One initially puzzling aspect of this solution is that it allocates the initial inventory of 200 units to meet demand in period 4 rather than in period 1. The explanation is that multiple optimal solutions exist and this solution is only one of them. However, all solutions result in the same production and anticipation inventory plans derived as follows:

PRODUCTION PLAN

Period	Regular-time Production	Overtime Production	Subcontracting	Total
1	6,500	—	—	6,500
2	6,500	400	—	6,900
3	6,500	1,300	—	7,800
4	8,000	1,600	1,000	10,600
5	7,000	—	—	7,000
6	4,400	—	—	4,400

ANTICIPATION INVENTORY

Period	Beginning Inventory Plus Total Production Minus Demand	Anticipation (Ending) Inventory
1	200 + 6,500 − 5,000	1,700
2	1,700 + 6,900 − 6,000	2,600
3	2,600 + 7,800 − 9,000	1,400
4	1,400 + 10,600 − 12,000	0
5	0 + 7,000 − 7,000	0
6	0 + 4,400 − 4,000	400

Discussion Questions

1. A particular linear programming maximization problem has the following less-than-or-equal-to constraints: (1) raw materials, (2) labor hours, and (3) storage space. The optimal solution occurs at the intersection of the raw materials and labor hours constraints, so those constraints are binding. Management is considering whether to authorize overtime. What useful information could the linear programming solution provide to management in making this decision? Suppose a warehouse becomes available for rent at bargain rates. What would management need to know in order to decide whether to rent the warehouse? How could the linear programming model be helpful?
2. Linear programming and the transportation method promise optimal solutions. However, wise managers sometimes, after seeing the optimal results such as in Figures D.8 or D.12, might decide to implement different plans. How do you explain such decision making?

Problems

The OM Explorer and POM for Windows software is available to all students using the 10th edition of this textbook. Go to **www.pearsonhighered.com/krajewski** to download these computer packages. Check with your instructor on how best to use it. For linear programming problems with more than two variables, and for transportation problems, use POM for Windows to solve your model formulations. The emphasis in these cases is on modeling problems and interpreting computer output.

1. The Sports Shoe Company is a manufacturer of basketball and football shoes. The manager of marketing must decide the best way to spend advertising resources. Each football team sponsored requires 120 pairs of shoes. Each basketball team requires 32 pairs of shoes. Football coaches receive $300,000 for shoe sponsorship, and basketball coaches receive $1,000,000. The manager's promotional budget is $30,000,000. The company has a limited supply (4 liters, or 4,000 cubic centimeters) of flubber, a rare and costly compound used in promotional athletic shoes. Each pair of basketball shoes requires 3 cc of flubber, and each pair of football shoes requires 1 cc. The manager wants to sponsor as many basketball and football teams as resources will allow.
   a. Create a set of linear equations to describe the objective function and the constraints.

b. Use graphic analysis to find the visual solution.

c. What is the maximum number of each type of team that the company can sponsor?

2. A business student at Nowledge College must complete a total of 65 courses to graduate. The number of business courses must be greater than or equal to 23. The number of nonbusiness courses must be greater than or equal to 20. The average business course requires a textbook costing \$60 and 120 hours of study. Nonbusiness courses require a textbook costing \$24 and 200 hours of study. The student has \$3,000 to spend on books.

a. Create a set of linear equations to describe the objective function and the constraints.

b. Use graphic analysis to find the visual solution.

c. What combination of business and nonbusiness courses minimizes total hours of study?

d. Identify the slack or surplus variables.

3. In Problem 2, suppose that the objective is to minimize the cost of books and that the student's total study time is limited to 12,600 hours.

a. Use graphic analysis to determine the combination of courses that minimizes the total cost of books.

b. Identify the slack or surplus variables.

4. Mile-High Microbrewery makes a light beer and a dark beer. Mile-High has a limited supply of barley, limited bottling capacity, and a limited market for light beer. Profits are \$0.20 per bottle of light beer and \$0.50 per bottle of dark beer.

a. The following table shows resource availability of products at the Mile-High Microbrewery. Use the graphic method of linear programming to maximize profits. How many bottles of each product should be produced per month?

	PRODUCT		
Resource	**Light Beer (x_1)**	**Dark Beer (x_2)**	**Resource Availability (per month)**
Barley	0.1 gram	0.6 gram	2,000 grams
Bottling	1 bottle	1 bottle	6,000 bottles
Market	1 bottle	—	4,000 bottles

b. Identify any constraints with slack or surplus.

5. The plant manager of a plastic pipe manufacturer has the opportunity to use two different routings for a particular type of plastic pipe. Routing 1 uses extruder A, and routing 2 uses extruder B. Both routings require the same melting process. The following table shows the time requirements and capacities of these processes:

	TIME REQUIREMENTS (HR/100 FT)		
Process	**Routing 1**	**Routing 2**	**Capacity (hr)**
Melting	1	1	45
Extruder A	3	0	90
Extruder B	0	1	160

Each 100 feet of pipe processed on routing 1 uses 5 pounds of raw material, whereas each 100 feet of pipe processed on routing 2 used only 4 pounds. This difference results from differing scrap rates of the extruding machines. Consequently, the profit per 100 feet of pipe processed on routing 1 is \$60 and on routing 2 is \$80. A total of 200 pounds of raw material is available.

a. Create a set of linear equations to describe the objective function and the constraints.

b. Use graphic analysis to find the visual solution.

c. What is the maximum profit?

