✓CONCEPT CHECK
How do you know just by looking that $(x-2)^2 = -4$ has complex but not real solutions?

OBJECTIVE
2 Solving by Completing the Square

Notice from Examples 3 and 4 that, if we write a quadratic equation so that one side is the square of a binomial, we can solve by using the square root property. To write the square of a binomial, we write perfect square trinomials. Recall that a perfect square trinomial is a trinomial that can be factored into two identical binomial factors.

Perfect Square Trinomials	*Factored Form*
$x^2 + 8x + 16$	$(x+4)^2$
$x^2 - 6x + 9$	$(x-3)^2$
$x^2 + 3x + \frac{9}{4}$	$\left(x+\frac{3}{2}\right)^2$

Notice that for each perfect square trinomial in x, **the constant term of the trinomial is the square of half the coefficient of the x-term.** For example,

$$x^2 + 8x + 16 \qquad\qquad x^2 - 6x + 9$$

$$\frac{1}{2}(8) = 4 \text{ and } 4^2 = 16 \qquad \frac{1}{2}(-6) = -3 \text{ and } (-3)^2 = 9$$

The process of writing a quadratic equation so that one side is a perfect square trinomial is called **completing the square.**

EXAMPLE 5 Solve $p^2 + 2p = 4$ by completing the square.

Solution First, add the square of half the coefficient of p to both sides so that the resulting trinomial will be a perfect square trinomial. The coefficient of p is 2.

$$\frac{1}{2}(2) = 1 \quad \text{and} \quad 1^2 = 1$$

Add 1 to both sides of the original equation.

$$p^2 + 2p = 4$$
$$p^2 + 2p + 1 = 4 + 1 \quad \text{Add 1 to both sides.}$$
$$(p+1)^2 = 5 \quad \text{Factor the trinomial; simplify the right side.}$$

We may now use the square root property and solve for p.

$$p + 1 = \pm\sqrt{5} \quad \text{Use the square root property.}$$
$$p = -1 \pm \sqrt{5} \quad \text{Subtract 1 from both sides.}$$

Notice that there are two solutions: $-1 + \sqrt{5}$ and $-1 - \sqrt{5}$.

PRACTICE
5 Solve $b^2 + 4b = 3$ by completing the square.

EXAMPLE 6 Solve $m^2 - 7m - 1 = 0$ for m by completing the square.

Solution First, add 1 to both sides of the equation so that the left side has no constant term.

$$m^2 - 7m - 1 = 0$$

Answer to Concept Check:

21. $(3x - 1)^2 = -16$

22. $(4y + 2)^2 = -25$

23. $(z + 7)^2 = 5$

24. $(x + 10)^2 = 11$

25. $(x + 3)^2 + 8 = 0$

26. $(y - 4)^2 + 18 = 0$

Add the proper constant to each binomial so that the resulting trinomial is a perfect square trinomial. Then factor the trinomial.

27. $x^2 + 16x +$ ______

28. $y^2 + 2y +$ ______

29. $z^2 - 12z +$ ______

30. $x^2 - 8x +$ ______

31. $p^2 + 9p +$ ______

32. $n^2 + 5n +$ ______

33. $x^2 + x +$ ______

34. $y^2 - y +$ ______

MIXED PRACTICE

Solve each equation by completing the square. These equations have real number solutions. See Examples 5 through 7.

35. $x^2 + 8x = -15$

36. $y^2 + 6y = -8$

37. $x^2 + 6x + 2 = 0$

38. $x^2 - 2x - 2 = 0$

39. $x^2 + x - 1 = 0$

40. $x^2 + 3x - 2 = 0$

41. $x^2 + 2x - 5 = 0$

42. $x^2 - 6x + 3 = 0$

43. $y^2 + y - 7 = 0$

44. $x^2 - 7x - 1 = 0$

45. $3p^2 - 12p + 2 = 0$

46. $2x^2 + 14x - 1 = 0$

47. $4y^2 - 2 = 12y$

48. $6x^2 - 3 = 6x$

49. $2x^2 + 7x = 4$

50. $3x^2 - 4x = 4$

51. $x^2 + 8x + 1 = 0$

52. $x^2 - 10x + 2 = 0$

53. $3y^2 + 6y - 4 = 0$

54. $2y^2 + 12y + 3 = 0$

55. $2x^2 - 3x - 5 = 0$

56. $5x^2 + 3x - 2 = 0$

Solve each equation by completing the square. See Examples 5 through 8.

57. $y^2 + 2y + 2 = 0$

58. $x^2 + 4x + 6 = 0$

59. $y^2 + 6y - 8 = 0$

60. $y^2 + 10y - 26 = 0$

61. $2a^2 + 8a = -12$

62. $3x^2 + 12x = -14$

63. $5x^2 + 15x - 1 = 0$

64. $16y^2 + 16y - 1 = 0$

65. $2x^2 - x + 6 = 0$

66. $4x^2 - 2x + 5 = 0$

67. $x^2 + 10x + 28 = 0$

68. $y^2 + 8y + 18 = 0$

69. $z^2 + 3z - 4 = 0$

70. $y^2 + y - 2 = 0$

71. $2x^2 - 4x = -3$

72. $9x^2 - 36x = -40$

73. $3x^2 + 3x = 5$

74. $10y^2 - 30y = 2$

Use the formula $A = P(1 + r)^t$ to solve Exercises 75 through 78. See Example 9.

75. Find the rate r at which \$3000 compounded annually grows to \$4320 in 2 years.

76. Find the rate r at which \$800 compounded annually grows to \$882 in 2 years.

77. Find the rate at which \$15,000 compounded annually grows to \$16,224 in 2 years.

78. Find the rate at which \$2000 compounded annually grows to \$2880 in 2 years.

Neglecting air resistance, the distance $s(t)$ in feet traveled by a freely falling object is given by the function $s(t) = 16t^2$, where t is time in seconds. Use this formula to solve Exercises 79 through 82. Round answers to two decimal places.

79. The Petronas Towers in Kuala Lumpur, completed in 1998, are the tallest buildings in Malaysia. Each tower is 1483 feet tall. How long would it take an object to fall to the ground from the top of one of the towers? (*Source:* Council on Tall Buildings and Urban Habitat, Lehigh University)