ENGINEERING ECONOMIC AND COST ANALYSIS

THIRD EDITION

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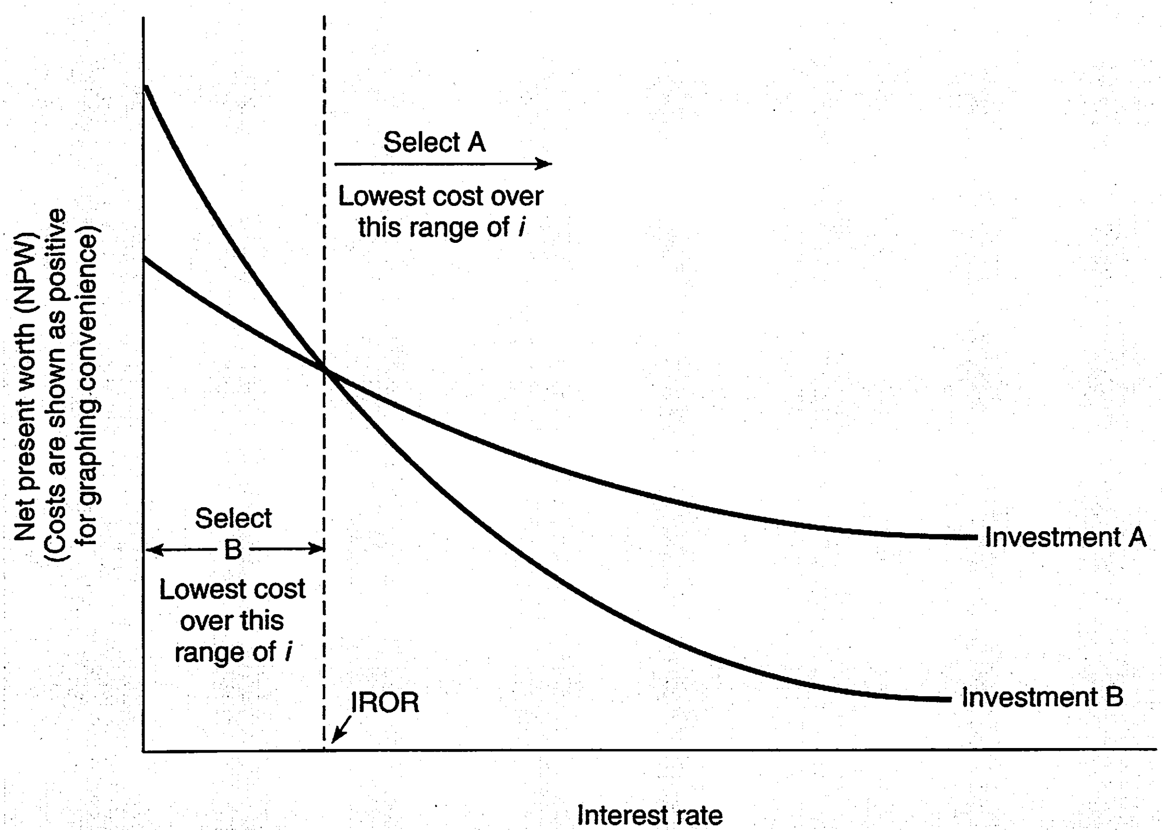
CHAPTER **11**

Incremental Rate of Return (IROR) on Required Investments



KEY EXPRESSIONS IN THIS CHAPTER

IROR = Incremental Rate of Return, the interest rate of return on each increment of added investment.

MARR = Minimum Attractive Rate of Return, the minimum acceptable rate of return. The MARR represents alternative outside investment opportunities. Investment opportunities yielding *less* than the MARR are normally considered neither competitive nor worthwhile.

RATE OF RETURN ON THE INCREMENT OF INVESTMENT

Circumstances, often beyond our control, frequently require us to make sizable expenditures that do *not* produce income or add to profit. For instance, new environmental regulations may require an electric power plant to install additional pollution-control devices, a construction contractor may receive orders from OSHA to buy new safety equipment, or a private car owner may need a new set of tires for the car. None of these expenditures produce income by themselves, but all are necessary expenditures required for the continued operation of some important function.

Competitive alternatives usually exist for fulfilling most needs in our relatively free economy. The power plant can consider several alternative methods of pollution control, the contractor can select from competing safety equipment, and the car owner can choose from a wide selection of tires. To remain competitive, the alternatives with higher first costs frequently have either longer lives, lower operating and maintenance costs, higher resale values, greater capacity, or some combination thereof. Can the car owner afford to pay \$125 more in first cost for a set of tires in order to extend the expected life from 24 months to 36 months? Even though there is no direct dollar income or "return" resulting from this type investment, the incremental rate of return (IROR) method is well suited and often used to find the most economical alternative.

