

$$\begin{aligned}
 Z &= \frac{x - \mu}{\sigma} \\
 &= \frac{28 - 24}{\sqrt{5}} \\
 &= 1.79
 \end{aligned}$$

0.4633; thus,

$$P(x \leq 28) = 0.9633$$

QUESTIONS

- 9-1. Why is CPM/PERT a popular and widely applied project scheduling technique?
- 9-2. What is the purpose of a CPM/PERT network?
- 9-3. Why are dummy activities used in a CPM/PERT network?
- 9-4. What is the critical path, and what is its importance in project planning?
- 9-5. What is slack, and how is it computed?
- 9-6. How are the mean activity times and activity variances computed in probabilistic CPM/PERT analysis?
- 9-7. How is total project variance determined in CPM/PERT analysis?
- 9-8. What is the purpose of project crashing analysis?
- 9-9. Describe the process of manually crashing a project network.
- 9-10. Which method for determining activity time estimates, deterministic or probabilistic, do you perceive to be preferable? Explain.
- 9-11. Explain how a Gantt chart differs from a CPM/PERT network, and indicate the advantage of the latter.
- 9-12. Discuss the relationship of direct and indirect costs in project management.
- 9-13. Describe the limitations and disadvantages of CPM/PERT.

INTERNET EXERCISES

WEBLINKS

- 9-14. Describe the difference between activity-on-node and activity-on-arrow project networks.
- 9-15. Identify and briefly describe the major elements of project management.
- 9-16. Select an everyday "project" you are familiar with such as a class project, preparing a meal, making a pizza, repairing your car. Develop a list of the activities, a CPM/PERT network (with time estimates), and a work breakdown schedule for the project.
- 9-17. Prepare a WBS for a spaghetti with meatballs dinner that includes a Caesar salad, a loaf of Italian bread, and wine. (Include the different components of the dinner at the upper level and the various detailed work activities required by each component at the lower level.)
- 9-18. Write a paper summarizing an actual project reported on in the magazine *PM Network*.
- 9-19. Describe and discuss the cultural differences between the United States and a country of your choice that might affect the management of a project in this foreign country.

PROBLEMS

WILEYPLUS

- 9-1. Construct a Gantt chart for the project described by the following set of activities, and indicate the project completion time:

Activity	Activity Predecessor	Time (Weeks)
1	—	5
2	—	4
3	1	3
4	2	6

- 9-2. Construct a Gantt chart for the project described by the following set of activities, and indicate the project completion time and the available slack for each activity:

Activity	Activity Predecessor	Time (weeks)
1	—	3
2	—	7
3	1	2
4	2	5
5	2	6
6	4	1
7	5	4

- 9-3. Use the project activities that follow to determine the following:
 - a. Construct a Gantt chart; indicate the project completion time and slack for each activity.
 - b. Construct the CPM/PERT network, compute the length of each path in the network, and indicate the critical path.

Activity	Activity Predecessor	Time (weeks)
1	—	4
2	—	7
3	1	8
4	1	3
5	2	9
6	3	5
7	3	2
8	4, 5, 6	6
9	2	5

- 9-4. Construct a network from the information in the following table and identify all the paths in the network, compute the length of each, and indicate the critical path.

period prior to his flight was quite hectic and time was critical, since several other famous pilots of the day were also planning transatlantic flights. Once Ryan Aircraft was contracted to build the *Spirit of St. Louis*, it took only a little over 2½ months to construct the plane and fly it to New York for the takeoff. If CPM/PERT had been available to Charles Lindbergh, it no doubt would have been useful in helping him plan this project. Use your imagination and assume that a CPM/PERT network, as shown in the figure below, with the following estimated activity times, was developed for the flight.

bids primarily on contracts for government-related activities to produce parts for things such as military aircraft, weapons systems, and the space program. The company is bidding on a contract to produce a component part for the fuselage assembly in a new space shuttle. A major criterion for selecting the winning bid besides low cost is the time required to produce the part. However, if the company is awarded the contract it will be held strictly to the completion date specified in the bid, and any delays will result in severe financial penalties. In order to determine the project completion time to put in its bid, the company has identified the project activities, precedence relationships, and activity times shown in the following table:

Time Estimates (days)

