COMPARING DIFFERENT LIFE SPANS

If the alternatives have different life spans, the IROR may be found by several methods.

1. *Use equivalent net annual worth.* Formulate the equation for each alternative in terms of equivalent annual worth instead of present worth. (Remember that when using equivalent annual costs for comparing alternatives with different life spans, the inference is that replacement costs equal original costs.)
2. *Assume multiple equal replacements.* Find the present worth of enough future replacements of each alternative to reach a common life span.

Example Illustrating Annual Costs

When determining the IROR, annual costs often are easier to use than present worth in cases where different life spans occur and the lowest common multiple happens to be a large or awkward figure to work with. Remember that using annual costs to compare alternatives with unequal life spans implies equal replacement costs, unless special provisions to the contrary are made. The following example illustrates this situation.

EXAMPLE 11.3

Three grades of roofing are available to replace a residential roof, and you are asked to determine which is the more economical. Data on the roofs are presented as follows:

1. 15-year roof costs \$1,800 plus \$100 per year for end-of-year maintenance.
2. 20-year roof costs \$2,200 plus \$50 per year for end-of-year maintenance.
3. 25-year roof costs \$3,000 and requires no maintenance.
   a. Plot a graph of A versus i for each alternative.
   b. On the graph, show which ranges of i favor which alternatives.

Discussion: The least common multiple for the n values (lives) in this problem is 300 years, which renders the present worth method of finding IROR cumbersome and tedious. The annual cost method is much simpler.

Solution. Find i for the IROR in terms of annual costs. A graph of the values of annual costs versus i is shown in Figure 11.8. The equations for each alternative are found in terms of annual worth. Notice the use of an alternative format for this solution. Instead of the variables being listed in a separate *column* for each alternative, they are formulated in a separate *row* for each alternative. Both formats yield identical results. You may want to use only one of

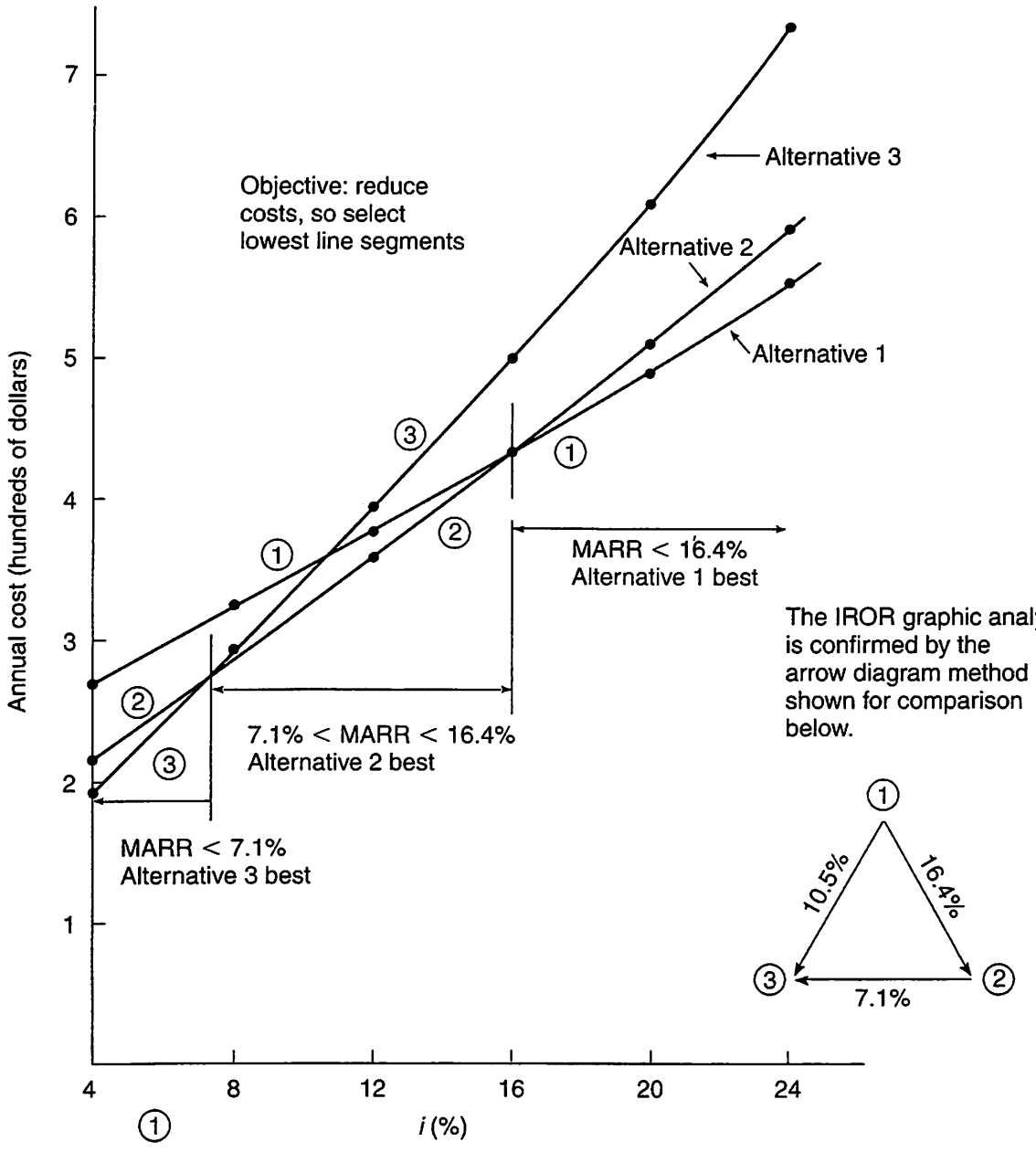


Figure 11.8 Graph of annual costs versus rate of return for Example 11.3. *Note:* Costs are shown as positive values in this example.

these two formats in order to maintain consistency.

Alternative 1

$$A_1 = \$100 + \$1{,}800(A/P, i, 15)$$

Alternative 2

$$A_2 = \$50 + \$2{,}200(A/P, i, 20)$$

Alternative 3

$$A_3 = \$3{,}000(A/P, i, 25)$$

The solutions can be found by solving simultaneous equations in three pairs, and finding the respective three values of i for which

$$A_1 = A_2$$
$$A_1 = A_3$$
$$A_2 = A_3$$

These solutions yield the following matrix of values:

	2	3
1	16.4%	10.5%
2		7.1%

Conclusion. The graph, arrow diagram, and matrix all indicate that

For MARR > 16.4 percent, use alternative 1.
For 16.4% > MARR > 7.1 percent, use alternative 2.
For MARR < 7.1 percent, use alternative 3.

