8.14. Jim Matthews, vice president for marketing of the J. R. Nickel Company, is planning advertising campaigns for two unrelated products. These two campaigns need to use some of the same resources. Therefore, Jim knows that his decisions on the levels of the two campaigns need to be made jointly after considering these resource constraints. In particular, letting x_1 and x_2 denote the levels of campaigns 1 and 2, respectively, these constraints are $4x_1 + x_2 \leq 20$ and $x_1 + 4x_2 \leq 20$.

In facing these decisions, Jim is well aware that there is a point of diminishing returns when raising the level of an advertising campaign too far. At that point, the cost of additional advertising becomes larger than the increase in net revenue (excluding advertising costs) generated by the advertising. After careful analysis, he and his staff estimate that the net profit from the first product (including advertising costs) when conducting the first campaign at level x_1 would be $3x_1 - (x_1 - 1)^2$ in millions of dollars. The corresponding estimate for the second product is $3x_2 - (x_2 - 2)^2$.

Letting P be total net profit, this analysis led to the following nonlinear programming model for determining the levels of the two advertising campaigns:

$$\text{Maximize} \quad P = 3x_1 - (x_1 - 1)^2 + 3x_2 - (x_2 - 2)^2$$

subject to

$$4x_1 + x_2 \leq 20$$
$$x_1 + 4x_2 \leq 20$$

and

$$x_1 \geq 0 \quad x_2 \geq 0$$

a. Construct tables to show the profit data for each product when the level of its advertising campaign is $x_1 = 0, 1, 2, 2.5, 3, 4, 5$ (for the first product) or $x_2 = 0, 1, 2, 3, 3.5, 4, 5$ (for the second product).

b. Use these profit data to draw rough-hand a smooth profit graph for each product. (Note that these profit graphs start at negative values when $x_1 = 0$ or $x_2 = 0$ because the products would lose money if there is no advertising to support them.)

c. On the profit graph for the first product, draw an approximation of this profit graph by inserting a dashed-line segment between the profit at $x_1 = 0$ and $x_1 = 2$, between the profit at $x_1 = 2$ and $x_1 = 4$, and between the profit at $x_1 = 4$ and $x_1 = 5$. Then do the same on the profit graph for the second product with $x_2 = 0, 2, 4, 5$.

E* *d.* Use separable programming with the approximation of the profit graphs obtained in part *c* to formulate an approximate linear programming model on a spreadsheet for Jim Matthews's problem. Then solve this model. What does this solution say the levels of the advertising campaigns should be? What would the total net profit from the two products be?

E* *e.* Repeat parts *c* and *d* except using $x_1 = 0, 2, 2.5, 3, 5$ and $x_2 = 0, 3, 3.5, 4, 5$ for the approximations of the

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