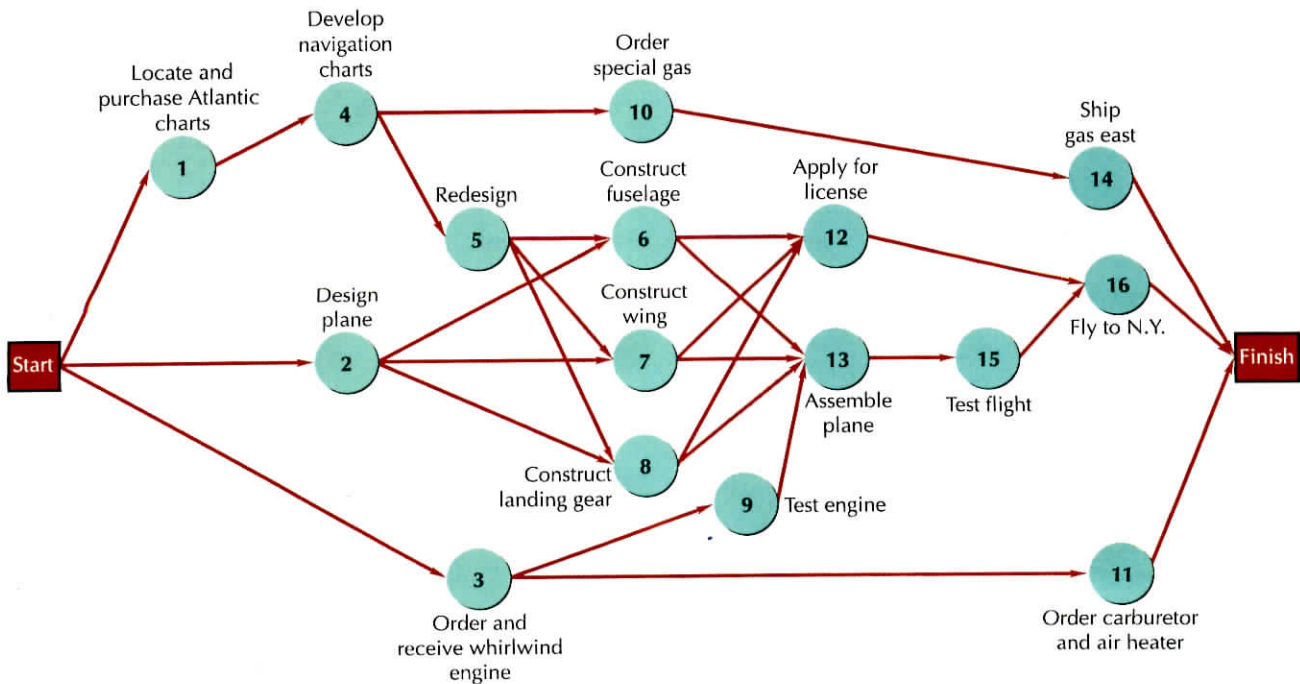
Activity	a	m	b
1	1	3	5
2	4	6	10
3	20	35	50
4	4	7	12
5	2	3	5
6	8	12	25
7	10	16	21
8	5	9	15
9	1	2	2
10	6	8	14
11	5	8	12
12	5	10	15
13	4	7	10
14	5	7	12
15	5	9	20
16	1	3	7

Time Estimates (Weeks)

Activity	Activity Predecessor	a	m	b
a	—	3	5	9
b	a	2	5	8
c	a	1	4	6
d	a	4	6	10
e	b	2	8	11
f	b	5	9	16
g	c	4	12	20
h	c	6	9	13
i	d	3	7	14
j	d	8	14	22
k	f, g	9	12	20
l	h, i	6	11	15
m	e	4	7	12
n	j	3	8	16
o	n	5	8	10

Determine the expected project duration and variance and the probability of completing the project in 67 days.

If RusTech, Inc., wants to be 90% certain that it can deliver the part without incurring a penalty, what time frame should it specify in the bid?



ALONG THE SUPPLY CHAIN

Innovative "Cloud" Projects at Google, Facebook, and Apple

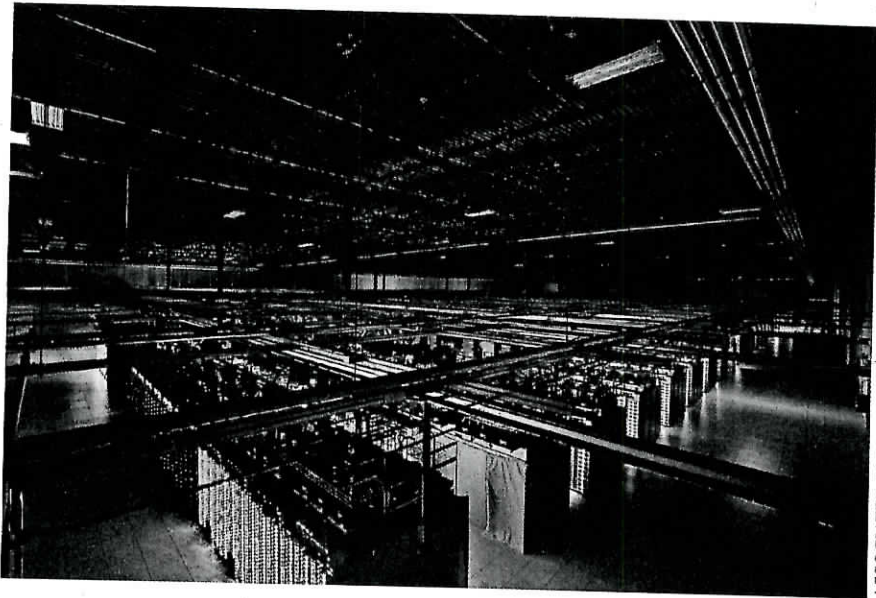
In recent years, Apple, Facebook, and Google have all opened new state-of-the-art data centers in North Carolina to support their cloud computing activities. Apple's new data center costing \$1 billion encompasses a half million square-foot facility; Google's new building that supports over 55,200 servers cost \$600 million; and Facebook's new 300,000 square-foot data center cost \$400 million.

Facebook, the social networking service and website with almost a billion active users, undertook its project to move 1 terabyte (i.e., 1 trillion bytes) of data from the company's old data center that was approaching capacity to a new warehouse. The project had to occur seamlessly without disrupting the service for Facebook users and the revenue-generating internal work groups that rely on the network, and it had to be completed quickly, within a little over three months from conception. Moving a terabyte of data, which is the equivalent of 250 billion "likes" on Facebook, in a small amount of time without disrupting the website was a daunting task for the project team. The project was implemented in two stages. The first stage involved constructing and setting up the infrastructure. Team members had to choose and test the hardware, make selections, and have it all shipped, installed, and stacked with the necessary wiring, cooling and pipes, which involved coordination between the project team, the hardware team, and vendors. Vendors had to move quickly to deliver the hardware in a short time, and the project team had to test the equipment around the clock to make sure it was compatible with the company's systems.

