Find each root. Assume that all variables represent nonnegative real numbers. See Example 4.

29. $-\sqrt[4]{16}$
30. $\sqrt[5]{-243}$
31. $\sqrt[4]{-16}$
32. $\sqrt{-16}$
33. $\sqrt[5]{-32}$
34. $\sqrt[5]{-1}$
35. $\sqrt[5]{x^{20}}$
36. $\sqrt[4]{x^{20}}$
37. $\sqrt[6]{64x^{12}}$
38. $\sqrt[5]{-32x^{15}}$
39. $\sqrt{81x^4}$
40. $\sqrt[4]{81x^4}$
41. $\sqrt[4]{256x^8}$
42. $\sqrt{256x^8}$

Simplify. Assume that the variables represent any real number. See Example 5.

43. $\sqrt{(-8)^2}$
44. $\sqrt{(-7)^2}$
45. $\sqrt[3]{(-8)^3}$
46. $\sqrt[5]{(-7)^5}$
47. $\sqrt{4x^2}$
48. $\sqrt[4]{16x^4}$
49. $\sqrt[3]{x^3}$
50. $\sqrt[5]{x^5}$
51. $\sqrt{(x-5)^2}$
52. $\sqrt{(y-6)^2}$
53. $\sqrt{x^2+4x+4}$ (*Hint:* Factor the polynomial first.)
54. $\sqrt{x^2-8x+16}$ (*Hint:* Factor the polynomial first.)

MIXED PRACTICE

Simplify each radical. Assume that all variables represent positive real numbers.

55. $-\sqrt{121}$
56. $-\sqrt[3]{125}$
57. $\sqrt[3]{8x^3}$
58. $\sqrt{16x^8}$
59. $\sqrt{y^{12}}$
60. $\sqrt[3]{y^{12}}$
61. $\sqrt{25a^2b^{20}}$
62. $\sqrt{9x^4y^6}$
63. $\sqrt[3]{-27x^{12}y^9}$
64. $\sqrt[3]{-8a^{21}b^6}$
65. $\sqrt[4]{a^{16}b^4}$
66. $\sqrt[4]{x^8y^{12}}$
67. $\sqrt[5]{-32x^{10}y^5}$
68. $\sqrt[5]{-243x^5z^{15}}$
69. $\sqrt{\frac{25}{49}}$
70. $\sqrt{\frac{4}{81}}$
71. $\sqrt{\frac{x^{20}}{4y^2}}$
72. $\sqrt{\frac{y^{10}}{9x^6}}$
73. $-\sqrt[3]{\frac{z^{21}}{27x^3}}$
74. $-\sqrt[3]{\frac{64a^3}{b^9}}$
75. $\sqrt[4]{\frac{x^4}{16}}$
76. $\sqrt[4]{\frac{y^4}{81x^4}}$

If $f(x) = \sqrt{2x+3}$ and $g(x) = \sqrt[3]{x-8}$, find the following function values. See Example 6.

77. $f(0)$
78. $g(0)$
79. $g(7)$
80. $f(-1)$
81. $g(-19)$
82. $f(3)$
83. $f(2)$
84. $g(1)$

Identify the domain and then graph each function. See Example 7.

85. $f(x) = \sqrt{x} + 2$
86. $f(x) = \sqrt{x} - 2$
87. $f(x) = \sqrt{x-3}$; use the following table.

x	$f(x)$
3	
4	
7	
12	

88. $f(x) = \sqrt{x+1}$; use the following table.

x	$f(x)$
−1	
0	
3	
8	

Identify the domain and then graph each function. See Example 8.

89. $f(x) = \sqrt[3]{x} + 1$
90. $f(x) = \sqrt[3]{x} - 2$
91. $g(x) = \sqrt[3]{x-1}$; use the following table.

x	$g(x)$
1	
2	
0	
9	
−7	

92. $g(x) = \sqrt[3]{x+1}$; use the following table.

x	$g(x)$
−1	
0	
−2	
7	
[illegible]	

EXAMPLE 4 Simplify the following expressions.

a. $\sqrt[4]{81}$ **b.** $\sqrt[5]{-243}$ **c.** $-\sqrt{25}$ **d.** $\sqrt[4]{-81}$ **e.** $\sqrt[3]{64x^3}$

Solution

a. $\sqrt[4]{81} = 3$ because $3^4 = 81$ and 3 is positive.

b. $\sqrt[5]{-243} = -3$ because $(-3)^5 = -243$.

c. $-\sqrt{25} = -5$ because -5 is the opposite of $\sqrt{25}$.

d. $\sqrt[4]{-81}$ is not a real number. There is no real number that, when raised to the fourth power, is -81.

e. $\sqrt[3]{64x^3} = 4x$ because $(4x)^3 = 64x^3$.

PRACTICE

4 Simplify the following expressions.

a. $\sqrt[4]{10{,}000}$ **b.** $\sqrt[5]{-1}$ **c.** $-\sqrt{81}$ **d.** $\sqrt[4]{-625}$ **e.** $\sqrt[3]{27x^9}$

OBJECTIVE

5 Finding $\sqrt[n]{a^n}$ Where *a* Is a Real Number

Recall that the notation $\sqrt{a^2}$ indicates the positive square root of a^2 only. For example,

$$\sqrt{(-7)^2} = \sqrt{49} = 7$$

When variables are present in the radicand and it is *unclear whether the variable represents a positive number or a negative number*, absolute value bars are sometimes needed to ensure that the result is a positive number. For example,

$$\sqrt{x^2} = |x|$$

This ensures that the result is positive. This same situation may occur when the index is any *even* positive integer. When the index is any *odd* positive integer, absolute value bars are not necessary.

Finding $\sqrt[n]{a^n}$

If n is an *even* positive integer, then $\sqrt[n]{a^n} = |a|$.

If n is an *odd* positive integer, then $\sqrt[n]{a^n} = a$.

Sample of now problem should be answer

EXAMPLE 5 Simplify.

a. $\sqrt{(-3)^2}$ **b.** $\sqrt{x^2}$ **c.** $\sqrt[4]{(x-2)^4}$ **d.** $\sqrt[3]{(-5)^3}$

e. $\sqrt[5]{(2x-7)^5}$ **f.** $\sqrt{25x^2}$ **g.** $\sqrt{x^2+2x+1}$

Solution

a. $\sqrt{(-3)^2} = |-3| = 3$ When the index is even, the absolute value bars ensure that our result is not negative.

b. $\sqrt{x^2} = |x|$

c. $\sqrt[4]{(x-2)^4} = |x-2|$

d. $\sqrt[3]{(-5)^3} = -5$

e. $\sqrt[5]{(2x-7)^5} = 2x-7$ Absolute value bars are not needed when the index is odd.

f. $\sqrt{25x^2} = 5|x|$

g. $\sqrt{x^2+2x+1} = \sqrt{(x+1)^2} = |x+1|$