P = Paula goes to work.
Q = Quincy has to work a double shift.

And we can symbolize the entire claim like this:

~P → Q

Here is a truth table for this symbolization:

P	Q	~P	~P → Q
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

*Like the conjunction, conditionals in ordinary language can have more than the meaning we assign to the arrow. The arrow represents what is often called the "material conditional," conditionals that are true except when the antecedent is true and the consequent false.

Differences between material conditionals and the conditionals used in ordinary language have held the attention of logicians and philosophers for a long time and are still controversial. See, for example, Richard Bradley, "A Defence of the Ramsey Test," in the January 2007 issue of the philosophical journal *Mind* (Vol. 116, Number 461, pp. 1-21).

FIGURE 1 The Four Basic Truth-Functional Symbols

Negation (~)

Truth table:

P	~P
T	F
F	T

Closest English counterparts: "not," or "it is not the case that"

Conjunction (&)

Truth table:

P	Q	(P & Q)
T	T	T
T	F	F
F	T	F
F	F	F

Closest English counterparts: "and," "but," "while"

Disjunction (∨)

Truth table:

P	Q	(P ∨ Q)
T	T	T
T	F	T
F	T	T
F	F	F

Closest English counterparts: "or," "unless"

Conditional (→)

Truth table:

P	Q	(P → Q)
T	T	T
T	F	F
F	T	T
F	F	T

Closest English counterparts: "if . . . then," "provided that"

Notice that the symbolized claim ~P → Q is false in the *last* row of this table. That's because, here and only here, the antecedent, ~P, is true and its consequent, Q, is false. Notice that we work from the simplest parts to the most complex: The truth value of P in a given row determines the truth value of ~P, and that truth value in turn, along with the one for Q, determines the truth value of ~P → Q.

Consider another combination: "If Paula goes to work, then Quincy and Rogers will get a day off." This claim is symbolized this way:

P → (Q & R)

This symbolization requires parentheses in order to prevent confusion with (P → Q) & R, which symbolizes a different claim and has a different truth table. Our claim is a conditional with a conjunction for a consequent, whereas (P → Q) & R is a conjunction with a conditional as one of the conjuncts. The parentheses are what make this clear.

You need to know a few principles to produce the truth table for the symbolized claim P → (Q & R). First, you have to know how to set up all the possible combinations of true and false for the three simple claims P, Q, and R. In claims with only one letter, there were two possibilities, T and F. In claims with two letters, there were four possibilities. *Every time we add another letter, the number of possible combinations of T and F doubles, and so, therefore,*

does the number of rows in our truth table. The formula for determining the number of rows in a truth table for a compound claim is $r = 2^n$, where r is the number of rows in the table and n is the number of letters in the symbolization. Because the claim we are interested in has three letters, our truth table will have eight rows, one for each possible combination of T and F for P, Q, and R. Here's how we do it:

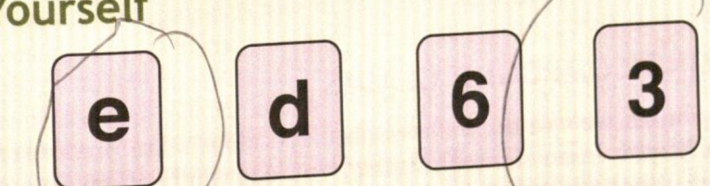
P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

The systematic way to construct such a table is to alternate Ts and Fs in the right-hand column, then alternate *pairs* of Ts and *pairs* of Fs in the next column to the left, then sets of *four* Ts and sets of *four* Fs in the next, and so forth. The leftmost column will always wind up being half Ts and half Fs.

The second thing we have to know is that the truth value of a compound claim in any particular case (i.e., any row of its truth table) depends entirely upon the truth values of its parts; and if these parts are themselves compound, their truth values depend upon those of their parts; and so on, until we get down to letters standing alone. The columns under the letters, which you have just learned to construct, will then tell us what we need to know. Let's build a truth table for P → (Q & R) and see how this works.

In Depth

Test Yourself



These cards are from a deck that has letters on one side and numbers on the other. They are supposed to obey the following rule: "If there is a vowel on one side, then the card has an even number on the other side."

Question: To see that the rule has been kept, which card(s) must be turned over and checked? (Most university students flunk this simple test of critical thinking.)