In stage two the data had to be transferred safely and quickly. The project team made a calculated gamble by not loading the huge amount of data onto thousands of servers and physically moving them; instead they ramped up the company's own private network and flowed the data directly to the new site, risking that flowing so much information in a short amount of time would disrupt Facebook's entire website. The data transfer was successfully achieved in a three-week period (with a network flow rate of one gigabyte per

second) by limiting and monitoring bandwidth throughout the transfer and constantly checking the flow to keep it in sync. The project was completed on time and with no delays or downtime. When the project was completed the new center could hold eight times more data than the previous location, it managed the data more efficiently, and it cut millions of dollars from Facebook's energy bills while reducing the center's ecological footprint.

One of the problems of such large data centers is the energy they consume; it is estimated that data centers, in general, consume approximately 1.5% of the world's electricity. Google, in constructing its data center, pioneered unique technology to reduce energy consumption by keeping it



ASSOCIATED PRESS

This Google Data Center in Hamina, Finland was converted from a Finnish paper mill at a cost of 350 million Euros. It is one of Google's most advanced and efficient data centers with a high-tech cooling system that uses sea water from the Bay of Finland.

hotter than what had been traditional in data centers. Typically, data centers are very chilly; however, Google keeps its data center warmer (around 77 degrees) by creating a "hot aisle," a space that takes hot air from the servers into water-filled coils, sending it outside the building to cool, then recirculating it back inside.

Can you think of any other, similar IT-related projects that a company like Amazon or Walmart might undertake?

Sources: Based on Sarah F. Gale, "A Closer Look, Facebook, Palo Alto, California," *PM Network* 24 (9); (September 2010), pp. 34-37; Patrick Thibodeau, "Apple, Google, Facebook Turn N.C. into Data Center Hub," *Computerworld* (June 3, 2011), www.computerworld.com; Jordan G. Teicher, "The Brain of the Beast: Google Reveals the Computers Behind the Cloud," *www.npr.org* (October 17, 2012).

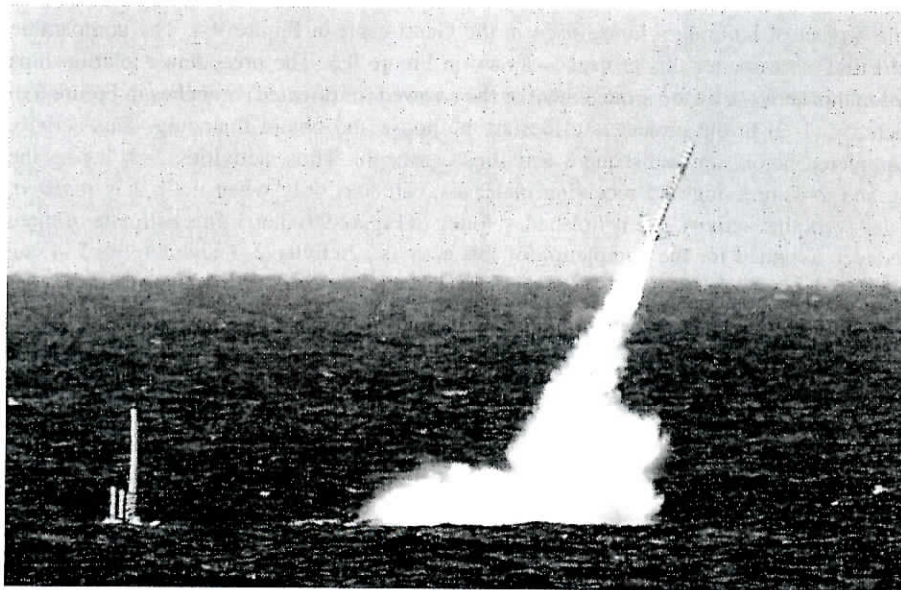
boxes) that might last several weeks, at the end of which a working product (i.e., the software) is delivered and demonstrated to the customer. This reduces the overall risk and allows the project to adapt to changes quickly. For a software or IT development project, an iteration might not result in a market-ready product, with multiple iterations required for release of the full (software) product or new features.

Team composition in an agile project is usually cross-functional and typically includes a customer representative. Face-to-face communication among team members is emphasized, and is frequent, usually daily (in short meetings called *scrums*), which helps to reveal problems as they arise. Team size is typically small (5–9 people) to simplify team communication and collaboration. When a team works in different locations, they maintain daily contact through videoconferencing, voice, email, and so on. At the end of each iteration, stakeholders and the customer representative review progress and re-evaluate priorities with a view to optimizing the ROI and ensuring the project is aligned with customer needs and goals.

The agile approach differs from traditional project management by responding to change rather than following a plan; by relying on individuals who interact instead of formal processes and tools; and by frequently communicating and collaborating with the customer. As global business environments have become subject to more and more rapid and constant change, the agile approach to project management is spreading beyond IT and software development into other sectors, particularly business environments that are dynamic and turbulent, such as technology-based companies, financial services, insurance, telecommunications, pharmaceuticals, utilities, and manufacturing. While the agile approach has benefited some companies, it also has limitations and is not universally applicable to project management, even if the projects involve significant change. It requires small groups of experienced, talented, cross-functional team members that work well together, and some companies cannot (nor need to) adapt to the absence of a formal planning process.