Example Illustrating Multiple Replacements

Example 11.4 illustrates how the IROR is found by multiple replacements when life spans differ among alternatives.

EXAMPLE 11.4

A firm of engineers is in need of additional office space. A landlord offers them suitable space with three alternative leasing plans:

1. A one-year lease for $50,000 paid in advance, with option to renew for five subsequent one-year leases under the same terms.
2. A two-year lease for $90,000 paid in advance, with option to renew two more times at no increase.
3. A three-year lease for $130,000 paid in advance, with option to renew one more time at no increase.
   a. Find which ranges of MARR favor which lease terms.
   b. If the firm can invest its working capital elsewhere in the business at a 20 percent pretax rate of return, which is the better selection of lease terms?

Solution. The one- and two-year leases are compared over the least common multiple of life spans, in this case a two-year period.

$$\text{two 1-year leases, } P_1 = -\$50{,}000 - \$50{,}000(P/F, i, 1)$$
$$\text{one 2-year lease, } P_2 = -\$90{,}000$$

The present worths of the two plans are equated and solved for i.

$$P_1 = P_2$$
$$-\$50{,}000 - \$50{,}000(P/F, i, 1) = -\$90{,}000$$
$$P/F = \frac{-\$90{,}000 + \$50{,}000}{-50{,}000}$$
$$\frac{1}{(1+i)} = 0.80$$
$$i = 0.25 = 25\%$$

In comparing the one- and two-year leases, the firm has an opportunity to spend an extra \$40,000 now in order to save spending \$50,000 a year from now. The effect is the same as investing \$40,000 now and receiving a \$50,000 return (benefit) in one year. Thus the rate of return on the incremental investment (IROR) is \$10,000/\$40,000 = 25 percent for the one-year period.

The one-year and three-year leases are compared over the least common multiple, in this case a three-year period, in the same manner as above:

$$\text{three 1-year leases, } P_3 = -\$50{,}000 - \$5{,}000(P/A, i, 2)$$
$$\text{one 3-year lease paid in advance, } P_1 = -\$130{,}000$$

Then equate P_3 to P_1 and solve as follows:

$$(P/A, i, 2) = \frac{130{,}000 - 50{,}000}{50{,}000} = 1.6000$$

Solving the P/A equation yields the factors listed below, which are then interpolated to find i.

i	$(P/A, i, 2)$
16%	1.6053
IROR	1.6000
17%	1.5853

$$i = \frac{1.6053 - 1.6000}{1.6053 - 1.5853} \times 1\% + 16\% = \underline{\underline{16.3\%}}$$

In comparing the two-year lease to the three-year lease, the shortest common time period is six years. The IROR between these alternatives is found as follows.

$$P_5 = -\$90{,}000 - \$90{,}000(P/F, i, 2) - \$90{,}000(P/F, i, 4)$$
$$P_6 = -\$130{,}000 - \$130{,}000(P/F, i, 3)$$

Equating P_5 to P_6 yields

$$P_6 - P_5 = -\$40{,}000 + \$90{,}000(P/F, i, 2) - \$130{,}000(P/F, i, 3) + \$90{,}000(P/F, i, 4)$$

Solving for $\Delta P = 0$, select trial i values. Try i at 8 percent.

$$\text{find } P_6 - P_5 = +\$114.10$$

This trial value of $i = 8$ percent yields a positive value of $P_6 - P_5$, so the next trial i should be higher, assuming the curve of i versus P declines to the right. Try $i = 9$ percent.

$$\text{find } P_6 - P_5 = -\$874.80$$

Interpolation yields

$$i = 8\% + \frac{114.10}{114.10 + 874.80} \times 1\% = \underline{\underline{8.1\%}}$$

The arrow diagram method provides a graphic solution process for this example, as shown in Figure 11.9.

For MARR > 25 percent, use the one-year leases.
For 25 percent > MARR > 8.1 percent, use two-year leases.
For MARR < 8.1 percent, use three-year leases.

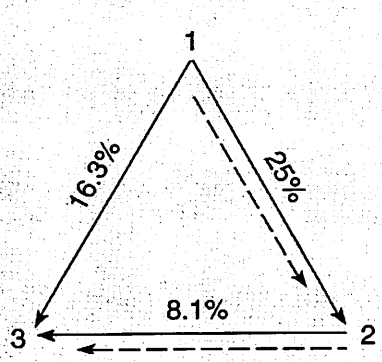


Figure 11.9 Arrow diagram for Example 11.4.

GRADIENTS

Changes in costs with the passage of time are a normal occurrence in real life and a reasonable mathematical simulation usually can be formulated by using gradients. The following example illustrates the use of gradients in IROR problems.

EXAMPLE 11.5

The state DOT has decided to construct a new road to bypass a congested area. They ask you to find the best of the three alternative routes from the data provided below.

Route	A	B	C
Initial cost ($)	1,500,000	1,300,000	1,600,000
Maintenance costs ($)	100,000/yr	120,000/yr	130,000/yr
Gradient maintenance costs ($)	12,000/yr/yr	12,000/yr/yr	12,000/yr/yr
Savings per vehicle (seconds)	25 s	15 s	40 s
Salvage value ($)	500,000	200,000	700,000
Estimated life (yr)	15	20	25

Each of the three routes is estimated to carry 5,000 vehicles per day for the first year, with a gradient of 500 vehicles per day for each year thereafter. The value of time for each vehicle is estimated at $3 per hour.

1. Find the incremental rate of return between the lowest cost route and the next lower cost alternative route.
2. The incremental rates of return for the other alternatives are as listed below. Draw an arrow diagram showing the three alternatives, including appropriate directions for each arrow between alternatives.

	A	C
B	find	16.7%
A		13.1%

3. Draw dashed arrows on the arrow diagram to illustrate the method of reaching the best alternative of the three if the MARR is 15 percent.

