

1. Use mathematical induction to prove that $\overline{z^n} = \overline{z}^n$ for all $n \geq 1$, where z is a complex number.
2. Let $p(x) = 6x^3 + kx^2 - (6 + 5i)x + 4 + 2i$, where k is an unknown complex number. It is given that $z_0 = -\frac{1}{2} - \frac{1}{2}i$ is a zero of $p(x)$.
 - (a) Find the value of k .
 - (b) Is $\overline{z_0}$ also a zero of $p(x)$? Justify your answer.
 - (c) Does your answer in (b) contradict Theorem 3.5 on Page 28 of the textbook? Explain.
 - (d) Find all zeros of $p(x)$.
3. Given that $2 - 3i$ is a zero of $f(x) = 2x^5 - 9x^4 + 32x^3 - 24x^2 + 38x - 39$, find all zeros of $f(x)$.

4. For each of the following polynomials:

- (a) Use Descartes' rules of signs to state the number of possible positive and negative zeros of the polynomial.
- (b) Use the bounds theorem to find bounds for zeros of the polynomial.
- (c) Taking the results of (a) and (b) into account, use the rational root theorem to list all possible rational zeros of the polynomial.
- (d) Find all roots of the equation.

(1) $p(x) = 24x^5 + 32x^4 - 26x^3 - 22x^2 + 6x + 4$

(2) $p(x) = 4x^5 + 20x^4 + 37x^3 + 52x^2 + 43x + 12$

5. Consider the polynomial $f(x) = 2015^2 + \sum_{n=1}^{2015} (-1)^n n^2 x^n$

- (a) Show that $f(x)$ must have at least one positive real root.
- (b) Show that $f(x)$ has no negative real roots.
- (c) Show that if x is any root of $f(x)$, then $|x| < 2$.

1.

Sketch the region bounded between the curves $y = \sin x$, $y = \cos x$, $x = 0$, $x = 2\pi$. Find the area of this region.

2.

Let R be the region bounded by the graphs of $y = x$ and $x = 4y - y^2$. Which is greater, the volume of the solid generated when R is revolved about the x -axis or the y -axis?

3.

Sketch the region R bounded by the curves $y = x^2$ and $y = \sqrt{x}$ between $x = 0$ and $x = 1$. Set up, but **DO NOT EVALUATE**, integrals that can be used to find the volume of the solid generated if R is revolved about the x -axis

(a) using cylindrical shells,

(b) using disks or washers.

4.

Evaluate.

(a) $\int (x^2 - 2x)e^{kx} dx$ (hint: use integration by part)

(b) $\int e^{2x} \sin 3x dx$ (hint: use integration by part)

(c) $\int \sqrt{\sin x} \cos^3 x dx$

(d) $\int \sin^6 x dx$.