9-19. On May 21, 1927, Charles Lindbergh landed at Le Bourget Field in Paris, completing his famous transatlantic solo flight. The preparation period prior to his flight was quite hectic and time was critical, since several other famous pilots of the day were also planning transatlantic flights. Once Ryan Aircraft was contracted to build the *Spirit of St. Louis*, it took only a little over 2½ months to construct the plane and fly it to New York for the takeoff. If CPM/PERT had been available to Charles Lindbergh, it no doubt would have been useful in helping him plan this project. Use your imagination and assume that a CPM/PERT network, as shown in the figure below, with the following estimated activity times, was developed for the flight.

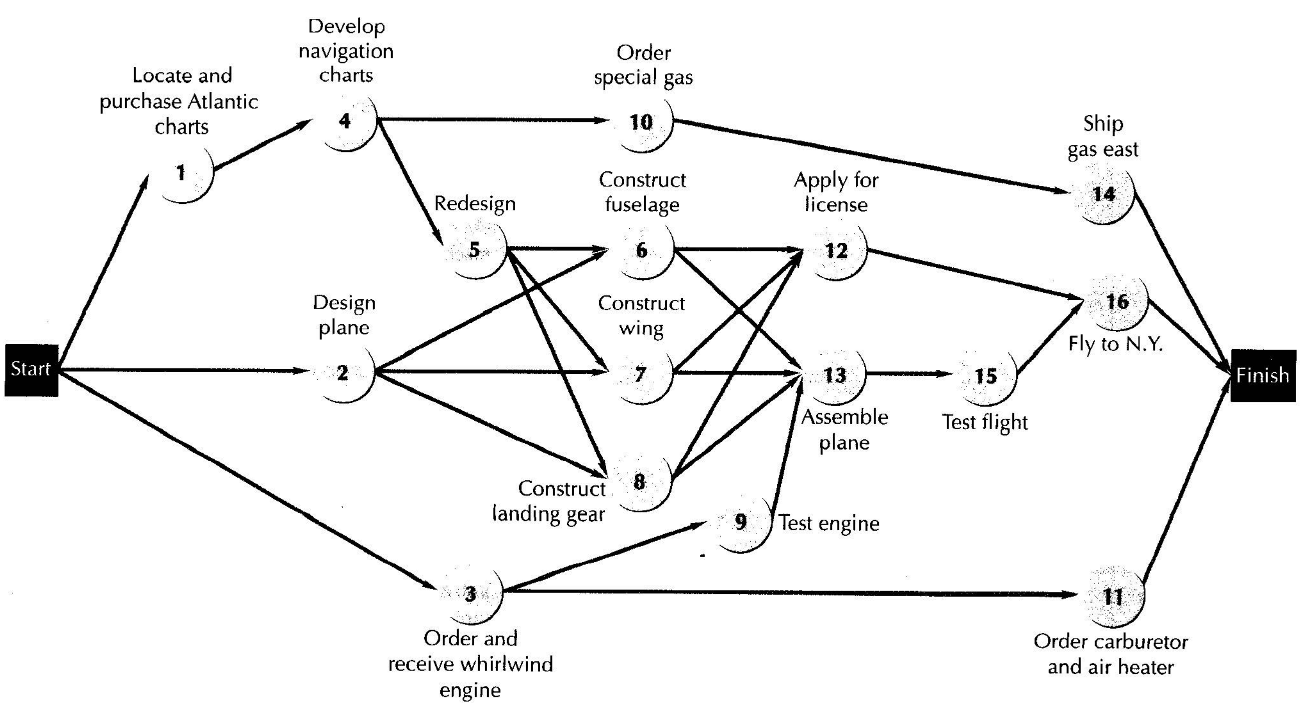
	Time Estimates (days)		
Activity	a	m	b
1	1	3	5
2	4	6	10
3	20	35	50
4	4	7	12
5	2	3	5
6	8	12	25
7	10	16	21
8	5	9	15
9	1	2	2
10	6	8	14
11	5	8	12
12	5	10	15
13	4	7	10
14	5	7	12
15	5	9	20
16	1	3	7

Determine the expected project duration and variance and the probability of completing the project in 67 days.