CPM/PERT

In 1956, a research team at E. I. du Pont de Nemours & Company, Inc., led by a du Pont engineer, Morgan R. Walker, and a Remington-Rand computer specialist, James E. Kelley Jr., initiated a project to develop a computerized system to improve the planning, scheduling, and reporting of the company's engineering programs (including plant maintenance and construction projects). The resulting network approach is known as the *critical path method (CPM)*. At the same time, the U.S. Navy established a research team composed of members of the Navy Special Projects Office,



The USS Florida, the nuclear powered missile submarine shown here launching a Tomahawk cruise missile, is a direct descendant of the USS George Washington, the first nuclear submarine of this type. In the late 1950s the Polaris Fleet Ballistic Missile Project included more than 250 prime contractors and 9000 subcontractors. The Navy Department credited PERT with bringing the Polaris missile submarine to combat readiness approximately two years ahead of the originally scheduled completion date.

development of the Polaris Missile Project (a ballistic missile-firing nuclear submarine). This network scheduling technique was named the *program evaluation and review technique*, or PERT. The Polaris project eventually included 23 PERT networks encompassing 3000 activities.

Both CPM and PERT are derivatives of the Gantt chart and, as a result, are very similar. There were originally two primary differences between CPM and PERT. With CPM a single estimate for activity time was used that did not allow for any variation in activity times—activity times were treated as if they were known for certain, or “deterministic.” With PERT, multiple time estimates were used for each activity that allowed for variation in activity times—activity times were treated as “probabilistic.” The other difference was related to the mechanics of drawing the project network. In PERT, activities were represented as arcs, or arrowed lines, between two nodes, or circles, whereas in CPM activities were represented as the nodes or circles. However, over time CPM and PERT have been effectively merged into a single technique conventionally referred to as CPM/PERT.

The advantage of CPM/PERT over the Gantt chart is in the use of a network to depict the precedence relationships between activities. The Gantt chart does not clearly show precedence relationships, a disadvantage that limited its use to small projects. The CPM/PERT network is a more efficient and direct means of displaying precedence relationships. In other words, in a network it is visually easier to see the precedence relationships, which makes CPM/PERT popular with managers and other users, especially for large projects with many activities.

THE PROJECT NETWORK

A CPM/PERT network consists of *branches* and *nodes*, as shown in Figure 9.5. When CPM and PERT were first developed, they employed different conventions for constructing a network. With CPM the nodes, or circles in Figure 9.5, represented the project activities. The arrows in between the nodes indicated the precedence relationships between activities. For the network in Figure 9.5, activity 1, represented by node 1, precedes activity 2, and 2 precedes 3. This approach to network construction is called **activity-on-node (AON)**. With PERT the opposite convention was taken. The branches represented the activities, and the nodes in between them reflected **events**, or points in time such as the end of one activity and the beginning of another. In this approach, referred to as **activity-on-arrow (AOA)**, the activities are normally identified by the node numbers at the start and end of an activity. For example, activity 1–2 precedes activity 2–3 in Figure 9.5. In this book, we will focus on the AON convention, but we will also provide an overview of AOA networks.

AOA NETWORK

To demonstrate how these components are used to construct the two types of network, we will use our example project of building a house used in the Gantt chart in Figure 9.4. The comparable AOA CPM/PERT network for this project is shown in Figure 9.6. The precedence relationships are reflected in this network by the arrangement of the arrowed (or directed) branches in Figure 9.6. The first activity (1–2) in the project is to design the house and obtain financing. This activity must be completed before any subsequent activities can begin. Thus, activities 2–3, laying the foundation, and 2–4, ordering and receiving materials, can start only when node 2 is *realized*, indicating the event that activity 1–2 is finished. (Notice in Figure 9.6 that a time estimate of three months has been assigned for the completion of this activity.) Activity 2–3 and activity 2–4 can

CPM/PERT uses a network to depict the precedence relationships among activities.

Activity-on-node (AON):

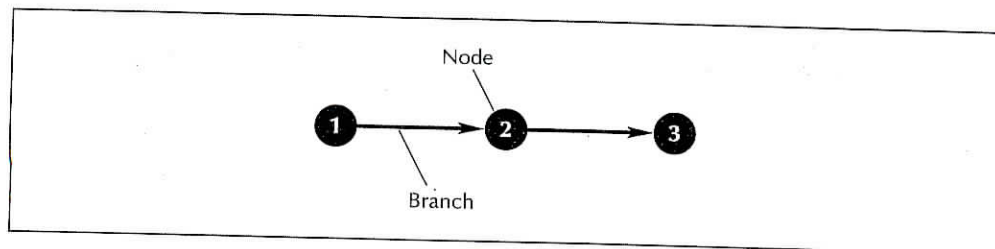
nodes represent activities and arrows show precedence relationships.

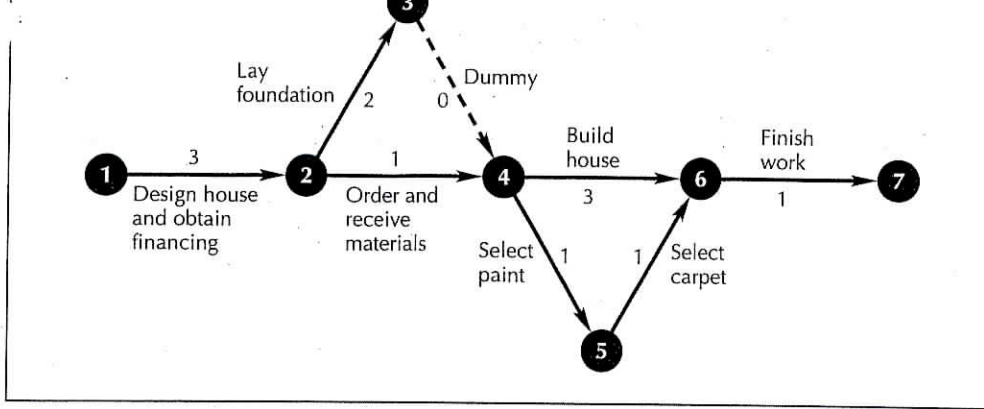
Events: completion or beginning of an activity.