P	Q	R	Q & R	P → (Q & R)
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

The three columns at the left, under P, Q, and R, are our *reference columns*, set up just as we discussed previously. They determine what goes on in the rest of the table. From the second and third columns, under the Q and the R, we can fill in the column under Q & R. Notice that this column contains a T only in the first and fifth rows, where both Q and R are true. Next, from the column under the P and the one under Q & R, we can fill in the last column, which is the one for the entire symbolized claim. It contains Fs only in rows two, three, and four, which are the only ones where its antecedent is true and its consequent is false.

What our table gives us is a *truth-functional analysis* of our original claim. Such an analysis displays the compound claim's truth value, based on the truth values of its simpler parts.

If you've followed everything so far without problems, that's great. If you've not yet understood the basic truth table idea, however, as well as the truth tables for the truth-functional symbols, then by all means stop now and go back over this material. You should also understand how to build a truth table for symbolizations consisting of three or more letters. What comes later builds on this foundation, and as with any construction project, without a strong foundation the whole thing collapses.

A final note before we move on: Two claims are **truth-functionally equivalent** if they have exactly the same truth table—that is, if the Ts and Fs in the column under one claim are in the same arrangement as those in the column under the other. Generally speaking, when two claims are equivalent, one can be used in place of another—truth-functionally, they each imply the other.

Okay. It's time now to consider some tips for symbolizing truth-functional claims.

SYMBOLIZING COMPOUND CLAIMS

Most of the things we can do with symbolized claims are pretty straightforward; that is, if you learn the techniques, you can apply them in a relatively clear-cut way. What's less clear-cut is how to symbolize a claim in the first place. We'll cover a few tips for symbolization in this section and then give you a chance to practice with some exercises.

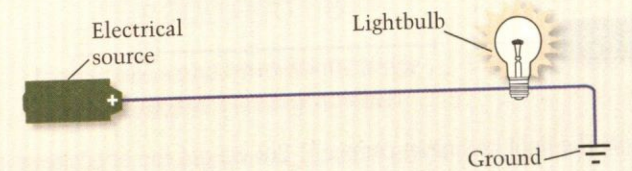
Remember, when you symbolize a claim, you're displaying its truth-functional structure. The idea is to produce a version that will be truth-functionally equivalent to the original informal claim—that is, one that will

In Depth

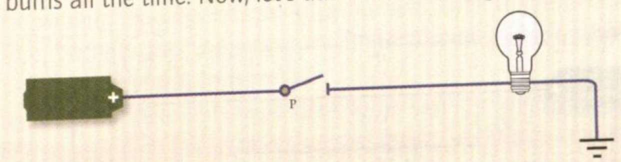
Truth-Functional Logic and Electrical Circuits

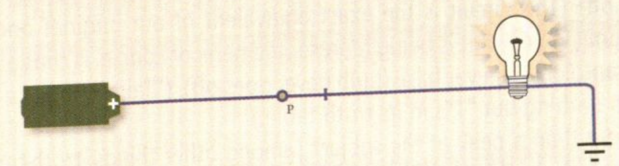
We mentioned at the beginning of the chapter that truth-functional logic is the basis of digital computing. This is because, translated into hardware systems, "true" and "false" become "on" and "off." Although there's a lot more to it than this, we can illustrate in a crude way a little of how this works.

Let's construct a simple electrical circuit from an electrical source to a ground and put a lightbulb in it somewhere, like this:

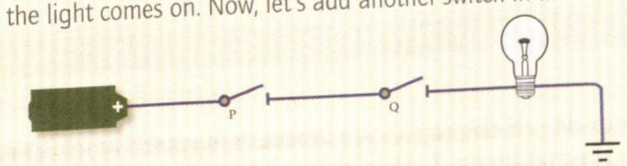


In this situation, the light burns all the time. Now, let's add a switch and give it a name, "P," like so:



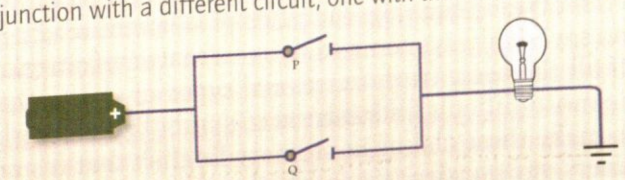


(Switch P represents a sentence that can be true or false, just as the switch can be open or closed.) When the switch is open (corresponding to false), in the second drawing, the light doesn't come on, but when it's closed (corresponding to true) in the third drawing the light comes on. Now, let's add another switch in the same line and call it "Q":



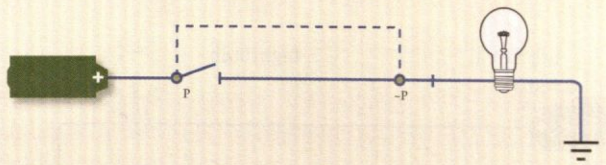
This simple circuit is analogous to a simple conjunction, "P & Q," because *both* switches must be closed for the bulb to come on, just as both conjuncts have to be true in order for the conjunction to be true. So, although there are four possible combinations for the switches (open + open, open + closed, closed + open, closed + closed), only one of them causes the bulb to burn, just as there is only one T in the truth table for conjunction.