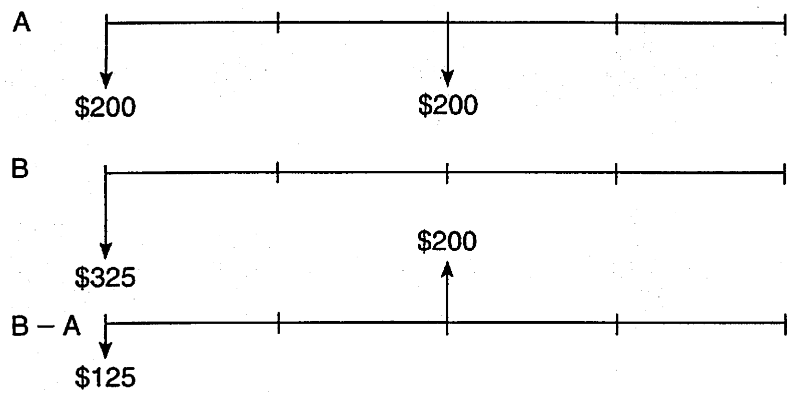
Instead of focusing on the rate of return earned by *every* dollar of investment, as in the rate of return method, the IROR method looks at the rate of return earned by each *additional* optional dollar of investment *above some required minimum.* Thus the IROR is the return on each optional incremental dollar of capital cost investment.

The problem now assumes the following form. If \$1 *must* be invested with no measurable return expected, and an additional (incremental) \$1 *may* be added to this investment with a positive return (savings in O & M, longer life, etc.) expected on the second \$1, should the optional second \$1 be invested in addition to the mandatory first \$1? The following example illustrates this situation.

EXAMPLE 11.1 **(Given: Must purchase either A or B; find the incremental rate of return on the increment of cost)**

A car owner has worn-out tires and no alternative transportation. He can buy a set of tires expected to last two years for \$200, or a set of tires expected to last four years for \$325 (\$125 more). Which is the better alternative?

Solution. The solution may be found through IROR. First clarify the problem situation by drawing the cash flow diagram of each alternative and the table of required expenditures A, optional additional expenditures B, and the incremental difference, B − A. For simplicity, assume that over a four-year period of time two sets of the two-year tires would be needed, and that the replacement cost for the two-year tires at EOY 2 remains at $200 and does not change. The increment of investment at PTZ is the difference in the first cost of



n	A	B	B – A
0	−200	−325	−125
1	0	0	0
2	−200	0	+200

the alternative sets of tires, or $325 – $200 = $125. If the extra $125 is paid out now, then there will be no need to buy another set of tires at the EOY 2, and a savings of $200 will occur at that point in time. Thus an investment of an incremental $125 now will produce a savings (or return) of $200 at EOY 2. The ROR on the incremental investment is called the IROR, and in this simplified case can be calculated as

$$F/P = (1+i)^n$$
$$\$200/\$125 = (1+i)^2$$
$$i = 1.6^{0.5} - 1 = 0.2649 = \underline{\underline{26.5\%}} = \text{IROR}$$

An extra $125 invested in the four-year tires yields the equivalent of 26.5 percent rate of return. Thus, the IROR on the increment of investment is 26.5 percent.

Question: Could this problem be solved by the ROR, PW, AW, or FW methods?

Answer 1, ROR: It could not be solved by the ROR (rate of return) method. Since there is no income, the ROR would be infinitely negative.

Answer 2, PW, AW, or FW: The problem can be "solved" by any of these methods. However, the lower cost alternative (better buy) will be to buy the $325 tires if the interest rate is less than 26.5 percent, and then switch to the

\$200 tires if an interest rate of higher than 26.5 percent is used. If exactly 26.5 percent interest is used, then the equivalent costs will be identical. For instance, examine the PW of each of the two alternatives at 10 percent.

$$\text{PW of two sets of 2-yr tires at } i = 10\%$$
$$P = -\$200 - \$200\underbrace{(P/F, 10\%, 2)}_{0.82645} =$$
$$P = -\$200 - \$165.29 = \underline{\underline{-\$365.29}}$$
$$\text{PW of one set of 4-yr tires, } P = \underline{\underline{-\$325.00}}$$

Conclusion. The one set of *four-year* tires is the better buy by a significant margin.

Now try a solution at $i = 30$ percent.

$$\text{PW of two sets of 2-yr tires at } i = 30\%$$
$$P = \$200 + \$200(P/F, 30\%, 2) =$$
$$P = -\$200 - \$118.34 = \underline{\underline{-\$318.34}}$$
$$\text{PW of one set of 4-yr tires at } i = 30\%$$
$$P = \underline{\underline{-\$325}}$$

Conclusion. The selection has reversed, and now the *two sets of two-year* tires are the better buy by a significant margin. If the PW were found at $i = 26.5$ percent, the PW values of the two alternatives would be equal.

Regarding the AW and FW methods, all three methods (PW, AW, FW) are related by constants. Therefore, whichever alternative costs less by one method also costs less by the other two methods. (An exception occurs when $n = \infty$, and the FW becomes infinitely large, whereas the PW and AW values normally do not.)

The *"do nothing" alternative.* Note that in Example 11.1 it was assumed that the car *must* be kept in operation and tires of some description *must* be purchased. The car owner could not afford to do nothing; therefore the "do nothing" alternative was not available. In this case if there were *no* negative consequences from doing nothing, then the "do nothing" alternative of not buying any tires at all would have been the more economical choice, since no costs are incurred.

