Name:

Inner Product Space Lab

MATH 3400: LINEAR ALGEBRA ***Spring 2014 Section C1***

1. Find all vectors in R^4 that are orthogonal to both

$$\begin{bmatrix}1\\1\\2\\-2\end{bmatrix} \text{ and } \begin{bmatrix}2\\1\\1\\2\end{bmatrix}.$$

2. Let $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ where

$$\boldsymbol{v}_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix}0\\2\\-1\\-1\end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix}-3\\1\\1\\1\end{bmatrix}$$

   a. Show that S is an orthogonal set.
   b. Determine an orthonormal set T so that span S =span T.
   c. Find a nonzero vector $\boldsymbol{v}_4$orthogonal to S.

3. Let $S = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$, where

$$\boldsymbol{u}_1 = \begin{bmatrix}1/3\\2/3\\2/3\end{bmatrix}, \quad \boldsymbol{u}_2 = \begin{bmatrix}2/3\\-2/3\\1/3\end{bmatrix}, \quad \boldsymbol{u}_3 = \begin{bmatrix}2/3\\1/3\\-2/3\end{bmatrix}.$$

   a. Show that S is an orthonormal basis for R^3.
   b. Express $\boldsymbol{w} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ as a linear combination of the vectors in S.
   c. Let V=span$\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$. Determine the orthogonal projection of $\boldsymbol{w} = \begin{bmatrix}1\\2\\-1\end{bmatrix}$ onto V and compute the distance from $\boldsymbol{w}$ to V.

4. Use the Gram-Schmidt process to determine an orthonormal basis for

$$W = \text{span}\left\{\underset{b_1}{\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}, \underset{b_2}{\begin{bmatrix}2\\1\\-2\\0\end{bmatrix}}, \underset{b_3}{\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}\right\}$$

$v_1 = b_1$

$v_2 = b_2 - \frac{(b_2, v_1)}{(v_1, v_1)} v_1$

$v_3 = b_3 - \frac{(b_3, v_1)}{(v_1, v_1)} v_1 \quad \frac{(b_3, v_2)}{(v_2, v_2)} v_2$

5. Find a basis for the orthogonal complement of

$$W = \text{span}\left\{\begin{bmatrix}0\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\\1\end{bmatrix}\right\}$$