9-20. RusTech Tooling, Inc., is a large job shop operation that builds machine tools and dies to manufacture parts for specialized items. The company bids primarily on contracts for government-related activities to produce parts for things such as military aircraft, weapons systems, and the space program. The company is bidding on a contract to produce a component part for the fuselage assembly in a new space shuttle. A major criterion for selecting the winning bid besides low cost is the time required to produce the part. However, if the company is awarded the contract it will be held strictly to the completion date specified in the bid, and any delays will result in severe financial penalties. In order to determine the project completion time to put in its bid, the company has identified the project activities, precedence relationships, and activity times shown in the following table:

Activity	Activity Predecessor	Time Estimates (Weeks) a	m	b
a	—	3	5	9
b	a	2	5	8
c	a	1	4	6
d	a	4	6	10
e	b	2	8	11
f	b	5	9	16
g	c	4	12	20
h	c	6	9	13
i	d	3	7	14
j	d	8	14	22
k	f, g	9	12	20
l	h, i	6	11	15
m	e	4	7	12
n	j	3	8	16
o	n	5	8	10

If RusTech, Inc., wants to be 90% certain that it can deliver the part without incurring a penalty, what time frame should it specify in the bid?



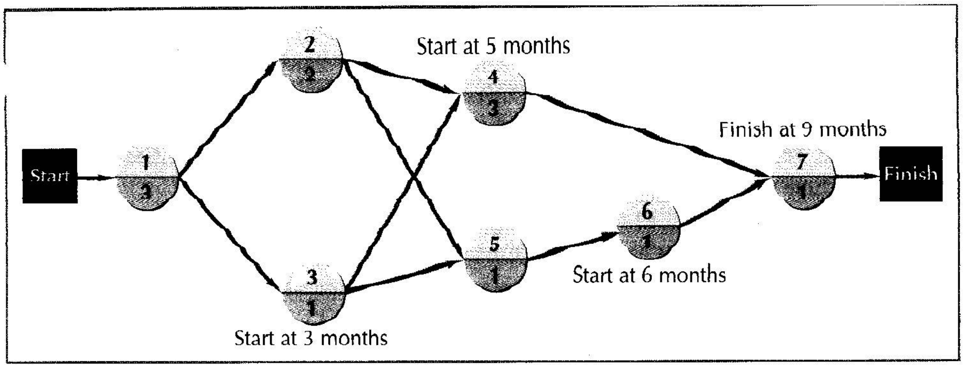


Figure 9.9
Activity Start Times

Now consider the activity following node 4. Using the same logic as before, activity 7 cannot start until after eight months (five months at node 4 plus the three months required by activity 4) or after seven months. Because all activities preceding node 7 must be completed before activity 7 can start, the earliest this can occur is eight months. Adding one month for activity 7 to the start time at node 7 gives a project duration of nine months. This is the time of the longest path in the network—the critical path.

This brief analysis demonstrates the concept of a critical path and the determination of the minimum completion time of a project. However, this was a cumbersome method for determining a critical path. Next, we discuss a mathematical approach to scheduling the project activities and determining the critical path.

ACTIVITY SCHEDULING

In our analysis of the critical path, we determined the earliest time that each activity could be finished. For example, we found that the earliest time activity 4 could start was five months. This time is referred to as the **earliest start time**, and it is expressed symbolically as **ES**. In order to show the earliest start time on the network as well as some other activity times, we will develop in

Earliest start time (ES): the earliest time an activity can start.

