

7. (Perpetual call) A perpetual option is one that never expires. (Such an option must be of American style.) Use Exercise 6 to show that the value of a perpetual call on a non-dividend-paying stock is  $C = S$ .
8. (A surprise  $\oplus$ ) Consider a deterministic cash flow stream  $(x_0, x_1, x_2, \dots, x_n)$  with all positive flows. Let  $PV(r)$  denote the present value of this stream at an interest rate  $r$ .
- (a) If  $r$  decreases, does  $PV(r)$  increase or decrease?
- (b) Solve the Simplico gold mine problem with  $r = 4\%$  and find that the value of the lease is \$22.1 million. Can you explain why the value decreased relative to its value with  $r = 10\%$ ?
9. (My coin) There are two propositions: (a) I flip a coin. If it is heads, you are paid \$3; if it is tails, you are paid \$0. It costs you \$1 to participate in this proposition. You may do so at any level, or repeatedly, and the payoffs scale accordingly. (b) You may keep your money in your pocket (earning no interest). Here is a third proposition: (c) I flip the coin three times. If at least two of the flips are heads, you are paid \$27; otherwise zero. How much is this proposition worth?
10. (The happy call) A New York firm is offering a new financial instrument called a "happy call." It has a payoff function at time  $T$  equal to  $\max(.5S, S - K)$ , where  $S$  is the price of a stock and  $K$  is a fixed strike price. You always get something with a happy call. Let  $P$  be the price of the stock at time  $t = 0$  and let  $C_1$  and  $C_2$  be the prices of ordinary calls with strike prices  $K$  and  $2K$ , respectively. The fair price of the happy call is of the form

$$C_H = \alpha P + \beta C_1 + \gamma C_2.$$

Find the constants  $\alpha$ ,  $\beta$ , and  $\gamma$ .

11. (You are a president) It is August 6. You are the president of a small electronics company. The company has some cash reserves that will not be needed for about 3 months, but interest rates are very low. Your chief financial officer (CFO) tells you that a progressive securities firm has an investment that guarantees no losses and allows participation in upward movements of the stock market. In fact, the total rate of return until the third week of November is to be determined by the formula  $\max(0, .25r)$ , where  $r$  is the rate of return on the S&P 100 stock index during the 3-month period (ignoring dividends). The CFO suggests that this conservative investment might be an ideal alternative to participation in the interest rate market and asks for your opinion. You pick up *The Wall Street Journal* and make a few simple calculations to check whether it is, in fact, a good deal. Show these calculations and the conclusion. Data: A stock index is 14.2. There are two call options for November with strikes 410 and 420 and prices 13 and 7.5. The yield on November Treasury bills is 3.11.
12. (Simplico invariance) If the Simplico mine is solved with all parameters remaining the same except that  $u = 1.2$  is changed to  $u = 1.3$ , the value of the lease remains unchanged to within three decimal places. Indeed, quite wide variations in  $u$  and  $d$  have almost no influence on the lease price. Give an intuitive explanation for this.
13. (Change of period length  $\oplus$ ) A stock has volatility  $\sigma = .30$  and a current value of \$36. An American put option on this stock has a strike price of \$40, and expiration is in 5 months. The interest rate is 8%. Find the value of this put using a binomial lattice with 1-month intervals. Repeat using a lattice with half-month intervals.
14. (Average value Complexico  $\oplus$ ) Suppose that the price received for gold extracted from time  $k$  to  $k + 1$  is the average of the price of gold at these two times; that is,  $(g_k + g_{k+1})/2$ .

However, costs are incurred at the beginning of the period whereas revenues are received at the end of the period. Find the value of the Complexico mine in this case.

15. ("As you like it" option) Consider the stock of Examples 14.3 and 14.4, which has  $\sigma = .20$  and an initial price of \$62. The interest rate is 10%, compounded monthly. Consider a 5-month option with a strike price of \$60. This option can be declared, after exactly 3 months, by the purchaser to be either a European call or a European put. Find the value of this "as you like it" option.
16. (Tree harvesting  $\oplus$ ) You are considering an investment in a tree farm. Trees grow each year by the following factors:

Year	1	2	3	4	5	6	7	8	9	10
Growth	1.6	1.5	1.4	1.3	1.2	1.15	1.1	1.05	1.02	1.01

The price of lumber follows a binomial lattice with  $u = 1.20$  and  $d = .9$ . The interest rate is constant at 10%. It costs \$2 million each year, payable at the beginning of the year, to lease the forest land. The initial value of the trees is \$5 million (assuming they were harvested immediately). You can cut the trees at the end of any year and then not pay rent after that. (For those readers who care, we assume that cut lumber can be stored at no cost.)

- (a) Argue that if the rent were zero, you would never cut the trees as long as they were growing.
- (b) With rent of \$2 million per year, find the best cutting policy and the value of the investment opportunity.
17. (Coin market) There is a market for bets on the outcome of a coin toss. The possible outcomes are heads, tails, and edge. There are three assets traded in that market:
- Asset A pays \$1 independent of the outcome.  
 Asset B pays \$1.50 for a head and \$0 for tails and \$1 if the coin lands on its edge.  
 Asset C pays \$10 if and only if the coin lands on its edge (and \$0 otherwise).
- The prices of those assets are always constant and are fixed at \$1. We assume that the payments to the winning bidders take place immediately after the outcome is determined.
- (a) What is the implied risk-free interest rate in this market?  
 (b) What are the risk-neutral probabilities of each of the possible coin toss outcomes (heads, tails, edge)?
18. (High interest) Consider the stock and bond lattices on the left of Figure 14.18. Each has unit cost of \$1 with the payoffs shown at the end nodes. There is a new asset available—a derivative of the first two—with the payoffs shown at the end of two periods, as indicated in the lattice on the right side of the figure.

- (a) Find the value of this new asset.  
 (b) What is the deterministic payoff that should be obtained at the end of two periods?  
 (c) Show how to hedge this new asset by trading the stock and the bond in such a way that the deterministic payoff is obtained. [Hint: Construct a lattice with payoffs 16,