**Math 220**

Written Homework 1

Some instructions.

- Set up each problem. Use diagrams/pictures of the regions and/or solids involved. For volume and work problems, sketch the slices (slabs) used, as in class.

- Show your work, of course, but beyond that: Write neatly and clearly, and give all necessary explanation. Try to write so that a good calculus student, seeing the problem for the first time, could understand everything in your solution.

- Give exact answers, reasonably simplified. (Sometimes a decimal approximation the answer is of interest, and can help you to see whether your answer seems to have the right order of magnitude, but the exact answer is required for this assignment.)

---

1. Revolving the semicircle $y = \sqrt{r^2 - x^2}$ about the x-axis results in a sphere of radius r. Use the method in Section 6.5 to verify that the surface area of the sphere is $4\pi r^2$.

2. Let R be the region in the first quadrant under the curve $y = \sin x$ over the interval $0 \le x \le \pi$. Find the volume of the solid obtained by revolving R around the line $y = -1$.

   [Hint: You should find that your integrand involves $\sin^2 x$. I suggest using the trig identity $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$ to make it into something you can evaluate.]

3. The diagram shows coordinate axes with units in feet. The region in the first quadrant under the curve $y = 5\sqrt{4-x}$ forms the front end of a water tank as shown in the diagram. (The bottom and left side of the tank are rectangles 18 ft long, and the cross-sections of the tank parallel to the xy-plane are the same shape as the front end.)

   There is water in the tank; the depth of the water is H feet.

   (a) Sketch a horizontal cross section of the tank, representing a thin slab of water. Explain how to find the volume of your slab, and find this volume. Next, explain how to find the work that would be done to lift that slab of water to the top of the tank, and find that work. (Remember the weight density of water: $62.4\,\text{lbs/ft}^3$.)

   (b) Use your work in (a) to find the work required to pump all the water to the top of the tank. (Your answer will be in terms of H.) If the tank is completely full of water, how much work is required to pump it all to the top?
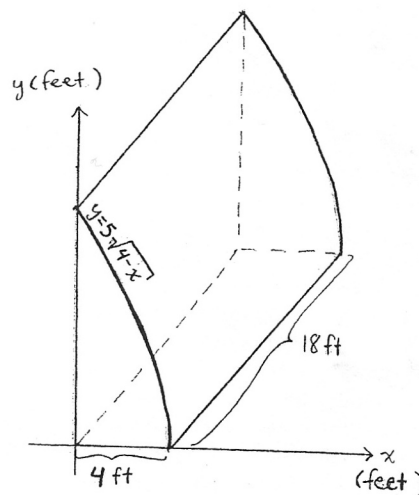
4. Find the volume of the solid whose base is the circle $x^2 + y^2 = 1$ and whose cross sections perpendicular to the x-axis are squares.

5. Let R be the region under the sine curve as in problem #2. Find the centroid of R.