Activity-on-arrow (AOA):

arrows represent activities and nodes are events for points in time.

Figure 9.5
Network Components





occur concurrently; neither depends on the other, and both depend only on the completion of activity 1-2.

When the activities of laying the foundation (2-3) and ordering and receiving materials (2-4) are completed, then activities 4-5 and 4-6 can begin simultaneously. However, before discussing these activities further, notice activity 3-4, referred to in the network as a dummy.

A **dummy** activity is inserted into the network to show a precedence relationship, but it does not represent any actual passage of time. Activities 2-3 and 2-4 have the precedence relationship shown in Figure 9.7a. However, in an AOA network, two or more activities are not allowed to share the same starting and ending nodes. Instead, activity 3-4 is inserted to give two activities separate end nodes and, thus, two separate identities as shown in Figure 9.7b. Notice, however, that a time of zero months has been assigned to activity 3-4. The dummy activity shows that activity 2-3 must be completed prior to any activities beginning at node 4, but it does not represent the passage of time.

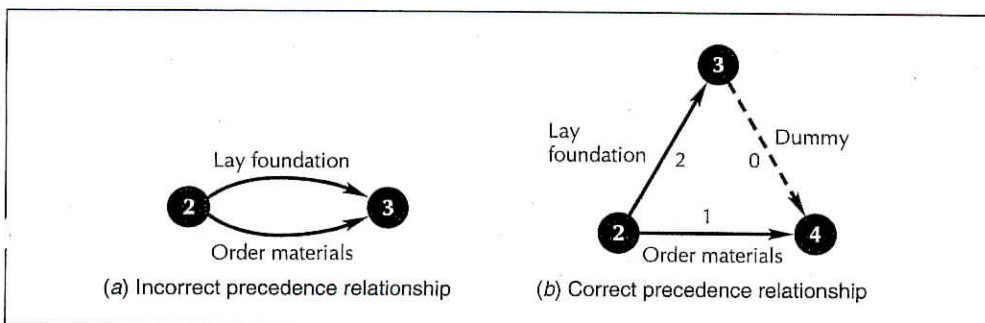
Returning to the network in Figure 9.6, we see that two activities start at node 4. Activity 4-6 is the actual building of the house, and activity 4-5 is the search for and selection of the paint for the exterior and interior of the house. Activity 4-6 and activity 4-5 can begin simultaneously and take place concurrently. Following the selection of the paint (activity 4-5) and the realization of node 5, the carpet can be selected (since the carpet color depends on the paint color). This activity can also occur concurrently with the building of the house (activity 4-6). When the building is completed and the paint and carpet are selected, the house can be finished (activity 6-7).

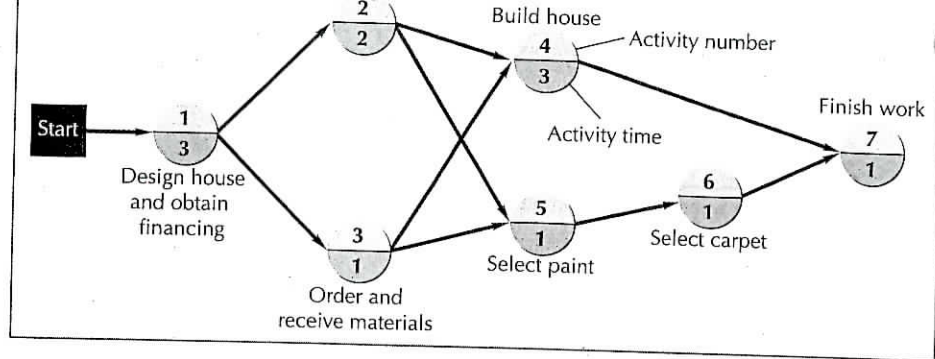
Dummy: two or more activities cannot share the same start and end nodes.

AON NETWORK

Figure 9.8 shows the comparable AON network to the AOA network in Figure 9.6 for our house building project. Notice that the activities and activity times are on the nodes and not on the activities as they were previously with the AOA network. The branches or arrows

Figure 9.7
Concurrent Activities





simply show the precedence relationships between the activities. Also, notice that there is no dummy activity; dummy activities are not required in an AON network since two activities will never be confused because they have the same start and end nodes. In general, both of the two methods accomplish the same thing, and the one that is used is usually a matter of individual preference. However, for our purposes the AON network has one distinct advantage—it is the convention used in the popular *Microsoft Project* software package, and because we want to demonstrate how to use this software, we will use the AON convention in this chapter.

THE CRITICAL PATH

A network path is a sequence of connected activities that runs from the start to the end of the network. The network in Figure 9.8 has several paths through it. In fact, close observations of this network show four paths, identified as A, B, C, and D:

- A: 1 - 2 - 4 - 7
- B: 1 - 2 - 5 - 6 - 7
- C: 1 - 3 - 4 - 7
- D: 1 - 3 - 5 - 6 - 7

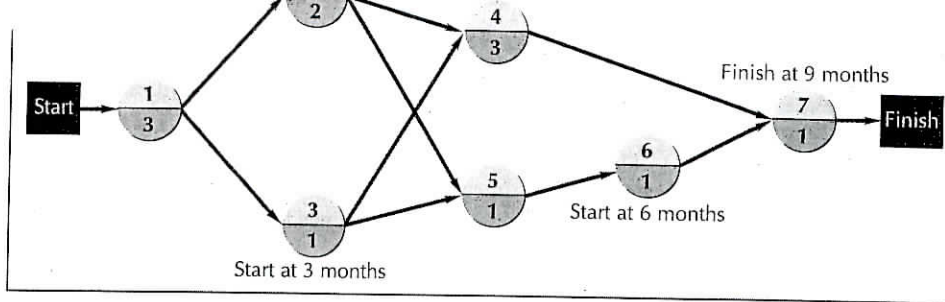
The project cannot be completed (i.e., the house cannot be built) sooner than the time required by the longest path in the network, in terms of time. The path with the longest duration of time is referred to as the **critical path**.