Which set of tires should the owner buy? The better alternative depends upon the owner's Minimum Attractive Rate of Return (MARR). If the incremental \$125 can be invested elsewhere at a higher rate than 26.5%, then buy the less-expensive tires and invest the \$125 elsewhere at the higher rate. If the MARR is less than 26.5%, then invest the incremental \$125 in the more expensive tires.

MARR, THE MINIMUM ATTRACTIVE RATE OF RETURN

In addition to the discussion of the MARR in Chapter 1, the following points should be reviewed at this time. By establishing a MARR, the following assumptions are made:

1. Sufficient funds are available to purchase the highest cost alternative under consideration, should the investment be justifiable.
2. If one of the lower cost alternatives is selected, the excess funds (the difference in cost between the selected lower cost alternative and the highest cost alternative) are assumed to be invested elsewhere earning the MARR.

Thus, the MARR involves consideration of opportunities to invest the *total available* funds, and not just the funds utilized in any particular alternative. The following paragraphs and examples illustrate these points.

OPTIONAL ORIENTATION FOR IROR SOLUTION MATRIX

Most of the IROR problems in this chapter are laid out so that each alternative investment (A, B, C, etc.) heads up a separate column, with the rows occupied by different types of payment items (cost new, O & M, etc.). Alternatively, the problems can be solved just as easily using a layout with the alternatives in the rows and the payment items in the columns.

The first part of Example 11.2 illustrates the use of alternatives placed in the columns. In Example 8.2 Continued the orientation is reversed for convenience, and the alternatives appear in the rows.

COMPARISON WITH EQUAL LIFE SPANS

The following example illustrates an IROR comparison of alternatives all having the same life span.

EXAMPLE 11.2 **(Given two alternative sets of P, A, F, n, find IROR)**

Assume a coal-burning electric power generating plant must select one pollution-control device from two suitable types available having the characteristics listed below. The "do nothing" alternative is not available.

Pollution-control device	A	B
Cost new	$1,000,000	$1,300,000
Operating costs	$ 300,000/yr	$ 265,000/yr
Life, *n*	20 yr	20 yr
Salvage value	$ 50,000	$ 60,000

Recommend the preferable alternative based on the incremental rate of return (IROR).

Solution. *Note:* Neither of these alternatives produces income for the owner except the small salvage value at EOY 20. Therefore, there is no need to determine the ROR, since it is obviously some large negative number. The problem now is to compare the extra benefits available from alternative B (save $35,000 per year in operating costs plus the small increase in salvage value) to the extra $300,000 initial cost of alternative B. To compare, first find the *incremental* rate of return (IROR) on the extra $300,000 investment in B, and then compare that IROR with the MARR.

Comparing the two alternatives, the lowest initial investment is required by alternative A, at $1,000,000. Alternative B requires $1,300,000 capital outlay ($300,000 more than B). This raises the basic question, Since a minimum of $1,000,000 must be spent on pollution control anyway, should an extra $300,000 be invested in alternative B in order to save $35,000 per year in operating costs, plus receive $10,000 more in salvage value at EOY 20? Will the lower operating cost and higher salvage value of B more than compensate for the extra $300,000 initial cost of alternative B? To answer these questions an incremental rate of return (IROR), or rate of return on the increment of investment, is calculated. As the name implies, the method is similar to that used for rate of return, but only the *increments* of cost and saving (or income) need be considered. For this example, the columns are for the alternatives and the rows are for cost items. The IROR can be found by the following procedure, illustrated in Example 11.2.

1. Determine which alternative has the lower first cost, place the name of that alternative at the top of the left column and list all the payment items for that alternative in the same left column.
2. Find the next higher-first-cost alternative and list the corresponding payment items in a column just to the right of the previous list.
3. Subtract the cost of the left-hand column from the cost of the right-hand column. This is done by simply changing the signs in the left column and adding the figures in the left column to those in the right column. Write an equivalent-worth equation for each type of payment item in the

list, using your choice of present worth, annual worth, or future worth equations.

4. Select a trial i value for the IROR, use it in each equation, and sum the results of all the equations. If the sum turns out to be a positive value, then the trial i selected is too low (assuming the graph is normal and curves down to the right). Therefore, try a higher trial value of i, and repeat until the sum becomes negative. (If the second trial sum of the equations turns out to be a higher positive value, the graph may curve up to the right, or there may be an error in the problem setup or calculations.)
5. After finding the i values for which the equivalent-worth values bracket the zero point, interpolate to find the actual IROR value of i.

For Example 11.2, the stepwise procedure is followed below.

Step 1. Alternative A is found to have the lower first cost of \$1,000,000 compared to \$1,300,000 for alternative B. The payment items for A are listed in the left-hand column.

Payment item	Alternative A
Cost new	−\$1,000,000
Operating	−\$ 300,000
Salvage value	+\$ 50,000

Step 2. The payment items for the next higher alternative B are listed in a column just to the right.

Payment item	Alternative A	Alternative B
Cost new	−\$1,000,000	−\$1,300,000
Operating	−\$ 300,000	−\$ 265,000
Salvage value	+\$ 50,000	+\$ 60,000

Step 3. The items in column A (the left column) are subtracted from column B by changing the sign and adding. Present worth is selected for the equivalent-worth equations (either AW or FW would serve equally well) and appear as shown.