We can represent disjunction with a different circuit, one with the switches wired in parallel rather than in series:

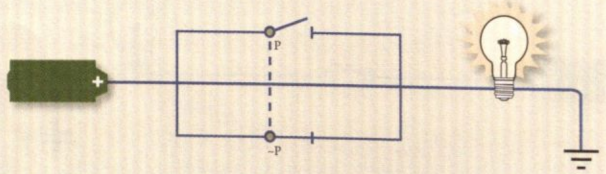


In this case, if *either* the P switch or the Q switch is on, the bulb will light up. So, it lights up in three of the four possible combinations of open/closed for the two switches, just as the disjunction "P ∨ Q" is true in three of the rows in its truth table.

We complicate our circuit-making chores somewhat when we bring in negation. If we have a switch labeled "~P," for example, we just treat it the same as if it were "P": It's either open or closed. But if our circuit contains a switch, P, and another switch, ~P, then we have to connect them (we'll do it with a dotted line), indicating that these switches are always opposite; when one closes, the other automatically opens. Now we get two interesting results: When two switches that are "negations" of each other are wired in series like this:



we have a dysfunctional circuit: The light can never come on! But we get the opposite result when we wire the two negation switches in parallel:



Here, the light can never go off! (This circuit is the exact equivalent of our original one, in which there were no switches at all.) In truth-functional logic, what is being represented here, of course, is that a contradiction is never true (bulb never comes on), and a tautology is never false (bulb never goes off). ("Tautology" is a traditional and somewhat fancy word for a sentence with nothing but "T"s in its truth table.)

This gives you nothing more than a peek at the subject (among other things, truth-functional logic can help us design circuits that are the simplest possible for doing a certain job—i.e., for being on and off under exactly the right circumstances); unfortunately, we don't have room to go further into the subject here. An Introduction to Computer Science class would be the best next step.

be true under all the same circumstances as the original and false under all the same circumstances. Let's go through some examples that illustrate a few of the most frequently encountered symbolization problems.

"If" and "Only If"

In symbolizing truth-functional claims, as in translating categorical claims in Chapter 8, nothing can take the place of a careful reading of what the claim in question says. It always comes down to a matter of exercising careful judgment.

Of all the basic truth-functional types of claim, the conditional is probably the most difficult for students to symbolize correctly. There are so many ways to make these claims in ordinary English that it's not easy to keep track. Fortunately, the phrases "if" and "only if" account for a large number of conditionals, so you'll have a head start if you understand their uses. Here are some general rules to remember:

Real Life

Truth-Functional Trickery

Using what you know about truth-functional logic, can you identify how the sender of this encouraging-looking notice can defend the claim (because it *is* true), even though the receiver is not really going to win one nickel?

You Have Absolutely Won
$1,000,000.00

If you follow the instructions inside and return the winning number!

Answer: Because there is not going to be any winning number inside (there are usually several *losing* numbers, in case that makes you feel better), the conjunction "You follow the instructions inside and [you] return the winning number" is going to be false, even if you do follow the instructions inside. Therefore, because this conjunction is the antecedent of the whole conditional claim, the conditional claim turns out to be true.

Of course, uncritical readers will take the antecedent to be saying something like "If you follow the instructions inside *by returning the winning number inside* (as if there were a winning number inside). These are the people who may wind up sending their own money to the mailer.

> The word "if," used alone, introduces the antecedent of a conditional. The phrase "only if" introduces the consequent of a conditional.

To put it another way: It's not the location of the part in a conditional that tells us whether it is the antecedent or the consequent; it's the logical words that identify it. Consider this example:

Moore will get wet *if* Parker capsizes the boat.

The "Parker" part of the claim is the antecedent, even though it comes *after* the "Moore" part. It's as though the claim had said,

If Parker capsizes the boat, Moore will get wet.

We would symbolize this claim as P → M. Once again, it's the word "if" that tells us what the antecedent is.

Parker will pay up *only if* Moore sinks the nine ball.