ALONG THE SUPPLY CHAIN

The Terminal 5 Project at Heathrow Airport

British Airport Authority's (BAA) terminal five (T5) at Heathrow Airport in London, completed in 2008, was one of Europe's largest construction projects, taking 5.5 years and 60,000 people to complete. Located between two runways in a space equal in size to Hyde Park in London, T5 has the largest single-span roof in Europe, made up of six sections requiring 10 months to lift into place, and 11 miles of baggage conveyor belt. The facility, which cost £4.3 billion to build, increased Heathrow's capacity by 35 million annual passengers. For the first time in a project of this size and complexity, off-site prefabrication of over 2800 component modules was used extensively. Assembled components were transported to the building site where they were bolted together, thereby reducing construction site time and disruption, and construction traffic at the airport. A web-based system enabled off-site project module manufacturers in Dover and Scotland to be coordinated though a *virtual factory.* The supply of material, equipments, and workflows were all monitored through this one integrated data system. All project participants could see when modules were in production, completed, delivered, and stored. The system was especially important for planning and coordinating deliveries between suppliers because of the high volume of deliveries and the limited on-site space involved. The collaborative nature of this project combined with the "virtual factory" web-based computer system, facilitated the performance of the integrated project teams, resulting in a reduction in construction times, a safer working environment, and better quality.

Discuss some of the unique problems that you think might exist for a project like this one that involves a facility with daily, heavy public usage.

Source: John Summers, "The Virtual Factory," *Quality World* 31 (10); (October 2005), pp. 24–28; and Patricia Curmi, "Terminal Velocity," *Quality World* 33, (8); (August 2007), pp. 17–21.

and the earliest finish time is

$$\begin{aligned} EF &= ES + t \\ &= 5 + 3 \\ &= 8 \text{ months} \end{aligned}$$

All the remaining earliest start and finish times are computed similarly. Notice in Figure 9.11 that the earliest finish time for activity 7, the last activity in the network, is nine months, which is the total project duration, or critical path time.

Companions to the earliest start and finish are the **latest start** and **latest finish times**, **LS** and **LF**. The latest start time is the latest time an activity can start without delaying the completion of the project beyond the project critical path time. For our example, the project completion time (and earliest finish time) at node 7 is nine months. Thus, the objective of determining latest times is to see how long each activity can be delayed without the project exceeding nine months.

Latest start time (LS): the latest time an activity can start without delaying critical path time.

Latest finish time (LF): the latest time an activity can be completed and still maintain the project critical path time.

In general, the latest start and finish times for an activity are computed according to the following formulas:

$$\begin{aligned} LS &= LF - t \\ LF &= \min \text{ (LS immediate following activities)} \end{aligned}$$

Whereas a forward pass through the network is made to determine the earliest times, the latest times are computed using a **backward pass**. We start at the end of the network at node 7 and work backward, computing the latest times for each activity. Since we want to determine how long each activity in the network can be delayed without extending the project time, the latest finish time at node 7 cannot exceed the earliest finish time. Therefore, the latest finish time at node 7 is nine months. This and all other latest times are shown in Figure 9.12.

Backward pass: determines latest activity times by starting at the end of a CPM/PERT network and working forward.

Starting at the end of the network, the critical path time, which is also equal to the earliest finish time of activity 7, is nine months. This automatically becomes the latest finish time for activity 7, or

$$LF = 9 \text{ months}$$

Using this value, the latest start time for activity 7 is

$$\begin{aligned} LS &= LF - t \\ &= 9 - 1 \\ &= 8 \text{ months} \end{aligned}$$

The latest finish time for activity 6 is the minimum of the latest start times for the activities following node 6. Since activity 7 follows node 6, the latest finish time is

$$\begin{aligned} LF_6 &= \min \text{ (LS following activities)} \\ &= 8 \text{ months} \end{aligned}$$