	Alternative A	Alternative B	
Cost new $P_1 =$	+\$1,000,000	−\$1,300,000	=
Operating $P_2 =$	(+ 300,000	− 265,000)$(P/A, i\%, 20)$	=
Salvage value $P_3 =$	(− 50,000	+ 60,000)$(P/F, i\%, 20)$	=

Step 4. Next, a trial value of i is assumed. In this case try $i = 9$ percent. This value is used in each equation and the results summed, as shown.

		Alternative A	Alternative B	B − A trial i value 9%
Cost new	$P_1 =$	+ \$1,000,000	−\$1,300,000 =	− \$300,000
Operating	$P_2 =$	(+ 300,000	− 265,000)(P/A, i%, 20) =	+ 319,499
Salvage value	$P_3 =$	(− 50,000	+ 60,000)(P/F, i%, 20) =	+ 1,784
Total				+ \$ 21,283

The total is a relatively small plus value, which indicates that 9 percent is a little lower than the sought-for IROR, but close. The next trial is made at $i = 11$ percent.

		Alternative A	Alternative B	B − A trial i values 9%	11%
Cost new	$P_1 =$	+ \$1,000,000	− \$1,300,000 =	−\$300,000	−\$ 300,000
Operating	$P_2 =$	(+ 300,000	− 265,000)(P/A, i%, 20) =	+ 319,499	+ 278,716
Salvage value	$P_3 =$	(− 50,000	+ 60,000)(P/F, i%, 20) =	+ 1,784	+ 1,240
Total				+\$ 21,283	−\$ 20,043

The resulting NPW now has changed from a positive to a negative value, indicating that the IROR value of i lies between 9 and 11 percent. Interpolation yields

$$i = 9\% + \frac{21{,}283}{21{,}283 + 20{,}043} \times 2\% = \underline{\underline{10.0\% = \text{IROR}}}$$

Thus, the extra \$300,000 required for alternate B over alternate A earns a rate of return (IROR) of 10.0 percent through lower operating costs and higher salvage value.

Assuming the firm has the additional \$300,000 ready for investment, the decision makers must now consider whether other alternative investments would yield more than the 10.0 percent return available through investing the incremental \$300,000 in alternative B. In response to this question a minimum attractive rate of return (MARR) is established. This MARR represents alternative investment opportunities of comparable risk and other attributes. Only comparable investment opportunities with a rate of return equal to or higher than the MARR are considered as worthwhile. Opportunities to invest at a rate *lower* than the MARR are usually rejected as not sufficiently profitable. If a lot of good opportunities occur to invest at the MARR rate or higher, and investment funds run short, the MARR can be raised to suit the current supply and demand for the firm's investment funds. (In addition to the rate of return, investment opportunities are evaluated for risk, compatibility with the investor's objectives, timing of cash flow, time required to service the investment, and other characteristics.) See Figure 11.1.

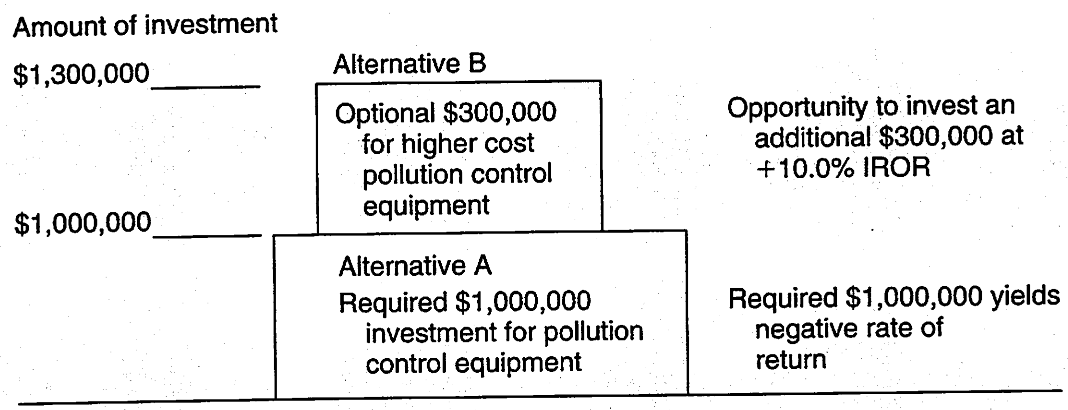


Figure 11.1 A $300,000 increment of investment earns 10.0% IROR.

COMPARING MULTIPLE ALTERNATIVES WHEN THE MARR IS KNOWN

Frequently more than two alternatives are available and need consideration. The IROR method is an excellent tool for analyzing such situations. The procedure for comparing multiple (more than two) alternatives is similar to the steps outlined previously in Example 11.2 for two alternatives. For illustration, the orientation is reversed, now with the alternatives occupying the rows, and the payment item names heading up the columns. The problem can be solved just as easily with either orientation.

1. List all the available alternatives, and sort them by first cost. The lowest first cost alternative is at the top of the list, and the list is placed in the left column so that each alternative is at the left end of its own row.