By summing the activity times (shown in Figure 9.8) along each of the four paths, we can compute the length of each path, as follows:

- Path A: 1 - 2 - 4 - 7
3 + 2 + 3 + 1 = 9 months
- Path B: 1 - 2 - 5 - 6 - 7
3 + 2 + 1 + 1 + 1 = 8 months
- Path C: 1 - 3 - 4 - 7
3 + 1 + 3 + 1 = 8 months
- Path D: 1 - 3 - 5 - 6 - 7
3 + 1 + 1 + 1 + 1 = 7 months

Because path A is the longest, it is the critical path; thus, the minimum completion time for the project is nine months. Now let us analyze the critical path more closely. From Figure 9.9 we can see that activity 3 cannot start until three months have passed. It is also easy to see that activity 4 will not start until five months have passed. The start of activity 4 is dependent on two activities leading into node 4. Activity 2 is completed after five months, but activity 3 is completed at the end of four months. Thus, we have two possible start times for activity 4, five months and four months. However, since the activity at node 4 cannot start until all preceding activities have been finished, the soonest node 4 can be realized is five months.

Critical path: the longest path through a network; it is the minimum project completion time.



Now consider the activity following node 4. Using the same logic as before, activity 7 cannot start until after eight months (five months at node 4 plus the three months required by activity 4) or after seven months. Because all activities preceding node 7 must be completed before activity 7 can start, the earliest this can occur is eight months. Adding one month for activity 7 to the start time at node 7 gives a project duration of nine months. This is the time of the longest path in the network—the critical path.

This brief analysis demonstrates the concept of a critical path and the determination of the minimum completion time of a project. However, this was a cumbersome method for determining a critical path. Next, we discuss a mathematical approach to scheduling the project activities and determining the critical path.

ACTIVITY SCHEDULING

In our analysis of the critical path, we determined the earliest time that each activity could be finished. For example, we found that the earliest time activity 4 could start was five months. This time is referred to as the **earliest start time**, and it is expressed symbolically as **ES**. In order to show the earliest start time on the network as well as some other activity times, we will develop in

Earliest start time (ES):
the earliest time an activity can start.

ALONG THE SUPPLY CHAIN

The Terminal 5 Project at Heathrow Airport

British Airport Authority's (BAA) terminal five (T5) at Heathrow Airport in London, completed in 2008, was one of Europe's largest construction projects, taking 5.5 years and 60,000 people to complete. Located between two runways in a space equal in size to Hyde Park in London, T5 has the largest single-span roof in Europe, made up of six sections requiring 10 months to lift into place, and 11 miles of baggage conveyor belt. The facility, which cost £4.3 billion to build, increased Heathrow's capacity by 35 million annual passengers. For the first time in a project of this size and complexity, off-site prefabrication of over 2800 component modules was used extensively. Assembled components were transported to the building site where they were bolted together, thereby reducing construction site time and disruption, and construction traffic at the airport. A web-based system enabled off-site project module manufacturers in Dover and Scotland to be coordinated through a

virtual factory. The supply of material, equipments, and workflows were all monitored through this one integrated data system. All project participants could see when modules were in production, completed, delivered, and stored. The system was especially important for planning and coordinating deliveries between suppliers because of the high volume of deliveries and the limited on-site space involved. The collaborative nature of this project combined with the "virtual factory" web-based computer system, facilitated the performance of the integrated project teams, resulting in a reduction in construction times, a safer working environment, and better quality.

Discuss some of the unique problems that you think might exist for a project like this one that involves a facility with daily, heavy public usage.

Source: John Summers, "The Virtual Factory," *Quality World* 31 (10); (October 2005), pp. 24–28; and Patricia Curmi, "Terminal Velocity," *Quality World* 33, (8); (August 2007), pp. 17–21.

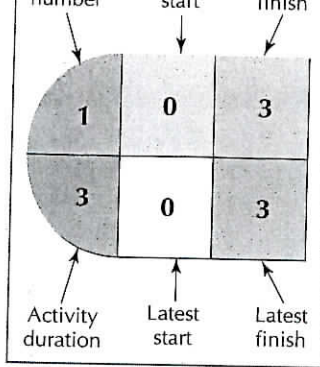


Figure 9.10
Node Configuration

Forward pass: starts at the beginning of a CPM/PERT network to determine the earliest activity times.

Earliest finish time (EF): is the earliest start time plus the activity time.

node 1, the first activity in our example project for designing the house and obtaining financing. To determine the earliest start time for every activity, we make a **forward pass** through the network. That is, we start at the first node and move forward through the network. The earliest start time for an activity is the maximum time in which all preceding activities have been completed—the time when the activity start node is realized.

The **earliest finish time (EF)** for an activity is simply the earliest start time plus the activity time estimate. For example, if the earliest start time for activity 1 is at time 0, then the earliest finish time is three months. In general, the earliest start and finish times for an activity are computed according to the following mathematical relationship.

$$ES = \text{maximum (EF) of immediate predecessors}$$

$$EF = ES + t$$

The earliest start and earliest finish times for all the activities in our project network are shown in Figure 9.11.