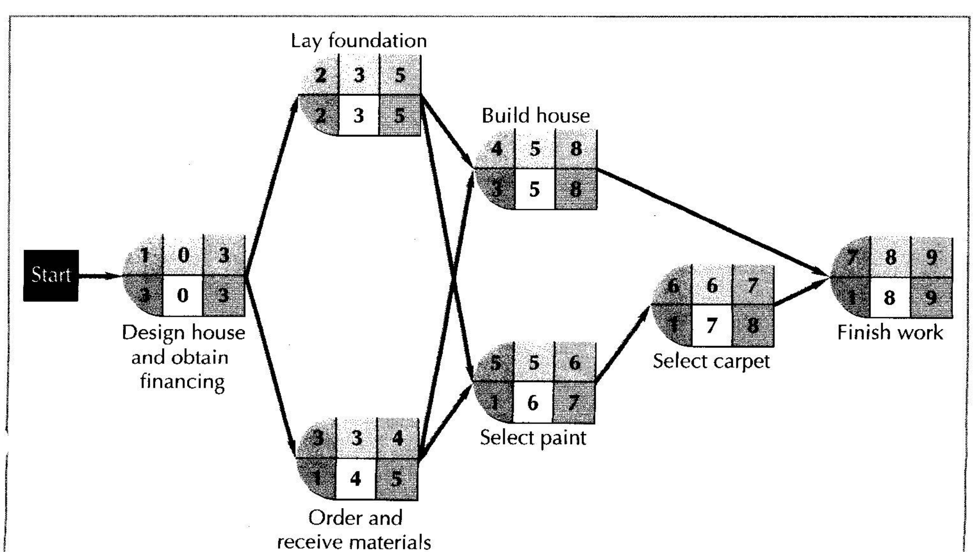


Figure 9.12
Latest Activity Start and Finish Times

TABLE 9.1 Activity Slack

Activity	LS	ES	LF	EF	Slack S
*1	0	0	3	3	0
*2	3	3	5	5	0
3	4	3	5	4	1
*4	5	5	8	8	0
5	6	5	7	6	1
6	7	6	8	7	1
*7	8	8	9	9	0

*Critical path.

of the Gantt chart in Figure 9.4. Slack is the amount of time an activity can be delayed without affecting the overall project duration. In effect, it is extra time available for completing an activity.

Slack, S, is computed using either of the following formulas:

$$S = \text{LS} - \text{ES}$$

or

$$S = \text{LF} - \text{EF}$$

For example, the slack for activity 3 is

$$\begin{aligned} S &= \text{LS} - \text{ES} \\ &= 4 - 3 \\ &= 1 \text{ month} \end{aligned}$$

If the start of activity 3 were delayed for one month, the activity could still be completed by month 5 without delaying the project completion time. The slack for each activity in our example project network is shown in Table 9.1. Table 9.1 shows there is no slack for the activities on the critical path (marked with an asterisk); activities not on the critical path have slack.

Notice in Figure 9.12 that activity 3 can be delayed one month and activity 5 that follows it can be delayed one more month, but then activity 6 cannot be delayed at all even though it has one month of slack. If activity 3 starts late at month 4 instead of month 3, then it will be completed at month 5, which will not allow activity 5 to start until month 5. If the start of activity 5 is delayed one month, then it will be completed at month 7, and activity 6 cannot be delayed at all without exceeding the critical path time. The slack on these three activities is called *shared slack*. This means that the sequence of activities 3–5–6 can be delayed two months jointly without delaying the project, but not three months.

Slack is beneficial to the project manager because it enables resources to be temporarily diverted from activities with slack and used for other activities that might be delayed for various reasons or for which the time estimate has proved to be inaccurate.

The times for the network activities are simply estimates, for which there is usually not a lot of historical basis (since projects tend to be unique undertakings). As such, activity time estimates are subject to quite a bit of uncertainty. However, the uncertainty inherent in activity time estimates can be reflected to a certain extent by using probabilistic time estimates instead of the single, deterministic estimates we have used so far.

Probabilistic Activity Times

Probabilistic time estimates reflect uncertainty of activity times.

In the project network for building a house in the previous section, all activity time estimates were single values. By using only a single activity time estimate, we are, in effect, assuming that activity times are known with certainty (i.e., they are deterministic). For example, in Figure 9.8, the time