2. At the top of each remaining column, place a payment item name, such as Cost New, O & M, Salvage, etc.

3. Fill in the matrix with the appropriate costs.

4. Begin with the top row, representing the alternative with the lowest first cost. Compare all the costs of this first alternative with the alternative in the next row lower (that has the second lowest first cost). Do this by assuming some i value, reducing each payment item to a common equivalent value by use of PW, FW or AW.

5. Find the IROR value for this first pair of alternatives.

6. Compare the IROR to the known MARR. If the IROR is higher, accept this second alternative. Then proceed to compare the second alternative to the third. If the IROR is lower than the MARR, reject the second alternative and compare the first to the third.

7. Continue making comparisons, rejecting any alternatives with IRORs that are less than the MARR. The last acceptable alternative is the preferred one.

EXAMPLE 11.2 (continued)

Assume that the power plant in Example 11.2 had five alternatives instead of the two listed. Assume further that each had the first costs listed below, and that the "do nothing" alternative was not a viable option in this case.

Alternative	First cost	Annual O & M costs	Salvage value at EOY 20
A	$1,000,000	$300,000	$50,000
B	1,300,000	265,000	60,000
C	900,000	325,000	40,000
D	1,150,000	270,000	60,000
E	1,400,000	260,000	65,000

The first step is to list the alternatives in order of ascending first cost, as follows:

Alternative	First cost	Annual O & M costs	Salvage value at EOY 20
C	$ 900,000	$325,000	$40,000
A	1,000,000	300,000	50,000
D	1,150,000	270,000	60,000
B	1,300,000	265,000	60,000
E	1,400,000	240,000	65,000

Then assume that additional data on operating costs, salvage values, and so forth are provided (not shown). From these values the 10 IROR values are derived (calculations not shown) and listed in matrix table form as follows:

Cost of alternative	Alternative	A	D	B	E	
$ 900,000	C	24.7% (A−C)	21.6% (D−C)	14.0% (B−C)	16.2% (E−C)	←This line shows IROR of A, D, B, E, compared to C
1,000,000	A		19.5% (D−A)	10.0% (B−A)	13.9% (E−A)	
1,150,000	D			−3.6% (B−D)	10.4% (E−D)	
1,300,000	B				2.3% (E−B)	
1,400,000	E					

The process of selecting the better alternative begins with the lowest cost alternative C at the top row of the matrix. The IROR values on the row to the right of C are examined and the largest one selected. This is the 24.7 percent that can be earned by investing $100,000(A − C) in addition to the required $900,000 expenditure for minimum first cost pollution-control device C. If the MARR is less than 24.7 percent, then alternative A is the better selection over C. If the MARR is greater than 24.7 percent, then C is the better alternative, and none of the other alternatives or rows need be examined. *Whenever the MARR exceeds the IROR between the lowest cost alternative and all other alternatives* (MARR > the highest IROR in the top row of the matrix), *the lowest cost alternative is the better investment.* Similarly, as we proceed down into the matrix, wherever the MARR exceeds all the values on any row, the alternative for any given row is the better investment. Assume that in Example 11.2 the MARR < 24.7 percent, and alternative A is temporarily accepted. Then is it profitable to invest more than the $1,000,000 needed for A?

Cost of alternative	Alternative	A	D	B	E	
$ 900,000	C	24.7%	21.6%	14.0%	16.2%	
1,000,000	A		19.5%	10.0%	13.9%	← This line shows IROR of D, B, E compared to A
1,150,000	D			−3.6%	10.4%	
1,300,000	B				2.3%	

If MARR < 24.7 percent, reject C and temporarily accept A, the higher cost alternative between A and C, and proceed down to line A. If MARR > 24.7 percent, select C and end search.

Proceed to row A and compare the IROR values to the right of A (D − A), (B − A), and (E − A). The highest value is 19.5 percent for the increment (D − A), which indicates that an additional investment of $150,000 required to purchase D over the $1,000,000 required for A would yield an IROR of 19.5 percent. If the MARR > 19.5 percent (but less than 24.7 percent found in row C), proceed no further; invest only in A at $1,000,000.

For MARR < 19.5 percent, alternative D should be accepted temporarily over both A and C, but it still must be compared with both B and E before a final choice can be made. The same procedure is followed when comparing D to B and E.

Cost of alternative	Alternative	A	D	B	E	
$ 900,000	C	24.7%	21.6%	14.0%	16.2%	
1,000,000	A		19.5%	10.0%	13.9%	
1,150,000	D			−3.6%	10.4%	← This line shows IROR of B, E compared to D
1,300,000	B				2.3%	

The results may be summarized from the table as follows:

For MARR > 24.7%, use alternative C.
For 24.7% > MARR > 19.5%, use alternative A.
For 19.5% > MARR > 10.4%, use alternative D.
For MARR < 10.4%, use alternative E.

Note: There are no values of MARR under which alternative B is the better investment.