The earliest start time for the first activity in the network (for which there are no predecessor activities) is always 0, or, $ES = 0$. This enables us to compute the earliest finish time for activity 1 as

$$EF = ES + t$$

$$= 0 + 3$$

$$= 3 \text{ months}$$

The earliest start for activity 2 is

$$ES = \max (\text{EF immediate predecessors})$$

$$= 3 \text{ months}$$

and the corresponding earliest finish time is

$$EF = ES + t$$

$$= 3 + 2$$

$$= 5 \text{ months}$$

For activity 3 the earliest start time (ES) is three months, and the earliest finish time (EF) is four months.

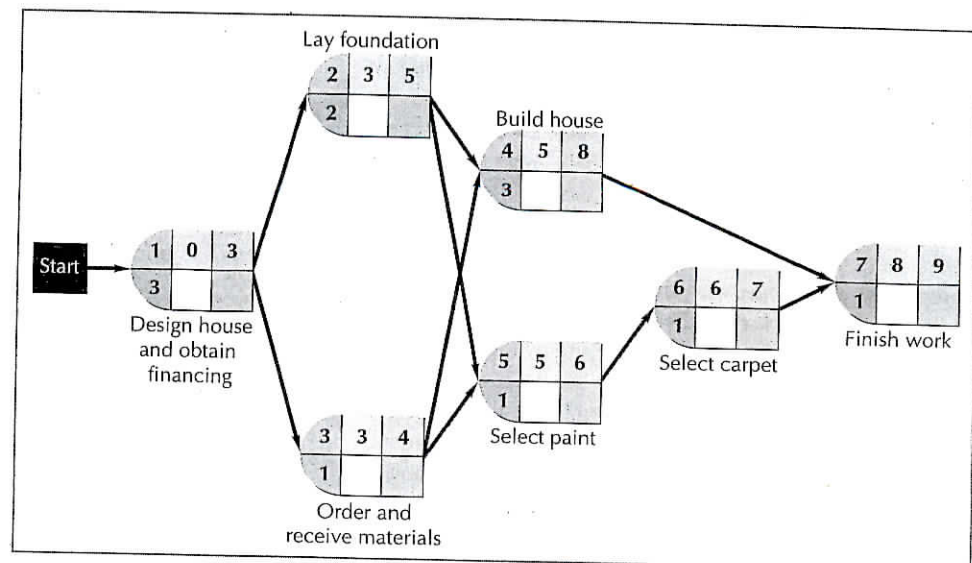
Now consider activity 4, which has two predecessor activities. The earliest start time is

$$ES = \max (\text{EF immediate predecessors})$$

$$= \max (5, 4)$$

$$= 5 \text{ months}$$

Figure 9.11
Earliest Activity Start and Finish Times



$$\begin{aligned}
 EF &= ES + t \\
 &= 5 + 3 \\
 &= 8 \text{ months}
 \end{aligned}$$

All the remaining earliest start and finish times are computed similarly. Notice in Figure 9.11 that the earliest finish time for activity 7, the last activity in the network, is nine months, which is the total project duration, or critical path time.

Companions to the earliest start and finish are the **latest start** and **latest finish times**, **LS** and **LF**. The latest start time is the latest time an activity can start without delaying the completion of the project beyond the project critical path time. For our example, the project completion time (and earliest finish time) at node 7 is nine months. Thus, the objective of determining latest times is to see how long each activity can be delayed without the project exceeding nine months.

In general, the latest start and finish times for an activity are computed according to the following formulas:

$$\begin{aligned}
 LS &= LF - t \\
 LF &= \min (\text{LS immediate following activities})
 \end{aligned}$$

Whereas a forward pass through the network is made to determine the earliest times, the latest times are computed using a **backward pass**. We start at the end of the network at node 7 and work backward, computing the latest times for each activity. Since we want to determine how long each activity in the network can be delayed without extending the project time, the latest finish time at node 7 cannot exceed the earliest finish time. Therefore, the latest finish time at node 7 is nine months. This and all other latest times are shown in Figure 9.12.

Starting at the end of the network, the critical path time, which is also equal to the earliest finish time of activity 7, is nine months. This automatically becomes the latest finish time for activity 7, or

$$LF = 9 \text{ months}$$

Using this value, the latest start time for activity 7 is

$$\begin{aligned}
 LS &= LF - t \\
 &= 9 - 1 \\
 &= 8 \text{ months}
 \end{aligned}$$

The latest finish time for activity 6 is the minimum of the latest start times for the activities following node 6. Since activity 7 follows node 6, the latest finish time is

$$\begin{aligned}
 LF_6 &= \min (\text{LS following activities}) \\
 &= 8 \text{ months}
 \end{aligned}$$

Latest start time (LS): the latest time an activity can start without delaying critical path time.

Latest finish time (LF): the latest time an activity can be completed and still maintain the project critical path time.

Backward pass: determines latest activity times by starting at the end of a CPM/PERT network and working forward.