Solution 1. Since the alternatives have different life spans, the equations are more easily formulated in terms of equivalent annual costs. Route B is the lowest cost and is subtracted from the next higher cost alternative route A, and the result is solved for the IROR value. Since savings are given in terms of seconds (s) rather than dollars, the first step is to find the annual dollar value of savings in time for each alternative route, calculated as

$$\text{savings route A:} \quad A_A = \$3/\text{hr} \times 5{,}000 \text{ vehicles/day} \times 365 \text{ days/yr} \times \frac{25\text{s/vehicle}}{3{,}600\text{s/hr}} = \$38{,}021/\text{yr}$$

$$\text{savings route B:} \quad A_B = \$3/\text{hr} \times 5{,}000 \text{ vehicles/day} \times 365 \text{ days/yr} \times \frac{15\text{s/vehicle}}{3{,}600\text{s/hr}} = \$22{,}813/\text{yr}$$

Then the gradient in savings per year is found as

$$\text{savings route A:} \quad G_A = \$3{,}802/\text{yr/yr}$$
$$\text{savings route B:} \quad G_B = \$2{,}281/\text{yr/yr}$$

The total equivalent annual costs plus savings are then formulated for each route as follows (all quantities are $\times$ \$1,000).

$$\text{A:} \quad A_A = -1{,}500(A/P, i, 15) + 500(A/F, i, 15) - 100 - 12(A/G, i, 15) + 38.021 + 3.802(A/G, i, 15)$$

$$\text{B:} \quad A_B = -1{,}300(A/P, i, 20) + 200(A/F, i, 20) - 120 - 12(A/G, i, 20) + 22.813 + 2.281(A/G, i, 20)$$

Then trial i values are substituted into the equations until a positive and a negative result of $A_A - A_B$ are obtained.

	For $i = 15\%$	For $i = 20\%$
A_A	$= -345.421$	-408.317
A_B	$= -355.070$	-406.469
$A_A - A_B$	$= +\ 9.649$	$-\ 1.848$

Thus, by interpolation,

$$\text{IROR} = i = 15\% + 5\% \times \frac{9.649}{9.649 + 1.848} = \underline{\underline{19.2\%}}$$

If problems of this type are encountered often, it is well to write or acquire a computer program to save time in finding solutions and ranging variables.

Solution 2. The three alternatives are represented by a three-sided polygon (triangle) with the lowest first cost alternative at the top (see Figure 11.10). The next higher first cost alternatives are represented by each corner in sequence, proceeding clockwise around the polygon. From every corner except the last, arrows are drawn from lower first cost to higher first cost.

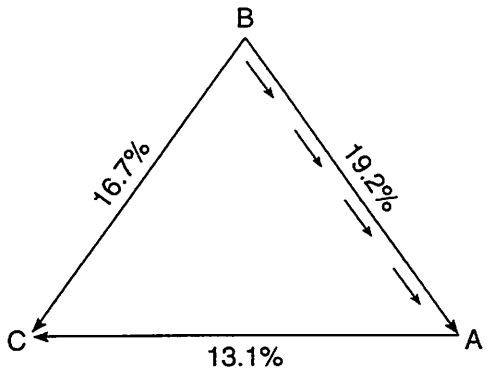


Figure 11.10 Arrow diagram for Example 11.5.

Solution 3

a. Begin at the lowest cost alternative B.

b. Find the highest IROR leading from this point (IROR = 19.2%).

c. Compare IROR with MARR (IROR = 19.2% > 15% = MARR); choose IROR and temporarily accept alternative A.

d. Find highest IROR leading from this point (IROR = 13.1%).

e. Compare IROR with MARR (IROR = 13.1% < 15% = MARR); reject alternative C, and stay with A.

Conclusion. With the MARR at 15 percent, route A should be constructed for $1,500,000 rather than route B for $1,300,000, since the extra $200,000 will earn an IROR of 19.2 percent. Route C should be rejected, since the extra $100,000 required over route A will only earn an IROR of 13.1 percent.

COMPARING ALTERNATIVES BY THE ROR VERSUS THE IROR

The following example illustrates some of the problems encountered if only the ROR method is used for comparing alternatives. In this example the relative rankings for the preferred alternatives reverse themselves as the i values are ranged from one value to the next.

EXAMPLE 11.6

The following information has been obtained on two alternative machines:

	Machine X	Machine Y
Initial cost ($)	200	400
Annualized income ($)	75	120
End-life salvage value ($)	50	150
Life (yr)	6	12

Which alternative should be selected?

Note: There are unequal lives involved, so we must either (a) assume multiple equal investments and benefits of machine X until the 12-year period is concluded, or (b) assume some different salvage value for machine Y after 6 years. Remember that considering each alternative on the basis of equivalent annualized worth *automatically* assumes multiple equal investments and benefits.

Solution. If we make assumption (a) above and calculate net annualized worth, we obtain the following:

Machine X

$$A = 75 - 200(A/P, i\%, 6) + 50(A/F, i\%, 6) = 0$$

try 30%	0.37839	$0.07839 = +3.24$
try 40%	0.46126	$0.06126 = -14.189$

by interpolation: $i = \underline{\underline{31.9\%}}$

Machine Y

$$A = 120 - 400(A/P, i\%, 12) + 150(A/F, i\%, 12) = 0$$

try 30%	0.31345	$0.01345 = -3.364$
try 25%	0.26845	$0.01845 = +15.39$

by interpolation: $i = \underline{\underline{29.1\%}}$

If the maximum ROR were the correct criterion, then the decision would be to choose machine X. But if we check the net annual worth of each machine at a specific value of interest, then we may or may not arrive at the same solution.

To illustrate, let us take a value for i of 20 percent and determine the net annual worth of each machine.

Net annual worth of machine X

$$A(X) = 75 - 200\underbrace{(A/P, 20\%, 6)}_{0.30072} + 50\underbrace{(A/F, 20\%, 6)}_{0.10072} = \underline{\underline{\$19.9/\text{yr}}}$$

Net annual worth of machine Y

$$A(Y) = 120 - 400\underbrace{(A/P, 20\%, 12)}_{0.225262} + 150\underbrace{(A/F, 20\%, 12)}_{0.025262} = \underline{\underline{\$33.7/\text{yr}}}$$

In this case machine Y has the greater net worth and thus would be selected. But this is a different answer than when the rate of return was examined, so we must conclude that utilizing the net annual worth for a specific interest rate can yield results very different from those found by simply calculating the ROR of each investment. Because different conclusions can be reached, some further analysis is needed. At this point, the major conclusion to be drawn is that choosing among multiple alternatives involves *more* than simply choosing the alternative with the highest ROR. A return of 20 percent on $10,000 yields a greater profit ($2,000) than a return of 100 percent on a $1 investment ($1 profit) (assuming the investments are mutually exclusive and nonduplicative). If machine X is chosen (earning 31.9 percent on this $200), then the remaining money available ($400 − $200 = $200 in this case) is assumed to earn some MARR (for example 20 percent). Now the complete question to be answered is whether $200 earning 31.9 percent plus $200 earning 20 percent is better or worse than $400 earning 29.1 percent (machine Y). Calculation of the net annual worth at 20 percent reveals that machine Y is the better investment, assuming an MARR of 20 percent. The IROR method considers the *differences* between the alternatives over the same number of years. Figure 11.11 shows the cash flows extended to provide a comparison over the same total life span.

$$\text{NPW} = -200 + 45\underbrace{(P/A, i\%, 12)}_{3.1903} + 150\underbrace{(P/F, i\%, 6)}_{0.20718} + 100\underbrace{(P/F, i\%, 12)}_{0.04293}$$

try 30% NPW = −21.02

try 25% NPW = +13.83 (3.7252; 0.26215; 0.06872)

by interpolation, IROR = 27.0%

This means the $200 *additional* investment earns 27 percent. If the MARR is 20 percent, then machine X would earn 31.9 percent, and the unused additional investment would presumably earn 20 percent, while machine Y ($400 investment) would earn 29.1 percent. A $400 investment earning 29.1 percent is better than a $200 investment at 31.9 percent plus another $200 investment at 20 percent (the MARR).

