In Exercises 1–31 use the Laplace transform to solve the initial value problem.

1. $y'' + 3y' + 2y = e^t, \quad y(0) = 1, \quad y'(0) = -6$

2. $y'' - y' - 6y = 2, \quad y(0) = 1, \quad y'(0) = 0$

3. $y'' + y' - 2y = 2e^{3t}, \quad y(0) = -1, \quad y'(0) = 4$

4. $y'' - 4y = 2e^{3t}, \quad y(0) = 1, \quad y'(0) = -1$

5. $y'' + y' - 2y = e^{3t}, \quad y(0) = 1, \quad y'(0) = -1$

6. $y'' + 3y' + 2y = 6e^t, \quad y(0) = 1, \quad y'(0) = -1$

7. $y'' + y = \sin 2t, \quad y(0) = 0, \quad y'(0) = 1$

8. $y'' - 3y' + 2y = 2e^{3t}, \quad y(0) = 1, \quad y'(0) = -1$

9. $y'' - 3y' + 2y = e^{4t}, \quad y(0) = 1, \quad y'(0) = -2$

10. $y'' - 3y' + 2y = e^{3t}, \quad y(0) = -1, \quad y'(0) = -4$

11. $y'' + 3y' + 2y = 2e^t, \quad y(0) = 0, \quad y'(0) = -1$

12. $y'' + y' - 2y = -4, \quad y(0) = 2, \quad y'(0) = 3$

13. $y'' + 4y = 4, \quad y(0) = 0, \quad y'(0) = 1$

14. $y'' - y' - 6y = 2, \quad y(0) = 1, \quad y'(0) = 0$

15. $y'' + 3y' + 2y = e^t, \quad y(0) = 0, \quad y'(0) = 1$

16. $y'' - y = 1, \quad y(0) = 1, \quad y'(0) = 0$

17. $y'' + 4y = 3 \sin t, \quad y(0) = 1, \quad y'(0) = -1$

18. $y'' + y' = 2e^{3t}, \quad y(0) = -1, \quad y'(0) = 4$

19. $y'' + y = 1, \quad y(0) = 2, \quad y'(0) = 0$

20. $y'' + y = t, \quad y(0) = 0, \quad y'(0) = 2$

21. $y'' + y = t - 3 \sin 2t, \quad y(0) = 1, \quad y'(0) = -3$

22. $y'' + 5y' + 6y = 2e^{-t}, \quad y(0) = 1, \quad y'(0) = 3$

23. $y'' + 2y' + y = 6 \sin t - 4 \cos t, \quad y(0) = -1, \quad y'(0) = 1$

24. $y'' - 2y' - 3y = 10 \cos t, \quad y(0) = 2, \quad y'(0) = 7$

25. $y'' + y = 4 \sin t + 6 \cos t, \quad y(0) = -6, \quad y'(0) = 2$

26. $y'' + 4y = 8 \sin 2t + 9 \cos t, \quad y(0) = 1, \quad y'(0) = 0$

27. $y'' - 5y' + 6y = 10e^t \cos t, \quad y(0) = 2, \quad y'(0) = 1$

28. $y'' + 2y' + 2y = 2t, \quad y(0) = 2, \quad y'(0) = -7$

29. $y'' - 2y' + 2y = 5 \sin t + 10 \cos t, \quad y(0) = 1, \quad y'(0) = 2$

30. $y'' + 4y' + 13y = 10e^{-t} - 36e^t, \quad y(0) = 0, \quad y'(0) = -16$

31. $y'' + 4y' + 5y = e^{-t}(\cos t + 3 \sin t), \quad y(0) = 0, \quad y'(0) = 4$

32. $2y'' - 3y' - 2y = 4e^t, \quad y(0) = 1, \quad y'(0) = -2$

33. $6y'' - y' - y = 3e^{2t}, \quad y(0) = 0, \quad y'(0) = 0$

34. $2y'' + 2y' + y = 2t, \quad y(0) = 1, \quad y'(0) = -1$

35. $4y'' - 4y' + 5y = 4 \sin t - 4 \cos t, \quad y(0) = 0, \quad y'(0) = 1$