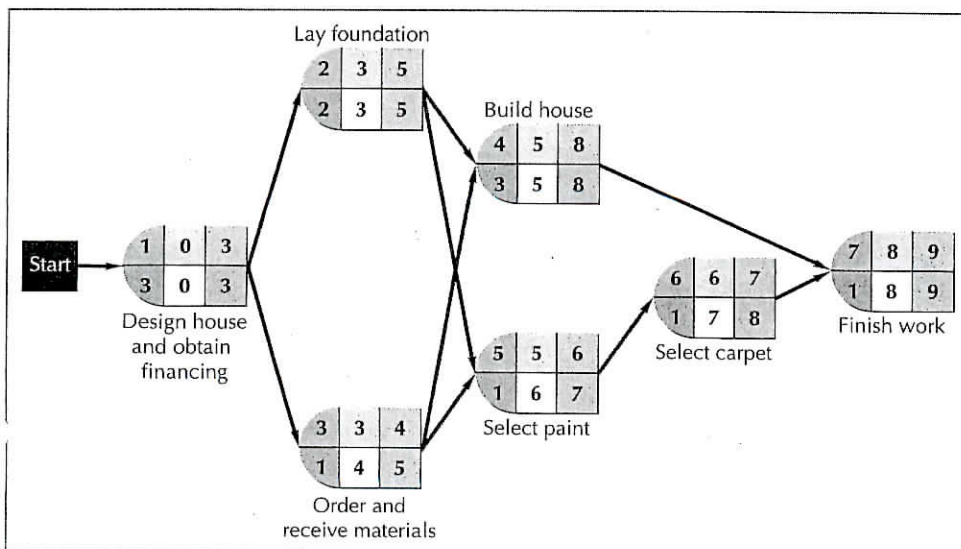


Figure 9.12
Latest Activity Start and Finish Times

$$\begin{aligned} LS &= LF - t \\ &= 8 - 1 \\ &= 7 \text{ months} \end{aligned}$$

For activity 4, the latest finish time (LF) is eight months, and the latest start time (LS) is five months; for activity 5, the latest finish time (LF) is seven months, and the latest start time (LS) is six months.

Now consider activity 3, which has two activities, 4 and 5, following it. The latest finish time is computed as

$$\begin{aligned} LF &= \min (\text{LS following activities}) \\ &= \min (5, 6) \\ &= 5 \text{ months} \end{aligned}$$

The latest start time is

$$\begin{aligned} LS &= LF - t \\ &= 5 - 1 \\ &= 4 \text{ months} \end{aligned}$$

All the remaining latest start and latest finish times are computed similarly. Figure 9.12 includes the earliest and latest start times and earliest and latest finish times for all activities.

ACTIVITY SLACK

The project network in Figure 9.12, with all activity start and finish times, highlights the critical path (1-2-4-7) we determined earlier by inspection. Notice that for the activities on the critical path, the earliest start times and latest start times are equal. This means that these activities on the critical path must start exactly on time and cannot be delayed at all. If the start of any activity on the critical path is delayed, then the overall project time will be increased. We now have an alternative way to determine the critical path besides simply inspecting the network. The activities on the critical path can be determined by seeing for which activities $ES = LS$ or $EF = LF$. In Figure 9.12 the activities 1, 2, 4, and 7 all have earliest start times and latest start times that are equal (and $EF = LF$); thus, they are on the critical path.

For activities not on the critical path for which the earliest and latest start times (or earliest and latest finish times) are not equal, *slack* time exists. We introduced slack with our discussion

A primary use of CPM/PERT is to plan and manage construction projects of all types, such as this Virginia Department of Transportation bridge construction project near Springfield, Virginia, where I-95 and I-395 converge.



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Activity	LS	ES	LF	EF	Slack S
*1	0	0	3	3	0
*2	3	3	5	5	0
3	4	3	5	4	1
*4	5	5	8	8	0
5	6	5	7	6	1
6	7	6	8	7	1
*7	8	8	9	9	0

*Critical path.

of the Gantt chart in Figure 9.4. Slack is the amount of time an activity can be delayed without affecting the overall project duration. In effect, it is extra time available for completing an activity.

Slack, S , is computed using either of the following formulas:

$$S = LS - ES$$

or

$$S = LF - EF$$

For example, the slack for activity 3 is

$$\begin{aligned} S &= LS - ES \\ &= 4 - 3 \\ &= 1 \text{ month} \end{aligned}$$

If the start of activity 3 were delayed for one month, the activity could still be completed by month 5 without delaying the project completion time. The slack for each activity in our example project network is shown in Table 9.1. Table 9.1 shows there is no slack for the activities on the critical path (marked with an asterisk); activities not on the critical path have slack.

Notice in Figure 9.12 that activity 3 can be delayed one month and activity 5 that follows it can be delayed one more month, but then activity 6 cannot be delayed at all even though it has one month of slack. If activity 3 starts late at month 4 instead of month 3, then it will be completed at month 5, which will not allow activity 5 to start until month 5. If the start of activity 5 is delayed one month, then it will be completed at month 7, and activity 6 cannot be delayed at all without exceeding the critical path time. The slack on these three activities is called *shared slack*. This means that the sequence of activities 3–5–6 can be delayed two months jointly without delaying the project, but not three months.

Slack is beneficial to the project manager because it enables resources to be temporarily diverted from activities with slack and used for other activities that might be delayed for various reasons or for which the time estimate has proved to be inaccurate.

The times for the network activities are simply estimates, for which there is usually not a lot of historical basis (since projects tend to be unique undertakings). As such, activity time estimates are subject to quite a bit of uncertainty. However, the uncertainty inherent in activity time estimates can be reflected to a certain extent by using probabilistic time estimates instead of the single, deterministic estimates we have used so far.

Probabilistic Activity Times

In the project network for building a house in the previous section, all activity time estimates were single values. By using only a single activity time estimate, we are, in effect, assuming that activity times are known with certainty (i.e., they are deterministic). For example, in Figure 9.8, the time

Probabilistic time estimates reflect uncertainty of activity times.