This claim is different. In this case, the "Parker" part is the antecedent because "only if" introduces the consequent of a conditional. This is truth-functionally the same as

Real Life

Damned If You Do, But If You Don't . . .

> The fearful, and unbelieving, and the abominable, and murderers, and whoremongers, and sorcerers, and idolators, and all liars, shall have their part in the lake which burneth with fire and brimstone.
>
> —Revelation 21:8

This came to us in a brochure from a religious sect offering salvation for the believer. Notice, though, that the passage from the Bible doesn't say that, if you believe, you *won't* go to hell. It says, if you don't believe, you *will* go to hell.

If Parker pays up (P), then Moore sunk (or must have sunk) the nine ball (M).

Using the letters indicated in parentheses, we'd symbolize this as

$P \rightarrow M$

Don't worry about the grammatical tenses; we'll adjust those so that the claims make sense. We can use "if" in front of a conditional's antecedent, or we can use "only if" in front of its consequent; we produce exactly equivalent claims in the two cases. As is the case with "if," it doesn't matter where the "only if" part of the claim occurs. The part of this claim that's about Moore is the consequent, even though it occurs at the beginning of this version:

Only if Moore sinks the nine ball will Parker pay up.

Once again: $P \rightarrow M$.

Exercise 9-1

Symbolize the following using the claim variables P and Q. (You can ignore differences in past, present, and future tense.)

1. If Quincy learns to symbolize, Paula will be amazed.
2. Paula will teach him if Quincy pays her a big fee.
3. Paula will teach him only if Quincy pays her a big fee.
4. Only if Paula helps him will Quincy pass the course.
5. Quincy will pass if and only if Paula helps him.

Claim 5 in the preceding exercise introduces a new wrinkle, the phrase "if and only if." Remembering our general rules about how "if" and "only if" operate separately, it shouldn't surprise us that "if and only if" makes both

antecedent and consequent out of the claim it introduces. We can make P both antecedent and consequent this way:*

$(P \rightarrow Q) \& (Q \rightarrow P)$

There are other ways to produce conditionals, of course. In one of its senses, the word "provided" (and the phrase "provided that") works like the word "if" in introducing the antecedent of a conditional. "Moore will buy the car, provided the seller throws in a ton of spare parts" is equivalent to the same expression with the word "if" in place of "provided."

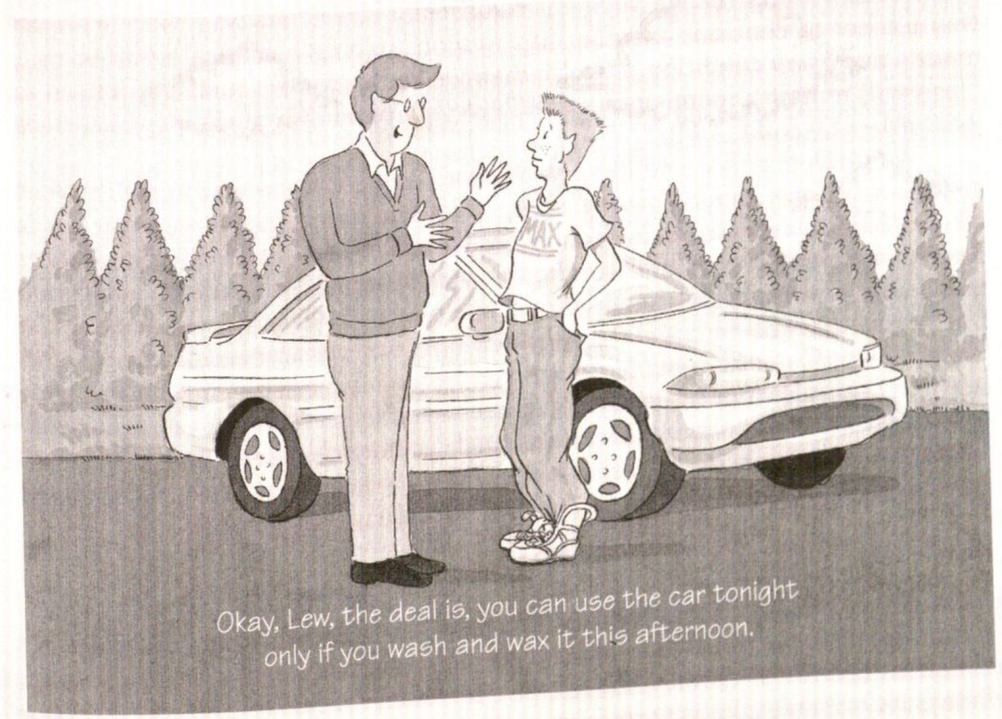
Necessary and Sufficient Conditions

Conditional claims are sometimes spelled out in terms of necessary and sufficient conditions. Consider this example:

The presence of oxygen is a necessary condition for combustion.