6. A manufacturer of textile dyes can use two different processing routings for a particular type of dye. Routing 1 uses drying press A, and routing 2 uses drying press B. Both routings require the same mixing vat to blend chemicals for the dye before drying. The following table shows the time requirements and capacities of these processes:

	TIME REQUIREMENTS (HR/KG)		
Process	**Routing 1**	**Routing 2**	**Capacity (hr)**
Mixing	2	2	54
Dryer A	6	0	120
Dryer B	0	8	180

Each kilogram of dye processed on routing 1 uses 20 liters of chemicals, whereas each kilogram of dye processed on routing 2 uses only 15 liters. The difference results from differing yield rates of the drying presses. Consequently, the profit per kilogram processed on routing 1 is \$50 and on routing 2 is \$65. A total of 450 liters of input chemicals is available.

a. Write the constraints and objective function to maximize profits.

b. Use the graphic method of linear programming to find the optimal solution.

c. Identify any constraints with slack or surplus.

7. The Trim-Look Company makes several lines of skirts, dresses, and sport coats. Recently, a consultant suggested that the company reevaluate its South Islander line and allocate its resources to products that would maximize contribution to profits and to overhead. Each product requires the same polyester fabric and must pass through the cutting and sewing departments. The following data were collected for the study:

	PROCESSING TIME (HR)		
Product	**Cutting**	**Sewing**	**Material (yd)**
Skirt	1	1	1
Dress	3	4	1
Sport coat	4	6	4

The cutting department has 100 hours of capacity, sewing has 180 hours of capacity, and 60 yards of material are available. Each skirt contributes \$5 to profits and overhead; each dress, \$17; and each sport coat, \$30.

a. Specify the objective function and constraints for this problem.

b. Use a computer package such as POM for Windows to solve the problem.

8. Consider Problem 7 further.

a. How much would you be willing to pay for an extra hour of cutting time? For an extra hour of sewing time? For an extra yard of material? Explain your response to each question.

b. Determine the range of right-hand-side values over which the shadow price would be valid for the cutting constraint and for the material constraint.

9. Polly Astaire makes fine clothing for big and tall men. A few years ago Astaire entered the sportswear market with the Sunset line of shorts, pants, and shirts. Management wants to make the amount of each product that will maximize profits. Each type of clothing is routed through two departments, A and B. The relevant data for each product are as follows:

	PROCESSING TIME (HR)		
Product	**Department A**	**Department B**	**Material (yd)**
Shirts	2	1	2
Shorts	2	3	1
Pants	3	4	4

Department A has 120 hours of capacity, department B has 160 hours of capacity, and 90 yards of material are available. Each shirt contributes $10 to profits and overhead; each pair of shorts, $10; and each pair of pants, $23.

a. Specify the objective function and constraints for this problem.

b. Use a computer package such as POM for Windows to solve the problem.

c. How much should Astaire be willing to pay for an extra hour of department A capacity? How much for an extra hour of department B capacity? For what range of right-hand values are these shadow prices valid?

10. The Butterfield Company makes a variety of knives. Each knife is processed on four machines. The processing times required are as follows. Machine capacities (in hours) are 1,500 for machine 1; 1,400 for machine 2; 1,600 for machine 3; and 1,500 for machine 4.

	PROCESSING TIME (HR)			
Knife	**Machine 1**	**Machine 2**	**Machine 3**	**Machine 4**
A	0.05	0.10	0.15	0.05
B	0.15	0.10	0.05	0.05
C	0.20	0.05	0.10	0.20
D	0.15	0.10	0.10	0.10
E	0.05	0.10	0.10	0.05

Each product contains a different amount of two basic raw materials. Raw material 1 costs $0.50 per ounce, and raw material 2 costs $1.50 per ounce. There are 75,000 ounces of raw material 1 and 100,000 ounces of raw material 2 available.

	REQUIREMENTS (OZ/UNIT)		
Knife	**Raw Material 1**	**Raw Material 2**	**Selling Price ($/unit)**
A	4	2	15.00
B	6	8	25.50
C	1	3	14.00
D	2	5	19.50
E	6	10	27.00

a. If the objective is to maximize profit, specify the objective function and constraints for the problem. Assume that labor costs are negligible.

b. Solve the problem with a computer package such as POM for Windows.

11. The Nutmeg Corporation produces three different products, each in a 1-pound can: Almond-Lovers Mix, Walnut-Lovers Mix, and the Thrifty Mix. Three types of nuts are used in Nutmeg's products: almonds, walnuts, and peanuts. Nutmeg currently has 350 pounds of almonds, 150 pounds of walnuts, and 1000 pounds of peanuts. Each of Nutmeg's products must contain a certain percentage of each type of nut, as shown in the following table. The table also shows the revenue per can as well as the cost per pound to purchase nuts.

	PERCENTAGE REQUIREMENTS PER CAN			
	Almonds	**Walnuts**	**Peanuts**	**Revenue per can**
Almond-Lovers Mix	80%	20%	0%	$8.00
Walnut-Lovers Mix	20%	80%	0%	$10.00
Thrifty Mix	10%	10%	80%	$4.50
Cost per pound	$4.50	$6.00	$3.00	

a. Given Nutmeg's current stock of nuts, how many cans of each product should be produced to maximize revenue?

b. Does the solution you developed in part a change if Nutmeg is interested in maximizing contribution margin (defined as revenue per unit – raw material cost)?

c. If 50 additional pounds of walnuts became available, how would your contribution-margin maximizing solution from part (b) change?

12. A problem often of concern to managers in processing industries is blending. The Nutmeg Corporation, from Problem 11, is considering a new product it intends to sell to active and health-concerned adults. This new product will be a 4-ounce package of nuts that conforms to specific heath requirements. First, the 4 ounce package can contain no more than 720 calories. It must deliver at least 20 grams of protein. Finally, the package must provide at least 15 percent of the adult daily requirement (ADR) of calcium and 20 percent of the ADR of iron. Nutmeg would like to use only almonds, walnuts, and peanuts in this new