Graphically, the selection process is as shown in Figure 11.2. Since an investment in pollution-control equipment of at least $900,000 is required, would you recommend an additional investment

of $100,000 in A that yields an IROR of 24.7 percent?
of $250,000 in D that yields an IROR of 21.6 percent?
of $400,000 in B that yields an IROR of 14.0 percent?
of $500,000 in E that yields an IROR of 16.2 percent?

If the MARR is < 24.7 percent, then A is temporarily selected, since it has the greatest rate of return at 24.7 percent. Having selected A, at a cost of $1,000,000 (see Figure 11.3) would you now recommend an additional

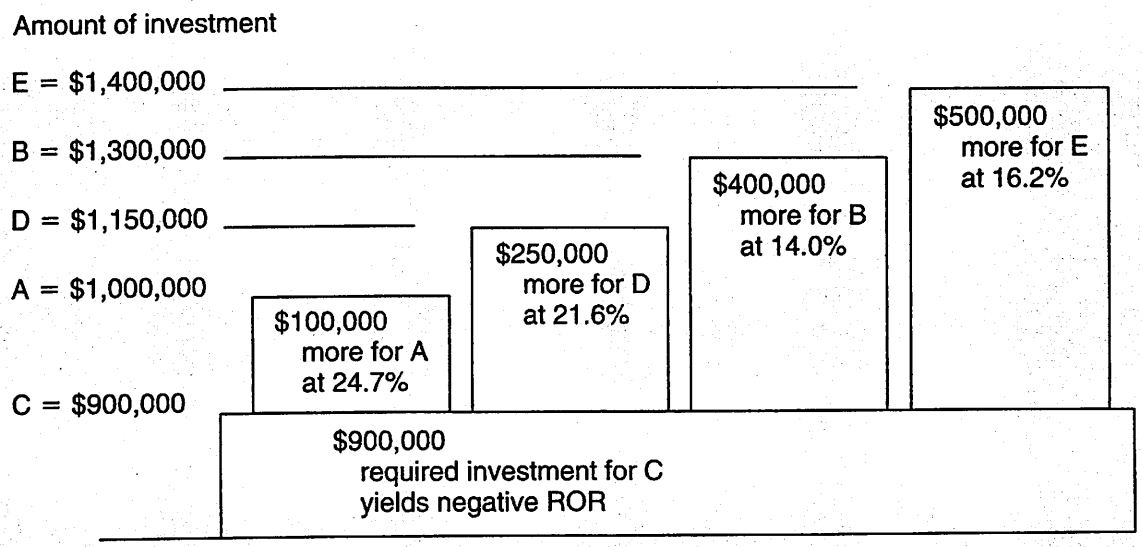


Figure 11.2 Alternative increments of investments with their respective IRORs, assuming a required initial investment in C.

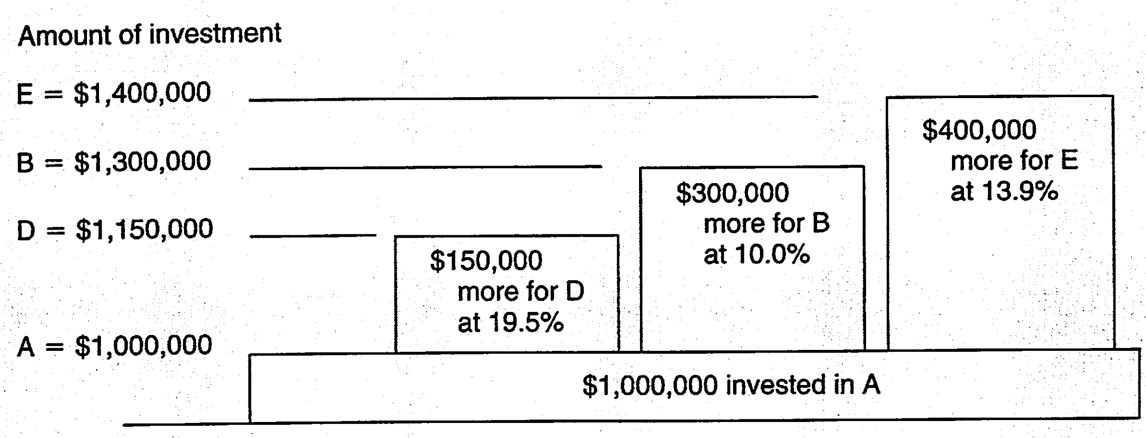


Figure 11.3 Alternative increments of investments with their respective IRORs, after committing at least $1,000,000 at level A.

investment of

$150,000 in D that yields an IROR of 19.5 percent?
$300,000 in B that yields an IROR of 10.0 percent?
$400,000 in E that yields an IROR of 13.9 percent?

If the MARR is < 19.5 percent, then alternative D is the logical selection since it has the greatest rate of return of those available alternatives at 19.5 percent. Having selected D, at a cost of $1,150,000, would you now recommend an additional investment of $150,000 in B that yields an IROR of −3.6 percent or $250,000 in E that yields an IROR of 10.4 percent? (See Figure 11.4).

If the MARR < 10.4 percent then E should be selected. The total investment pattern would now appear graphically as shown in Figure 11.5.