This tells us that we can't have combustion without oxygen, or "If we have combustion (C), then we must have oxygen (O)." Notice that *the necessary condition becomes the consequent of a conditional:* $C \rightarrow O$.

A sufficient condition *guarantees* whatever it is a sufficient condition for. Being born in the United States is a sufficient condition for U.S. citizenship—that's *all* one needs to be a U.S. citizen. *Sufficient conditions are expressed as the antecedents of conditional claims,* so we would say, "If Juan was born in the United States (B), then Juan is a U.S. citizen (C)": $B \rightarrow C$.



■ Comment: We often use "only if" when we mean to state both necessary and sufficient conditions, even though, literally speaking, it produces only the former. If Lew were a critical thinker, he'd check this deal more carefully before getting out the hose and bucket. See the text below.

*Many texts introduce a new symbol ("$P \leftrightarrow Q$") to represent "P if and only if Q." It works exactly like our version; i.e., it has the same truth table as "$(P \rightarrow Q) \& (Q \rightarrow P)$." Under some circumstances, the extra symbol provides some efficiencies, but for us it is unnecessary and would be merely something else to learn and remember.

On Language

Another "If" and "Only If" Confusion

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—A typical download caution

Presumably, they mean not "if" but "only if." Do you see why? In any case, this caution contains one heck of a weaseler (Chapter 5).

You should also notice the connection between "if" and "only if" on the one hand and necessary and sufficient conditions on the other. The word "if," by itself, introduces a sufficient condition; the phrase "only if" introduces a necessary condition. So the claim "X is a necessary condition for Y" would be symbolized "Y → X."

From time to time, one thing will be both a necessary and a sufficient condition for something else. For example, if Jean's payment of her dues to the National Truth-Functional Logic Society (NTFLS) guaranteed her continued membership (making such payment a sufficient condition) and there were no way for her to continue membership *without* paying her dues (making payment a necessary condition as well), then we could express such a situation as "Jean will remain a member of the NTFLS (M) if and only if she pays her dues (D)": (M → D) & (D → M).

We often play fast and loose with how we state necessary and sufficient conditions. A parent tells his daughter, "You can watch television only if you clean your room." Now, the youngster would ordinarily take cleaning her room as both a necessary and a sufficient condition for being allowed to watch television, and probably that's what a parent would intend by those words. But notice that the parent actually stated only a necessary condition; technically, he would not be going back on what he said if room cleaning turned out not to be sufficient for television privileges. Of course, he'd better be prepared for more than a logic lesson from his daughter in such a case, and most of us would be on her side in the dispute. But, literally, it's the necessary condition that the phrase "only if" introduces, not the sufficient condition.

"Unless"

Consider the claim "Paula will foreclose unless Quincy pays up." Asked to symbolize this, we might come up with ~Q → P because the original claim is equivalent to "If Quincy doesn't pay up, then Paula will foreclose." But there's an even simpler way to do it. Ask yourself, What is the truth table for ~Q → P? If you've gained familiarity with the basic truth tables by this time, you realize that it's the same as the table for P ∨ Q. And, as a matter of fact, you can treat the word "unless" exactly like the word "or" and symbolize it with a "∨".

"Either . . . Or"

Sometimes we need to know exactly where a disjunction begins; it's the job of the word "either" to show us. Compare the claims

Either P and Q or R

and

P and either Q or R.

These two claims say different things and have different truth tables, but the only difference between them is the location of the word "either"; without that word, the claim would be completely ambiguous. "Either" tells us that the disjunction begins with P in the first claim and Q in the second claim. So, we would symbolize the first (P & Q) ∨ R and the second P & (Q ∨ R).

The word "if" does much the same job for conditionals that "either" does for disjunctions. Notice the difference between

P and if Q then R

and

If P and Q then R.

"If" tells us that the antecedent begins with Q in the first example and with P in the second. Hence, the second must have P & Q for the antecedent of its symbolization.

In general, the trick to symbolizing a claim correctly is to pay careful attention to exactly what the claim says—and this often means asking yourself just exactly what would make this claim false (or true). Then, try to come up with a symbolization that says the same thing—that is false (or true) in exactly the same circumstances. There's no substitute for practice, so here's an exercise to work on.

