**Calculus Application Problem** Name ______________________

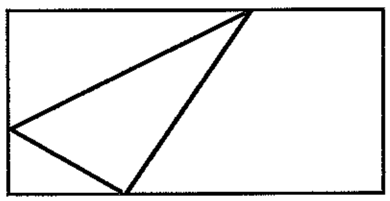
**Keep on Folding!** **Due at the beginning of class** __________

The problem we are going to solve is:

> **Given a sheet of paper 8.5" by 11", fold the top left corner down to a point on the bottom edge. Where should you place this corner to maximize the area of the triangle formed in the bottom left corner?**

Remember, the area of a triangle can be found by the geometry formula: **Area =** ____________________.

This formula tells us that the area is a function of the __________ and the __________ of the triangle. Let's do some measuring (and calculating!) and determine the area for some triangles. Take your 8.5" by 11" piece of paper and (holding it sideways) mark the bottom edge in 1" units and the left side of the paper in 0.5" units. (You only need to mark the bottom edge up to 8".)



Fold the top left corner down to the point on the bottom edge that measures 1". Estimate the height of the triangle to the nearest 0.1". ________ Now calculate the area of the triangle. __________. We are now going to let the base of the triangle grow longer.

What will happen to the height of the triangle? ______________________________________

What do you think will happen to the area of the triangle? ______________________________

Let's see if the above conjectures are correct. Fold the top left corner down to the indicated point on the bottom edge of the piece of paper. Estimate the height of each triangle (to the nearest 0.1 in). Then calculate the area of the triangle. Use the chart below to show your results.

| BASE | HEIGHT | AREA |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |

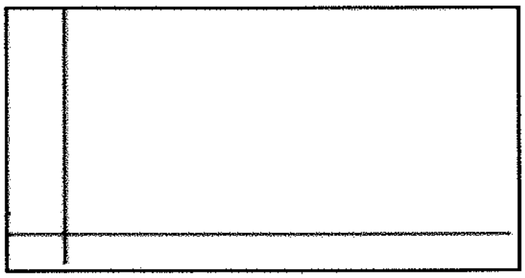
From your chart where does it appear that the maximum value of the area of the triangle occurs? The maximum area of the triangle is __________. It occurs when the base is __________ and the height is ____________.

**Solutions to the Problem:**

We want to determine the "exact value" of the base that will maximize the area of the triangle. We will do this two ways. First, a numerical/graphical solution (which does not require calculus), and then an analytical solution (which will require our calculus).

**1. The Numerical/Graphical Solution**

Let's look at the BASE, HEIGHT, and AREA data in graphical form. Using the values from the chart on the preceding page, put the values of the BASE in L1, the values of the HEIGHT in L2, and the values of the AREA in L3. We will first find a relationship between the BASE and the HEIGHT. Create a scatterplot of the BASE vs. HEIGHT (or L1 vs. L2). Set up your window to get a good view of your data points and plot the data. Draw a diagram of the data points below and indicate the window you used on your calculator.



[Xmin,Xmax]: ____________

[Ymin,Ymax]: ____________

What type of function do you think your data matches the closest? (It's not linear!) ______________________.
Using the regression option on your calculator, see if your guess was correct. Graph your regression equation. How does it fit?____________________ Write the results of the regression equation below. (Round values to 3 decimal places.) Do not write the equation in terms of x and y, but in terms of **B** (BASE) and **H** (HEIGHT).

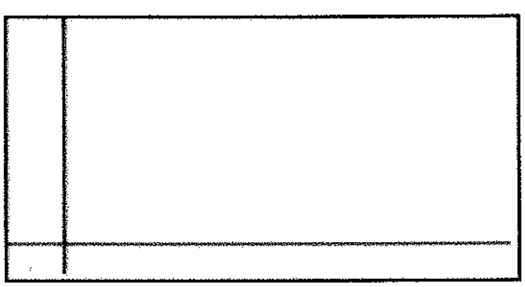
__________________________________________________

We are now ready to write our equation for the AREA of the triangle. Using the formula for the area of a triangle, the area **A** of the triangle (in terms of the base **B**) is:

A(B) = ____________________________________________________________________

The independent variable is ______________ and the dependent variable is _______________.

Before we graph this function, let's see the scatterplot of the relationship between the BASE (L1) of the triangle and the AREA (L3) of the triangle. Change the set up of the scatterplot, graph it, and show your graph below. Then, graph your area function to see how it fits.



Remember, we wanted to find where to fold the paper to maximize the area of the triangle. Use the graph of your "area function" to answer this question. The maximum AREA of the triangle is ________________ square inches. It occurs when the BASE of the triangle is ______________ inches, and the HEIGHT of the triangle is ________________ inches.

## 2. The Analytical Solution

In order to analytically find a function for the AREA **A** in terms of the BASE, again we first need to write a function for the HEIGHT **H** of the triangle in terms of the BASE **B**.

In the space below, draw a diagram of the paper with the top left corner folded down (similar to the figure on the first page). Label the base of the triangle **B**. Label the height of the triangle **H**. We need to write an expression for the <u>length of the hypotenuse of the triangle</u>. Your answer should be in terms of **H only**, not **B**.
(Hint: Unfold your paper to see how the hypotenuse relates to the height!)

The hypotenuse, in terms of the height **H**, is ________________

Use the Pythagorean Theorem and write an equation which shows the relationship between these three sides.

____________________ = _____________ + ________________

Using algebra, solve this equation for **H**. (It's not as hard as it looks!)

Write all decimals in the equations as <u>fractions</u>. **H** = ________________________________________

Compare this equation with the result of your quadratic regression equation on page 2. Although they may not appear similar, if you look closely at the coefficients you will see that they are almost the same (if you did your measurements accurately).

Using this form of the HEIGHT equation, rewrite the AREA **A** function in terms of the BASE **B**. Keep your coefficients in fraction form (not decimals) and distribute where appropriate to simplify the form of the function.

A(B) = __________________________________________________________________

= __________________________________________________________________

Sketch this function with the AREA vs. BASE scatterplot to see how this fits. (Of course it should fit well, since it is just another form of the other function.)

It's now time for our calculus! Explain <u>how</u> we can use the AREA function to analytically (not graphically) determine the value of the BASE that will maximize the AREA of the triangle.

______________________________________________________________________________

______________________________________________________________________________

Perform this calculus (and algebra) to determine the BASE that will maximize the AREA of the triangle. Compare your answer with the answer you obtained from the graphical solution.

**Results:**

The maximum AREA occurs when the BASE of the triangle is ____________ inches, and the HEIGHT of the triangle is ____________ inches. The maximum AREA of the triangle is ______________ square inches.