The answer (decision) depends upon the MARR. For this example, if the MARR is less than 27.0 percent (the IROR in this case), then machine Y is the better alternative. If the MARR is between 27.0 and 31.9 percent, then machine X is the best alternative. If the MARR is greater than 31.9 percent, then *neither* alternative should be selected.

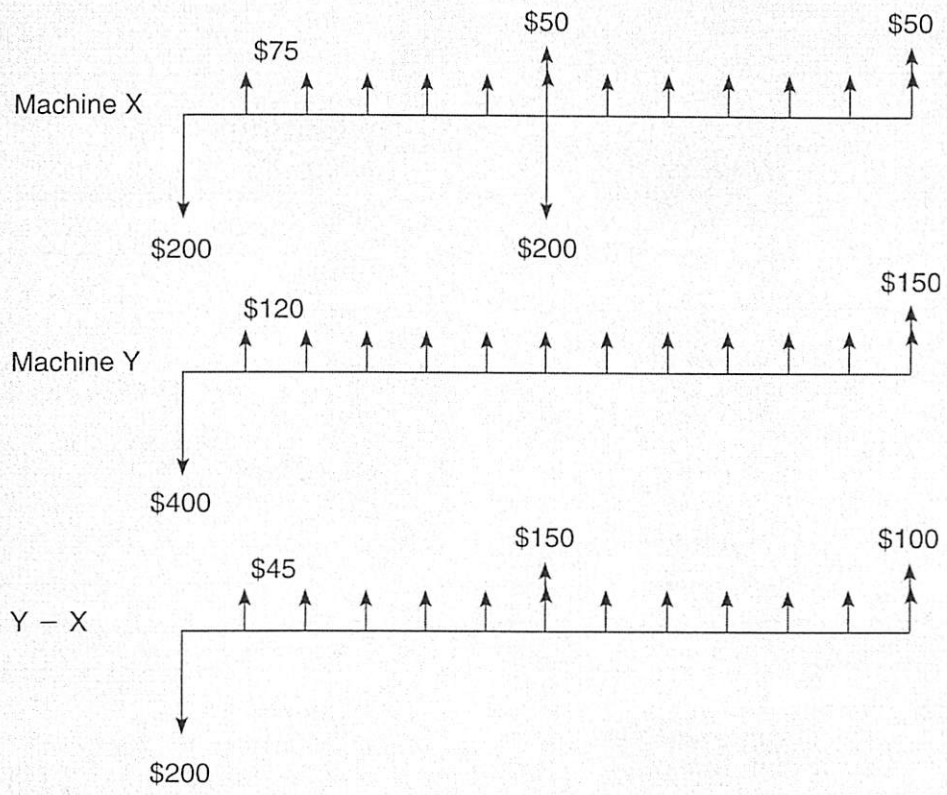


Figure 11.11 Cash flow diagram for Example 11.6.

COMPARING MULTIPLE ALTERNATIVES WHEN (1) THE MARR IS KNOWN, AND (2) THE "DO NOTHING" ALTERNATIVE IS AVAILABLE

Often there are a number of alternatives from which to choose, each being mutually exclusive but none required (the "do nothing" alternative is available). When this occurs, the time-consuming calculation of every alternative's ROR and the IROR for *every* difference between all possible combinations of alternatives becomes unnecessary *if* (1) the MARR can be established, and (2) the "do nothing" alternative is available. To analyze this type of multiple alternative, the following steps should be followed.

Step 1. List the alternatives *in order* of ascending first cost. The alternative with the lowest first cost is placed at the top of the list (or in the left column).

Step 2. Calculate the ROR for each alternative. Discard any alternative whose ROR is less than the MARR. (In those cases where there are no incomes and the "do nothing" alternative is not a possible choice, step 2 can be omitted because the ROR for all alternatives is less than zero, and the decision must be based on the IROR values.)

Step 3. Regroup the alternatives after discarding any unacceptable alternatives based on ROR.

Step 4. Stepwise determine the IROR for each difference in alternatives, beginning with the lowest cost alternative, discarding either (a) the lower cost alternative if the IROR is greater than the MARR, or (b) the higher cost alternative if the IROR is less than the MARR.

Step 5. Select the single remaining alternative after analyzing all alternatives.

The following example illustrates this method.

EXAMPLE 11.7

An engineering firm is considering five different small computers to aid the firm in running complicated and repetitive calculations. Analysis of the various machines, when applied to the firm's business, indicates the following cash flow information.

	F	G	H	I	J
First cost ($)	25,000	18,000	22,500	28,400	32,000
Annual net income ($)	5,000	3,800	4,800	6,000	6,600
Useful life (yr)	8	8	8	8	8

Which computer should be selected if the MARR is 12%?

Solution. First, arrange the alternatives in ascending order of first cost.

	G	H	F	I	J
First cost ($)	18,000	22,500	25,000	28,400	32,000
Annual net income ($)	3,800	4,800	5,000	6,000	6,600

Second, calculate the ROR of each alternative by setting the NPW = 0.

Computer G

$$\text{NPW} = -18{,}000 + 3{,}800(P/A, i\%, 8) = 0$$

$$\text{then } (P/A, i\%, 8) = 18{,}000/3{,}800 = 4.7368$$

$$\text{to find } i, \text{ try } 13\%, (P/A, 13\%, 8) = 4.7988$$

$$\text{try } 14\%, (P/A, 14\%, 8) = 4.6389$$

$$\text{then by interpolation, ROR} = 13.4\%$$

Computer H

$$\text{NPW} = -22{,}500 + 4{,}800(P/A, i\%, 8) = 0$$
$$\text{so } (P/A, i\%, 8) = 4.6875$$
$$\text{then by interpolation, ROR} = 13.7\%$$

Computer F

$$\text{NPW} = -25{,}000 + 5{,}000(P/A, i\%, 8) = 0$$
$$\text{so } (P/A, i\%, 8) = 5{,}000$$
$$\text{then by interpolation, ROR} = 11.8\%$$

Computer I

$$\text{NPW} = -28{,}400 + 6{,}000(P/A, i\%, 8) = 0$$
$$\text{so } (P/A, i\%, 8) = 4.7333$$
$$\text{then by interpolation, ROR} = 13.4\%$$

Computer J

$$\text{NPW} = -32{,}000 + 6{,}600(P/A, i\%, 8) = 0$$
$$\text{so } (P/A, i\%, 8) = 4.8485$$
$$\text{then by interpolation, ROR} = 12.7\%$$

Third, regroup the alternatives, after discarding those alternatives whose ROR does not meet the MARR (computer F in this case would be discarded).