Exercise 9-2

When we symbolize a claim, we're displaying its truth-functional structure. Show that you can figure out the structures of the following claims by symbolizing them. Use these letters for the first ten items:

P = Parsons signs the papers.
Q = Quincy goes (or will go) to jail.
R = Rachel files (or will file) an appeal.

Use the symbols ~, &, ∨, and →. We suggest that, at least at first, you make symbolization a two-stage process: First, replace simple parts of claims with letters; then, replace logical words with logical symbols, and add parentheses as required. We'll do an example in two stages to show you what we mean.

Example

If Parsons signs the papers, then Quincy will go to jail but Rachel will not file an appeal.

Stage 1: If P, then Q but ~R.
Stage 2: P → (Q & ~R)

▲ 1. If Parsons signs the papers then Quincy will go to jail, and Rachel will file an appeal.

▲ 2. If Parsons signs the papers, then Quincy will go to jail and Rachel will file an appeal.

3. If Parsons signs the papers and Quincy goes to jail then Rachel will file an appeal.

4. Parsons signs the papers and if Quincy goes to jail Rachel will file an appeal.

▲ 5. If Parsons signs the papers then if Quincy goes to jail Rachel will file an appeal.

6. If Parsons signs the papers Quincy goes to jail, and if Rachel files an appeal Quincy goes to jail.

7. Quincy goes to jail if either Parsons signs papers or Rachel files an appeal.

8. Either Parsons signs the papers or, if Quincy goes to jail, then Rachel will file an appeal.

9. If either Parsons signs the papers or Quincy goes to jail then Rachel will file an appeal.

10. If Parsons signs the papers then either Quincy will go to jail or Rachel will file an appeal.

For the next ten items, use the following letters:

C = My car runs well.
S = I will sell my car.
F = I will have my car fixed.

▲ 11. If my car doesn't run well, then I will sell it.

▲ 12. It's not true that, if my car runs well, then I will sell it.

13. I will sell my car only if it doesn't run well.

14. I won't sell my car unless it doesn't run well.

15. I will have my car fixed unless it runs well.

▲ 16. I will sell my car but only if it doesn't run well.

17. Provided my car runs well, I won't sell it.

18. My car's running well is a sufficient condition for my not having it fixed.

19. My car's not running well is a necessary condition for my having it fixed.

▲ 20. I will neither have my car fixed nor sell it.

Exercise 9-3

▲ Construct truth tables for the symbolizations you produced for Exercise 9-2. Determine whether any of them are truth-functionally equivalent to any others. (Answers to the items with triangles are provided in the answer section at the end of the book.)

TRUTH-FUNCTIONAL ARGUMENT PATTERNS (BRIEF VERSION)

This section is an alternative to the two sections that follow ("Truth-Functional Arguments" and "Deductions"). Those instructors who want to go into the subject of truth-functional logic in some depth should skip this section and cover the next two instead; they constitute a fairly thorough treatment and a concise introduction to symbolic logic.

For those who want briefer and more practical coverage of the subject, this section should suffice.

Three Common Valid Argument Patterns

Three forms of truth-functional argument are almost ubiquitous; they appear so frequently, and we are so accustomed to them, that we often make use of them almost without thinking. But it is important to understand and be able to recognize them because there are imposters that, because of superficial similarities, may look like valid argument patterns but are not.

First, we should recall what it means for an argument to be *valid*. To be valid, the truth of the argument's premises must guarantee the truth of its conclusion. Another way to say this is that it is impossible for the premises to be true while the conclusion is false. If it is even possible for the premises to be true without the conclusion being true, the argument is *invalid*.

Modus Ponens

Modus ponens ("in the affirmative mode," more or less) is a two-premise valid argument form, one premise of which is a conditional and the other of which is the antecedent of that conditional. The conclusion of the argument is the consequent of the conditional. (See page 298 if you need refreshing on the meanings of these terms.) So all cases of modus ponens fit this pattern:

If P then Q.
P.
―――――
Therefore Q.

You can see that one premise is the conditional: "If P then Q," and the other premise is the antecedent of that conditional: "P." The conclusion, "Q," is the consequent of the conditional. Every argument that has this form is valid. For example,

Example 1:

If the referee scored the fight in favor of Madderly, then Madderly wins the decision.
The referee did score the fight in favor of Madderly.
―――――
Therefore, Madderly wins the decision.