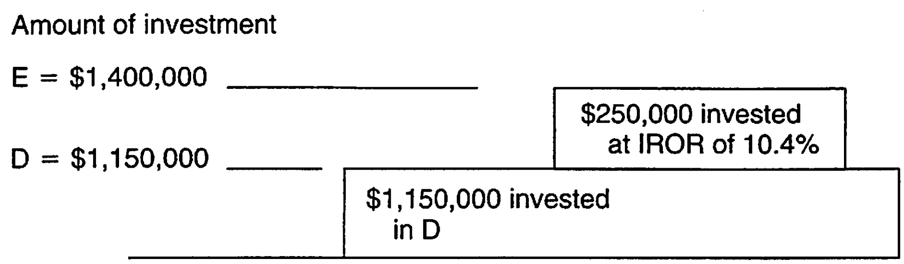


Figure 11.4 If alternative D is acceptable, should an additional $250,000 be invested in alternative E with an IROR of 10.4%?

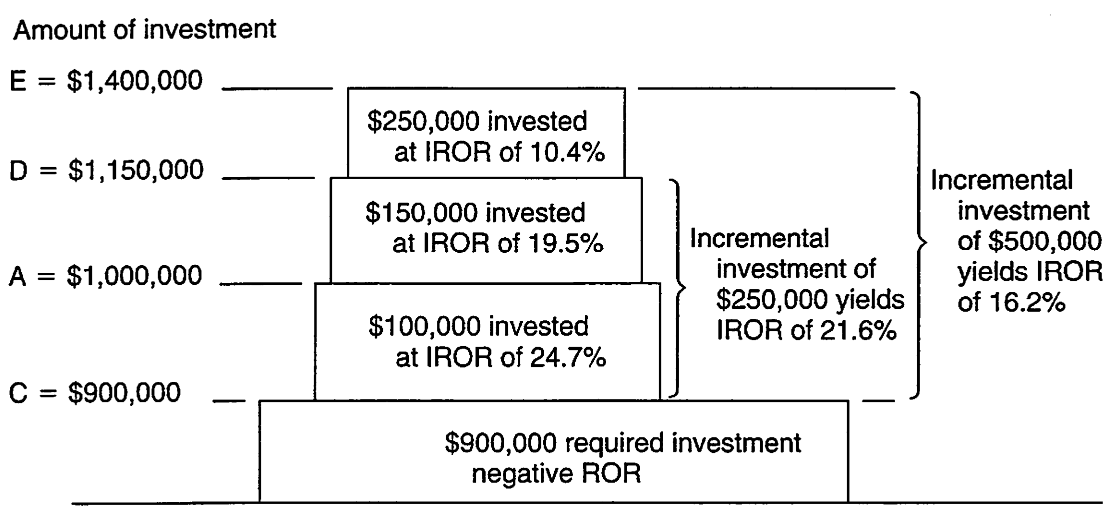


Figure 11.5 Total investment pattern.

SOLUTION BY ARROW DIAGRAM

Where a number of alternatives are available, a graphical solution using an arrow diagram has been devised by Dr. Gerald W. Smith of Iowa State University that greatly simplifies the selection process.

- The diagram is drawn as a closed geometric figure with the same number of corners as there are alternatives. Thus, using the data from the two matrix tables on page 297, the five alternatives of Example 11.2 are drawn as a pentagon. Six alternatives would be drawn as a hexagon, and so on.
- The first corner drawn always represents the alternative requiring the lowest capital investment (C in Example 11.2).
- Proceeding clockwise around the diagram, each successive corner represents the alternative with the next higher capital cost. The sides between the corners are tipped with arrows pointing toward the next higher cost and labeled with the corresponding return on incremental investment (IROR), as shown in Figure 11.6. Note that C

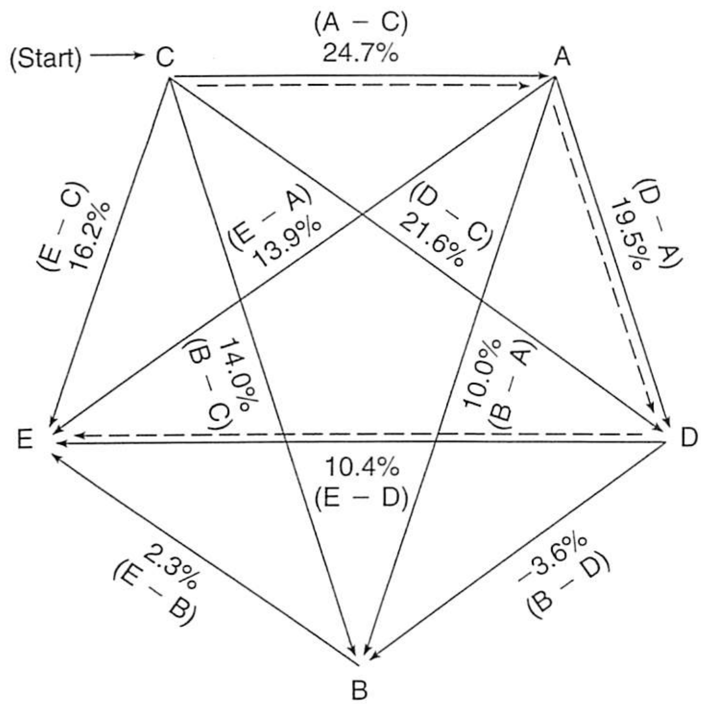


Figure 11.6 Arrow diagram for Example 11.2.

joins both A and E, since the lowest cost alternative will always be joined to two corners, the second lowest and the last (highest cost) alternative.