	G	H	I	J
First cost ($)	18,000	22,500	28,400	32,000
Annual net income ($)	3,800	4,800	6,000	6,600
ROR (%)	13.4	13.7	13.4	12.7

Fourth, in turn determine the IROR between each combination of paired alternatives, discarding the least desirable one of each pair.

Thus computer G can be considered as the comparison between computer G and the "do nothing" alternative. Since the IROR of computer G compared to "do nothing" is 13.4 percent, and this is greater than the MARR of 12 percent, therefore discard the "do nothing" alternative. Comparing H to G, the cash flow becomes

	H–G
Incremental first cost ($)	4,500
Incremental net income ($)	1,000
$-4{,}500 + 1{,}000(P/A, i\%, 8) = 0$	
$(P/A, i\%, 8) = 4.500$	

By interpolation, the IROR = 14.9 percent > 12 percent MARR; therefore, discard G. Next, compare I over H.

	I–H
Incremental first cost ($)	5,900
Incremental net income ($)	1,200
$-5{,}900 + 1{,}200(P/A, i\%, 8) = 0$	
$(P/A, i\%, 8) = 4.9167$	

By interpolation, the IROR = 12.3 percent > 12 percent MARR; therefore, discard H. Next, compare J over I.

	J–I
Incremental first cost ($)	3,600
Incremental net income ($)	600
$-3{,}600 + 600(P/A, i\%, 8) = 0$	
$(P/A, i\%, 8) = 6.000$	

By interpolation, the IROR = 6.9 percent < 12 percent MARR; therefore, discard J. The one remaining alternative, computer I, should be selected.

Note that if the net present worth of each alternative is calculated at the MARR of 12 percent, the best alternative would be computer I, as determined using IROR analysis, and the same answer would be obtained using net annual worth or net future worth, as illustrated in the following table.

	F	G	H	I	J
First cost ($)	25,000	18,000	22,500	28,400	32,000
Present worth of annual net income $A(P/A, 12\%, 8)$ ($)	24,838	18,878	23,845	29,806	32,787
Sum ($)	−162	878	1,345	1,406	787

The natural question is, Why expend the effort calculating IROR when net PW or net annual worth will often yield the same result with much less effort? Two important reasons for calculating IROR follow:

1. The IROR values show the limits of i within which each selection is valid. A selection that is valid for a wide range of i values on both sides of the MARR is a more convincing selection than one whose priority would change

if the MARR should move a couple of percentage points in tandem with normal market fluctuations.

2. A graph of the equivalent values versus i (which can be plotted from just the IROR values if need be) shows the amount of the differences between the equivalent worths of the competing alternatives for all values of i. This allows value judgments to be made by those who are considering buying the second or third most economical choice due to intangibles such as better dealer service, familiarity with the product, and so on. The graph shows just how much more the second or third alternatives will cost for each value of i, and this extra cost then can be weighed against the perceived intangible values.

SPREADSHEET APPLICATIONS

The calculations associated with making comparisons of investment opportunities using the incremental rate of return method are necessarily repetitive. Spreadsheets simplify the labor of making many similar calculations. Spreadsheets also offer the advantage of providing predefined formulas, such as the IRR function used in this example, which allow us to associate more than one set of values with a formula and calculate the interest rate of the incremental differences. By using simple macros, we can compare data and create text prints of decision parameters, as shown in row 41 of the following example, where we are comparing different interest-earning opportunities to our predefined minimum attractive rate of return, the MARR.

	A	B	C	D	E	F	G
1	EXAMPLE 11.7 MARR = 12%						
2	Solving our previous Example 11.7 using the EXCEL spreadsheet software						
3							
4	Order the investments, starting with the least first cost on the left side of the table.						
5	EOY	G	H	F	I	J	
6	0	−18000	−22500	−25000	−28400	−32000	
7	1	3800	4800	5000	6000	6600	
8	2	3800	4800	5000	6000	6600	
9	3	3800	4800	5000	6000	6600	
10	4	3800	4800	5000	6000	6600	
11	5	3800	4800	5000	6000	6600	
12	6	3800	4800	5000	6000	6600	
13	7	3800	4800	5000	6000	6600	
14	8	3800	4800	5000	6000	6600	
15							
16	IRR	13.4%	13.7%	11.8%	13.4%	12.7%	Calculate individual ROR.
17	Decision	Accept	Accept	Reject	Accept	Accept	Test against the MARR.
18							
19	We will temporarily accept the alternatives except for F, which does not provide the minimum ROR.						

(Spreadsheet continued)

	A	B	C	D	E	F	G
20	We now calculate incremental rates of returns for our investments, starting with the lowest first cost, G.						
21							
22	CELL CONTENTS: The following cell contents will allow us to evaluate the problem using a spreadsheet.						
23	Row 16 contents are typical of cell B16:				IRR(B6:B14)		
24	Row 17 contents are typical of cell B17:				IF(B16>=C1, Accept , Reject)		
25	Rows and columns for the table below are typical of cell B30:				C6-B6		
26	Row 40 contents are typical of cell B40:				IRR(B30:B38)		
27	Row 41 contents are typical of cell B41:				IF(B40>=C1, Accept , Reject)		
28							
29	EOY	H-G	I-H	J-I			
30	0	−4500	−5900	−3600			
31	1	1000	1200	600			
32	2	1000	1200	600			
33	3	1000	1200	600			
34	4	1000	1200	600			
35	5	1000	1200	600			
36	6	1000	1200	600			
37	7	1000	1200	600			
38	8	1000	1200	600			
39							
40	Delta IORR	14.9%	12.3%	6.9%	Calculate incremental rate of return (IROR).		
41	Decision	Accept	Accept	Reject	Test IROR against the MARR.		
42							
43	Therefore, computer I should be chosen over the other investment opportunities.						
44							

SUMMARY

In private as well as business life, many expenditures are required that do not yield a profitable return directly. Therefore, a negative rate of return often results, such as when purchasing tires for a car. However, when alternative candidates for purchase are available, an incremental rate of return (IROR) can be calculated on the increment of investment. The IROR is defined as the rate of return for an *additional* increment of investment. Finding the IRORs leads to finding which alternative is the preferred selection. To solve the IROR problems, the following procedures are employed.

