3. **The General Solution (Newton's Law of Cooling/Heating)**

We want to find a function **F(t)** that models the Fahrenheit temperature **F** of the probe at any time **t**, measured in seconds. Using a property of physics, called **Newton's Law of Cooling/Heating**, the temperature in an activity such as this can be modeled by an exponential function in the form:

$$\mathbf{F(t) = a \cdot b^t + c}.$$

4. **The Specific Solution**

First, to find an algebraic function that models our temperature vs. time data, it is clear that we need to write a piecewise function, one rule for the first (approximately) 35 seconds, and another for the last 35 (approximately) seconds. To find both of these rules, we will use the property of Newton's Law of Cooling.

In order to find a model of the form $\mathbf{F(t) = a \cdot b^t + c}$ for the first part of our data, we need to find the constants **a**, **b**, and **c**. We can find the constants **a** and **c** from the chart above (or by "tracing" on our scatterplot).

First we will find the constant **c**. According to Newton's Law of Cooling/Heating, and the data collected, the value of **c** would be approximately ____________.

To find **a**, record the temperature when t = 0. (0 , __________) Substitute this ordered pair, with the value of **c**, into our model $\mathbf{F(t) = a \cdot b^t + c}$, and solve for **a**. Show your work below.

a= ____________

To find **b**, the last constant in the model, we need another ordered pair. Let's use the ordered pair from the chart when t = 10 seconds.

Record this ordered pair. (10 , __________)

Substitute these values into the equation (with the values of **a** and **c**), and solve for the last unknown constant **b**. Show your work below. Round **b** to three decimal places. Then write the function **F(t)**.

b= ____________

F(t)= __________________

To check your work, graph your equation with your scatterplot to see how it fits the first part of the data. If it doesn't fit well, find your mistake!

The equation that fits the second part of the data is similar to the first equation. However, since we are beginning with a time other than t = 0, we need to apply a "horizontal shift" to our function. Therefore, the resulting form of this function is $\mathbf{F(t) = a \cdot b^{t-h} + c}$, where **h = 35**. Use this form, and the hints given for the first part of the function, and find a rule that fits this part of the data in the scatterplot. Show all of your work below. Again, graph this equation to check it.

a= ____________

b= ____________ (3 decimals)

c= ____________

F(t)= ____________________

Finally, combine the results of the two parts of your rule, and write a **piecewise function** that models the temperature vs. time data.

$$F(t) = \begin{cases} \\ \\ \end{cases}$$

5. **Working with the the Temperature Function F(t).**

Note: Do not use the data from the chart or the scatterplot, use your function to answer these questions.

a. Looking at the graph of the function **y = F(t)**, answer the following questions about the derivative function, **F'(t)**, which represents the **instantaneous rate of change** of the temperature at any time **t**.

i. When, if ever, is **F'(t)** positive? ______________________________

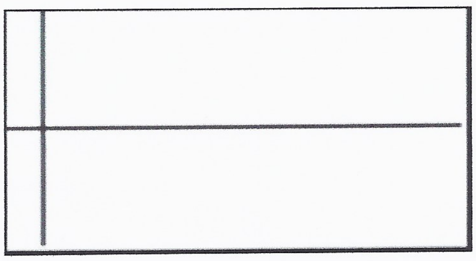
ii. When, if ever, is **F'(t)** negative? ______________________________

iii. When, if ever, is **F'(t)** undefined? ______________________________

iv. When, if ever, is **F'(t)** increasing in value? ______________________________

v. When, if ever, is **F'(t)** decreasing in value? ______________________________

b. From your answers above, sketch a graph of **y = F'(t)** below.



c. Analytically, using derivative properties, find the rule for **F'(t)**.

Hints:

1. **F'(t)** will also be a piecewise function.
2. Finding the derivative of the second part of the piecewise function requires an application of the **Chain Rule.**

Find the derivative of the two parts of **F(t)**. Show your work and write your final answer on the next page.

$$F'(t) = \begin{cases} \\ \\ \end{cases}$$

Verify your answer by graphing it on your calculator and comparing it to the graph drawn in part b above.

d. Using the function **F'(t)** above, how fast is the temperature changing when **t = 0, t = 10, and t = 45**? State the units with your answer.

F'(0) = ____________________ **F'(10)** = ______________________ **F'(45)** = ______________________

e. An important calculus property, called the **Mean Value Theorem** (used in proving many calculus properties), says that on the domain of a function where the graph is "smooth"; i.e. no breaks or cusps, there must exist "at least one t-value where the instantaneous rate of change is equal to the average rate of change".

i. Consider the the time interval [0 , 15]. What is the average rate of change of **F(t)** on this interval? Show the expression below that you are using in evaluating this value.

ii. Since the derivative of a function represents the instantaneous rate of change of the function at any t-value, set the derivative **F'(t)** equal to the value found in part i above, and solve for **t**. Show your work!

iii. Now consider the the time interval [40 , 50]. What is the average rate of change of **F(t)** on this interval? Show the expression below that you are using in evaluating this value.

iv. Find the t-value where the instantaneous rate of change is equal to the average rate of change on [40 , 50]. Show your work!