- To geographically determine the better alternative, start at the corner representing the lowest capital investment, in this case, C. Select the line with the highest rate of return leading from that starting corner, in this case the increment (A − C) or 24.7 percent.
- If the MARR is higher than this rate (MARR > 24.7 percent), then the lowest capital investment C, costing $900,000, is the better alternative.
- Assume for Example 11.2 that the MARR is lower (MARR < 24.7 percent). Then alternative A is temporarily the better choice, and we therefore proceed from corner A for our next comparison.
- Next, select the highest rate of return leading away from our first temporary selection (corner A), that is, 19.5 percent leading to corner D. Note that this was the better among three choices, 13.9 percent, 10.0 percent, and 19.5 percent. From D there is a choice between B, yielding −3.6 percent, and E, yielding 10.4 percent. It should also be noted here that there are only two choices from D; considering C is not a choice, because C is a lower, not a higher, cost alternative. If the MARR is lower then 10.4 percent (MARR < 10.4 percent) then accept E.
- No arrows lead away from E, and we therefore have no more comparison opportunity, indicating that E is the alternative with the highest capital investment.
- At whatever point, the IROR exceeds the MARR then temporarily accept the destination alternative at the point of the arrow. At whatever point the MARR

exceeds all IRORs leading from that point, they accept the alternative represented by that point as the better selection.

Reviewing the diagram:

If the MARR is greater than 24.7 percent, select the lowest cost alternative, C.
If the MARR is between 24.7 and 19.5 percent, then A is the better selection.
If the MARR is between 19.5 and 10.4 percent, then D is the better selection.
If the MARR is less than 10.4 percent, then E is the better selection.

OPPORTUNITY FOR THE "DO NOTHING" ALTERNATIVE (NO INVESTMENT)

In Example 11.2 if the power plant is relieved of the need to install pollution-control devices, then another alternative is available: the opportunity for no investment ("do nothing"). The pollution-control devices produce no direct income for the plant (although they undoubtedly benefit the area in many ways); instead they increase the plant's cost, which in turn results in a negative rate of return when each alternative is considered independently. Thus the "no investment" opportunity is now added to the arrow diagram as the lowest first cost option, at corner 0. The comparative returns on incremental investment are as indicated along the appropriate sides of the diagram in Figure 11.7. The graphic solution now begins at corner 0, and all arrows away from 0 yield negative rates of return. The diagram simply confirms what in this case is intuitively obvious, that (unless the MARR is lower than −5.8 percent) no investment from this group of alternatives is recommended.

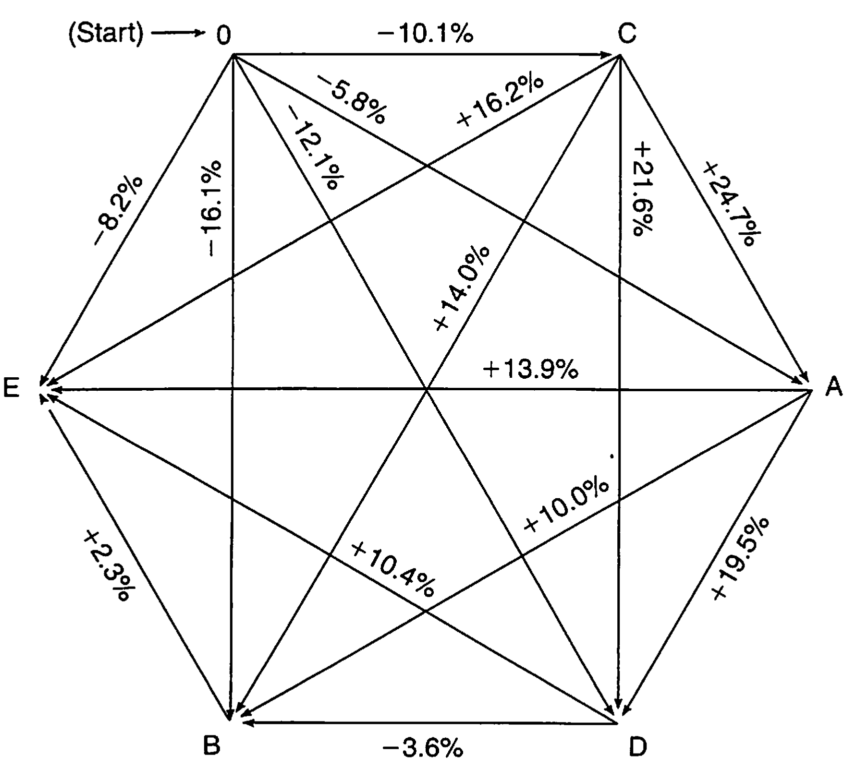


Figure 11.7 Arrow diagram with "do nothing" alternative.