Step 1. List the alternatives *in order* of ascending first cost, with the lowest first cost at the top of the list.

Step 2. Comparing each successive alternative in turn, determine the IROR for each increment of investment, beginning with the lowest cost alternative. Compare each IROR with the minimum attractive rate of return (MARR), if known. Immediately discard either (a) the lower cost alternative if the IROR

is greater than the MARR or (b) the higher cost alternative if the IROR is less than the MARR.

If the MARR is not known, or if more information is desirable, then all IRORs should be calculated, and the ranges of MARR values listed that are associated with each alternative.

PROBLEMS FOR CHAPTER 11, INCREMENTAL RATE OF RETURN

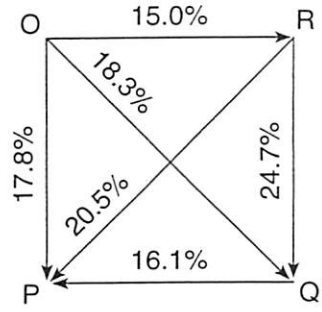
Problem Group A

A series of alternative investments is given, each with one or more P, A, F values, but all alternatives have the same n values.

A.1. Assume you are considering the following three investments. Using incremental rate of return analysis techniques, draw an arrow diagram and determine which alternative, if any, you should select (a) if MARR is 15 percent; (b) if MARR is 20 percent. The "do nothing" alternative is available.

	P	Q	R
Initial investment ($)	40,000	30,000	20,000
Annual net income ($)	7,400	5,700	3,200
Salvage value ($)	0	0	0
Life (yr)	20	20	20

[*Ans.* (a) select P; (b) select O (do nothing)]



A.2. Same as Problem A.1, except the life of each alternative is 30 years.

A.3. A sanitary sewer can be constructed using either a deep cut or a lift station, with the following costs estimated. Using incremental rate of return analysis techniques, determine which alternative should be selected for an MARR of 7 percent.

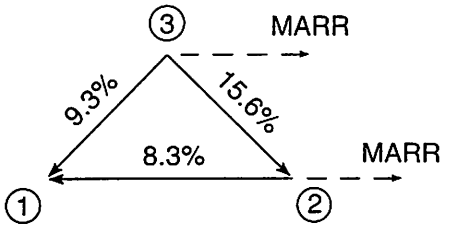
	Deep-cut alternative	Lift-station alternative
Installation cost of pipe ($)	275,000	235,000
Installation cost of lift station ($)	—	15,000
Annual maintenance costs ($)	400	1,800
Expected life (yr)	40	40

(*Ans.* IROR = 4.73% < 7%; select lift station)

A.4. Same as Problem A.3, except the expected life of the lift station is only 10 years, while the deep-cut life remains at 40 years.

A.5. A contractor asks your advice on selecting the best of three alternative financing methods for acquiring company pickup trucks. The alternatives are listed as follows.

1. Purchase the pickups now for $15,000 each. Sell after four years for $3,000 each.
2. Lease the pickups for four years at $3,600 per year paid in advance at the beginning of each year. Contractor pays all operating and maintenance costs, just as if she owned them.
3. Purchase the pickups on time payments, with $2,250 down payment and $3,960 per year at the end of each year. Sell after four years for $3,000 each.
   (a) Draw an arrow diagram showing the three alternatives.
   (b) Show which ranges of i favor which alternatives.



[*Ans.* (a) (b) if MARR > 15.6% select alternative 3; if 15.6% > MARR > 8.3% select alternative 2; if MARR < 8.3% select alternative 1]

A.6. A consulting firm finds it must provide reprints of a large number of items such as plans and specifications for its customers at a fixed nominal cost. It narrows the alternative means of providing this service for the next five years down to the following list of three.

1. Outside contract. A $1,000 deposit (returnable at EOY 5) is required to reserve these facilities. In addition, the fixed fee from customers is not enough to cover the cost of the work. Loss to the consultant amounts to $100 per year.
2. Lease the reproduction equipment, brand B. A deposit of $4,000 is required by the lessor, returnable upon termination of the lease. A cash flow profit of $320 per year for the reproduction work is assured under this plan.
3. Purchase brand C with guaranteed buy-back. This firm sells the reproduction equipment for $10,000 and guarantees to buy it back for 50 percent of the purchase price in five years. Calculations indicate this plan would yield a positive cash flow of $2,000 per year (not counting the purchase and resale cash flows).

In each case the study period is five years.

(a) Find the rate of return for each alternative.
(b) Find the incremental rate of return for each pair.

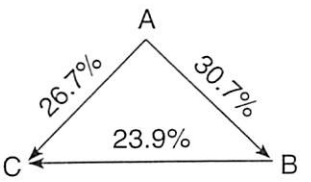
(c) Draw two arrow diagrams: (i) without the "do nothing" alternative; (ii) with the "do nothing" alternative.
(d) Analyze the diagrams and tell which alternative investment is best suited for each range of i.
(e) If the consulting firm is earning 20 percent on capital invested in the firm but must provide the reproduction service, which is the better alternative?
(f) If the consulting firm has the option of *not* providing the reproduction service, and it is earning 20 percent on capital, which is the better alternative?

A.7. Your client, a contractor, needs some new trucks to complete a project currently under contract. Analyze the three alternative types of trucks on the basis of the information listed below, and advise which is the best buy.

Type of truck	Cost new	Annual O & M cost	Annual increase in O & M costs	Resale value at EOY 4
A	$50,000	1.70/mi	$0.16/mi/yr	$10,000
B	56,000	1.60	0.16	15,000
C	65,000	1.50	0.09	15,000

The trucks will average 20,000 miles per year and will be sold at the end of four years for the resale values listed.
(a) Find the incremental rate of return between A and B. (*Hint:* Try i for B−A between 30 and 40 percent.)
(b) Assume the IROR of C−A is 26.7 percent and that of C−B is 23.9 percent. Draw an arrow diagram showing the IROR values and tell what ranges of MARR favor which alternatives.
(c) Show which alternative is recommended if the contractor's MARR = 15 percent.

[*Ans.* (a) B−A IROR = 30.7%; (b) if MARR > 30.7%, select A; if 30.7% > MARR > 23.9%, select B; if MARR < 23.9%, select C; (c) select C]



A.8. An OSHA inspector has just ruled that the lighting fixtures in your client's plant are inadequate to meet minimum standards. New lighting must be installed in order to stay in business. Three alternative installations are available with figures as tabulated below. In order to assist your client in making the best selection, develop the following information.
(a) Find the incremental rate of return between A and B.
(b) Assume the IROR of C−A is 17.1 percent and that of C−B is 32.4 percent. Draw an arrow diagram showing which alternative to choose for MARR = 5 percent.
(c) Show the break points (if any) and tell which ranges of MARR favor which alternatives.

Type	Cost new	Annual operating cost	Annual increase in operating cost	Salvage value at EOY 20
A. Mercury	$30,000	$14,000/yr	$2,000/yr/yr	$ 3,000
B. Fluorescent	45,000	13,000	1,900	13,000
C. Metal halide	54,000	10,000	1,800	3,000

(*Hint:* Find i, B−A between 10 and 20 percent.)

A.9. Assume that you are a consulting engineer, and your company car has just been totaled in wreck. You are now selecting a replacement car. Any of the three listed below is acceptable from a functional point of view.

(a) Find the incremental rate of return for each pair of alternatives.

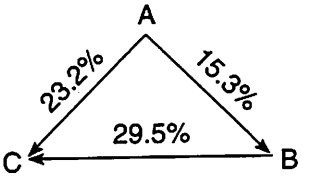
(b) Draw an arrow diagram showing which alternative to choose if MARR = 15 percent.

(c) Show the break points in the MARR (if any), and tell what ranges of MARR favor which alternatives.

Type	Cost new	Annual O & M cost	Annual increase O & M cost	Resale value at EOY 4
A. Electra	$15,000	51/mi	$0.042/mi/yr	$3,000
B. Monaco	17,100	48	0.048	4,500
C. Montego	19,500	45	0.027	4,500

The car will be driven about 20,000 miles per year and sold at the end of the fourth year. (*Hint:* Try i for B−A between 10 and 20 percent; C−A between 20 and 30 percent; C−B between 20 and 30 percent.)

[*Ans.* (a) − A + B 15.3%, − A + C 23.2%, − B + C 29.5%; (b) see diagram below; (c) if MARR > 23.2%, select A; if MARR < 23.2%, select C]



A.10. A friend of yours is considering the purchase of a home. The seller offers a choice of two options:

	Option A	Option B
Purchase price	$80,000	$78,000
Down payment	10,000	12,000
Mortgage	70,000	66,000
Interest rate on the mortgage	9%	9%
Payout period for mortgage	25 yr	25 yr

(a) What is the incremental rate of return on the difference between the two options? (*Hint:* Try between 15 and 25 percent.)

(b) Which option do you recommend and with what conditions (if any)?

Problem Group B

The n values of each alternative differ.

B.1. A subscription to a professional magazine is available at three different rates:

1 yr \$100
2 yr \$175
3 yr \$ 250

Assume you intend to subscribe probably for the rest of your professional career. Find the before-tax incremental rate of return on:

(a) The two-year subscription compared to the one-year.

(*Ans.* 33.3%)

(b) The three-year subscription compared to the one-year.

(*Ans.* 21.5%)

(c) If you can invest your money elsewhere at 20 percent, before taxes, which alternative is the better selection?

(*Ans.* Select the two-year subscription.)

B.2. A professional society whose dues are normally \$100 per year is offering life memberships for \$1,250. Assume you are a young engineer of 24, considering the life membership offer. If you stay active in the society until age 65 (41 years), what rate of return will the investment in life membership yield

(a) Compared to the \$100-per-year membership dues paid in advance, with the first year's dues payable right now?

(b) Compared to the \$100-per-year membership with dues paid in advance, but where the dues are expected to increase by \$5 per year every year (\$100 the first year, \$105 the second, etc.)?

B.3. The school board asks your help in determining which is the better alternative for obtaining portable classrooms.

1. One-year renewable leases, with \$1,000 per month paid in advance at the first of every month.

2. Purchase with guaranteed buy-back. The portable classrooms are purchased for \$100,000 each, and the vendor agrees to repurchase them from the school board at the end of three years for \$57,000, if the school board should want to sell them at that time.

3. Three-year lease for \$30,000 paid in advance.

Find the rate of return of:

(a) Alternative 2 compared to 1.

(*Ans.* $< 0.0\%$)

(b) Alternative 3 compared to 1.

(*Ans.* 13.0%)

(c) If a selection of the most economical alternative is made when funds may be borrowed at 6 percent, does the selection change if interest rates rise to 9 percent?

(*Ans.* No)

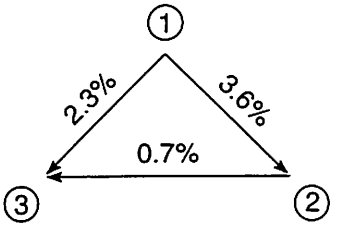
B.4. Three grades of roofing are available for a proposed office building, and you are asked to determine which is the most economical. Data on the roofs are as follows. (Find i in terms of A rather than PW.)

1. Fifteen-year roof costs $18,000 plus $1,000 per year for end-of-year maintenance.
2. Twenty-year roof costs $22,000 plus $500 per year for end-of-year maintenance.
3. Twenty-five-year roof costs $30,000 and requires no maintenance.
   (a) Plot a graph of A versus i for each alternative (i is the pretax rate at which the building owner can invest available funds).
   (b) On the graph, show which ranges of i favor which alternative.

B.5. Your firm has a large fleet of cars, and tires are a major expenditure. You are asked to determine which of the three alternatives listed below is the most economical. The vehicles average 2,000 miles per month. Assume i is the monthly rate of interest and is compounded monthly.

1. 20,000-mile tire costing $63 each.
2. 30,000-mile tire costing $87 each.
3. 40,000-mile tire costing $114 each.
   (a) Find the IROR for each combination of alternatives and draw an arrow diagram.
   (b) Find which ranges of MARR favor which alternatives.

   [*Ans.* (a) see diagram below; (b) for MARR > 3.6%, select 1; for 3.6 > MARR > 0.7%, select 2; for MARR < 0.7%, select 3)]



Problem Group C

Periodic and gradient payments.

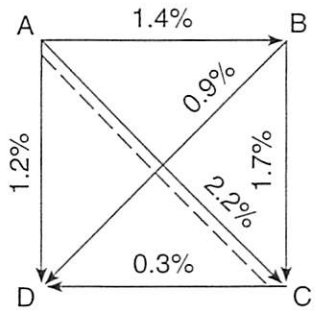
C.1. Assume that 18 months ago the state highway patrol purchased a new airplane for $55,000. Now the plane needs repairs. They need to decide whether to repair the plane and keep it, or trade it in on a new one. The "do nothing" alternative is not available. Whichever plane is selected will be kept for the next 60 months and then sold. The costs of each alternative follow:

	A: Keep and repair	B: Trade in on new
Market value if sold now, as is	$40,000	
Repairs needed now	6,500	
Dealer's advertised price for new plane		$70,000
Trade-in allowance for present plane, as is		48,000
Dealer financing on the difference		22,000
New-plane amortized loan, for $22,000 at 0.75%, 48 months first payment at EOM 1		
O & M for the first month (100 hr/mo.)	$10.00/hr	$8.90/hour
Gradient O & M for each succeeding month	$0.10/hr/month	$0.10/hr/month
Periodic overhauls at 4-month intervals, with the first at EOM 6 (geo. grad.)	$ 1,000	$ 850
Each periodic OH costs 4% more than the previous one		
Resale value at EOM 60	$30,000	$44,000

(a) Use the equivalent NAW to find the IROR. (*Hint:* Try 1 percent.)
(*Ans.* 1.35%).

(b) Assume other alternatives are available, represented by C and D. Draw the arrow diagram and show which alternative is favored if MARR = 1%.
(*Ans.* Select C.)

	B	C	D
A	find	2.2%	1.2%
B		1.7%	0.9%
C			0.3%



(c) Fill in the table below and show the MARR ranges favoring each alternative.
(*Ans.* > 2.2%→A; none→B;< 2.2% > 0.3%→C;< 0.3%→D)

MARR > _____ favors A
MARR < _____ but > _____ favors B
MARR < _____ but > _____ favors C
MARR < _____ favors D

C.2. A city water department is serving an urban area that is growing so large that the demand for water is about to exceed the capacity of the existing water main to supply it. The alternatives for the department include (a) installing a parallel water main, or (b) taking out the existing main and replacing it with a larger one. (The "do nothing" alternative is not available.) If the existing main is removed, it will have a net salvage value of $500,000 after all of the removal and cleanup costs. Both alternatives are expected to have 40 more years of useful life. Use the NAW method to determine which is the better alternative.

	A: Leave existing pipe in place; install parallel pipe	B: Replace with larger pipe
Cost of the one new parallel pipe only	$1,440K	
Cost of new larger pipe only		$3,000K
Government financing, available *only* for the larger pipe alternative, is a $1,800K loan at 5% amortized over 10 yr, with first payment due at EOY 1		
O & M costs	350K/yr	252K/yr
Gradient in O & M costs	10K/yr	10K/yr
Periodic maintenance at 3-yr intervals		
Each PM costs 10% more than the previous PM		
The first PM at EOY 5 costs	50K	150K
Salvage value at EOY 40	Zero	Zero

(a) Use the equivalent PW to find the IROR. (*Hint:* Try 12 percent.)

(b) Assume other alternatives, represented by C and D, are available. Draw the arrow diagram and show which alternative is favored if MARR = 10%.

	B	C	D
A	find	8.2%	9.3%
B		3.7%	8.9%
C			16.3%

(c) Fill in the table below and show the MARR ranges favoring each alternative.

MARR > _____ favors A
MARR < _____ but > _____ favors B
MARR < _____ but > _____ favors C
MARR < _____ favors D

C.3. Assume that you now own a two-year old car (present used car A) and are considering whether to keep it or buy a new car B. The data for each alternative are as follows:

	Present used car A	New car B
Current market value	$5,000	
Cost new: dealer's sticker price		$9,000
Dealer's trade-in allowance for present car		$5,800
Cash payment to dealer in addition to trade-in		$3,200
Resale value 36 months from now	$2,200	$3,000
Car will be driven	1,500 mi/month	1,500 mi/month
O & M costs (charged at EOM)	$0.14/mi	$0.09/mi
Gradient O & M costs	$0.0001/mi/month	$0.0001/mi/month
Periodic maintenance	$190/6,000 mi	$100/6,000 mi
First maintenance at EOM 4, last at EOM 36		
Gradient in periodic maintenance	$16/each/each	$6/each/each
Prestige value of new car over present used car	0	$15/month

(a) Find the IROR on the incremental investment in the new car B. Use the equivalent PW method. (*Hint*: Try $i = 2\%$ /month.)

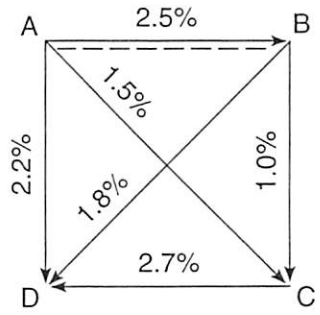
(*Ans.* 2.50%)

(b) Assume that other alternative new cars C and D are also under consideration and that the data tabulated below is correct. Assume D has the highest first cost, and C has the next highest first cost. **(i)** Draw an arrow diagram showing the four alternatives and the

direction of each arrow between alternatives. (**ii**) Show by dashed arrows the method of reaching the better alternative if the MARR is 2 percent per month.

(*Ans.* Select B.)

	B	C	D
A	find	1.5	2.2
B		1.0	1.8
C			2.7



C.4. Assume you are retained as a consulting engineer to a contractor who must either overhaul or replace his presently owned bulldozer. The present dozer is just two years old and was completely financed by a loan of $60,000, which is being paid off in six equal end-of-year installment payments with interest at 10 percent on the unpaid balance. This existing dozer could be sold now for $40,000 cash. There are two replacement dozers available that could handle the anticipated work load. Analyze the costs (shown below) of the three alternatives to determine which is the better investment.

Type	Overhaul and keep A	Replace with B
Initial cost		$40,000
Cost of overhaul	$10,000	
O & M costs	$ 7,500/yr	$ 9,000/yr
Gradient O & M costs	1,000/yr/yr	1,200/yr/yr
Major maintenance every 2 yr, including EOY 10	5,000/each	6,000/each
Savings to your firm compared to existing dozer	9,000/yr	9,000/yr
Gradient savings to your firm	1,400/yr/yr	1,400/yr/yr
Increment in major maintenance	1,100/each/each	1,250/each/each
Resale value of dozer at EOY 10	5,000	2,000
Estimated life	10 yr	10 yr

(**a**) Find the incremental rate of return between the lowest cost alternative and the next higher cost alternative. Use equivalent annual costs and income. (*Hint:* Try 25 percent.)

(**b**) Assume the incremental rates of return for the other alternatives are as listed below. Draw an arrow diagram showing the three alternatives and the direction of each arrow between the alternatives.

(**c**) Show by dashed arrows the method of reaching the better alternative of the three if the MARR is 15 percent.

	A	C
B	find	12.2
A		4.8

Problem Group D

Formulate your own problem.

D.1. Formulate a problem involving alternative methods of acquisition of goods or services of interest to you. Find the better alternative by the IROR method, and determine the ranges of i for which each alternative is feasible.