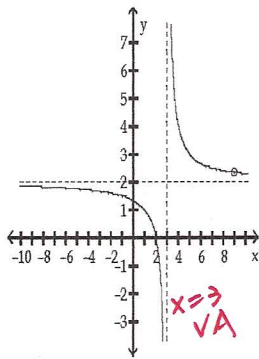


You are required to write neatly and show all work on the test body (2 pages). No credits will be given for answer only. Staple all your work together, No loose pages accepted.

1. For the following rational function graph, y-intercept is @ $(0, 4/3)$, find



$y=2$ (HA)

$x=3$ VA

read graph

$x=3$ VA
 $y=2$ HA
 empty Hole at $x=9$
 x intercept at $(2, 0)$
 y intercept at $(0, 4/3)$
 (given)

$$f(x) = \frac{2(x-2)(x-9)}{(x-3)(x-9)}$$

$$= \frac{(2x-4)(x-9)}{(x-3)(x-9)}$$

$(x-9)$: empty Hole @ 9
 $(x-2)$: x intercept at $(2, 0)$
 $(2x-4)$ or $2(x-2)$: $y=2$ HA
 y intercept $(0, 4/3)$

- a) Equation of the Vertical Asymptote $x=3$
 b) Equation of the Horizontal Asymptote $y=2$
 c) x -intercept $(2, 0)$
 d) Domain of this function exclude VA and empty Hole $(-\infty, 2) \cup (2, 9) \cup (9, \infty)$
 e) Equation of this rational function

2. For $f(x) = \frac{4x-11}{x-3} = 4 + \frac{1}{x-3}$

a) Find $D_f, R_f, f^{-1}, D_{f^{-1}}, R_{f^{-1}}$

b) verify $f^{-1} \circ f = x$ and verify $f \circ f^{-1} = x$

c) Graph function f , f^{-1} and $y=x$ on the same x-y plane

$$f^{-1} \circ f = f^{-1}\left(\frac{4x-11}{x-3}\right) = \frac{3\left(\frac{4x-11}{x-3}\right) - 11}{\left(\frac{4x-11}{x-3}\right) - 4} = \frac{\frac{3(4x-11) - 11(x-3)}{x-3}}{\frac{4x-11-4(x-3)}{x-3}}$$

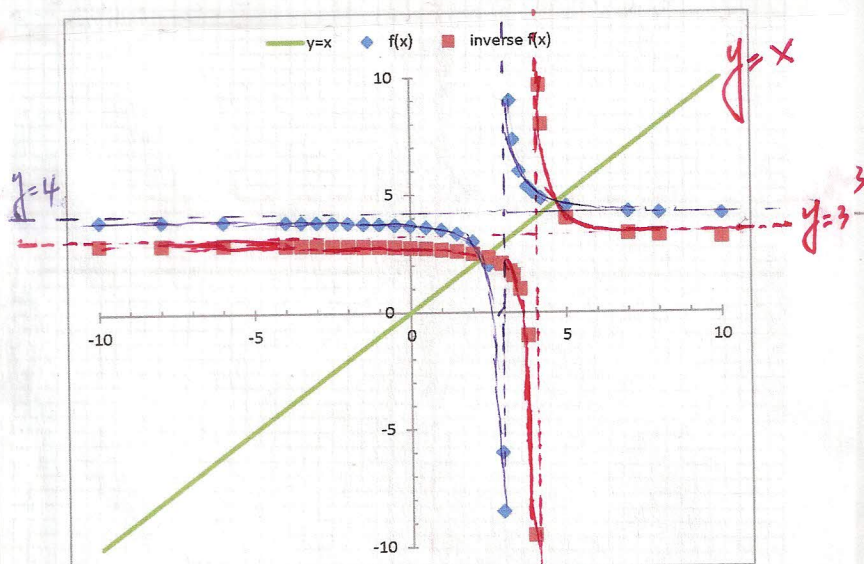
$$= \frac{12x-33-11x+33}{4x-11-4x+12} = \frac{x}{x-3} \cdot \frac{x-3}{x-3} = x$$

$$f \circ f^{-1} = f\left(f^{-1}(x)\right) = f\left(\frac{3x-11}{x-4}\right)$$

$$= \frac{4\left(\frac{3x-11}{x-4}\right) - 11}{\left(\frac{3x-11}{x-4}\right) - 3} = \frac{\frac{4(3x-11) - 11(x-4)}{x-4}}{\frac{3x-11-3(x-4)}{x-4}} = x$$

c) $f = \frac{4x-11}{x-3}$
 VA $x=3$
 HA $y=4$
 x intercept $(11/4, 0)$
 y intercept $(0, 11/3)$

$f^{-1} = \frac{3x-11}{x-4}$
 $x=4$
 $y=3$
 x intercept $(11/3, 0)$
 y intercept $(0, 11/4)$



$$\log_a x + \log_a y = \log_a xy$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^r = r \log_a x$$

$$\log_a x = \frac{\log x}{\log a}$$

$$\log_a 1 = 0 \quad \log_a a = 1$$

3. Given $\log_{10} 2 = m$ and $\log_{10} 3 = n$ find each logarithm by using m and n (Not Calculator data).

- a) $\log_{10} 1 = 0$
- b) $\log_{10} 18 = \log_{10} (3 \times 3 \times 2) = 2 \log_{10} 3 + \log_{10} 2 = 2n + m$
- c) $\log_{10} 5 = \log_{10} \frac{10}{2} = \log_{10} 10 - \log_{10} 2 = 1 - m$
- d) $\log_{10} \frac{2}{27} = \log_{10} \frac{2}{3 \times 3 \times 3} = \log_{10} 2 - \log_{10} 3 - \log_{10} 3 - \log_{10} 3 = m - 3n$
- e) $\log_{10} \sqrt[5]{8} = \log_{10} (2 \times 2 \times 2)^{\frac{1}{5}} = \log_{10} (2^3)^{\frac{1}{5}} = \log_{10} 2^{\frac{3}{5}} = \frac{3}{5} \log_{10} 2 = \left(\frac{3}{5}\right)(m)$
- f) $\log_3 2 = \frac{\log 2}{\log 3} = \frac{m}{n}$
- g) Find the pH of a solution with $[H_3O^+] = 2 \times 10^{-4}$ (pH value = $-\log_{10}[H_3O^+]$)

$$-\log_{10} [2 \times 10^{-4}]$$

$$= -(\log_{10} 2 + \log_{10} 10^{-4})$$

$$= -(\log_{10} 2 + (-4))$$

$$= -(m - 4) = 4 - m$$

4. Solve X

$$\begin{cases} a^{\log_a x} = x \\ \log_a a^x = x \\ e^{\ln x} = x \\ \ln e^x = x \\ 10^{\log x} = x \\ \log 10^x = x \end{cases}$$

- a) $\left(\frac{1}{5}\right)^{x-4} = (\sqrt[4]{5})^{3x+3}$
 $(5^{-1})^{x-4} = (5^{\frac{1}{4}})^{3x+3} \Rightarrow \frac{(-1)(x-4)}{1} = \frac{3x+3}{4} \Rightarrow -4x+16 = 3x+3$
 $x = \frac{13}{7}$
- b) $x = e^{\ln(2e-2+\pi)}$
 $x = 2e - 2 + \pi$
- c) $\log_x \frac{121}{49} = -2 \Rightarrow x^{-2} = \frac{121}{49} \Rightarrow x^2 = \frac{49}{121} \Rightarrow x = \pm \frac{7}{11}$
 $x = \frac{7}{11}$ only (because x is the base, $x > 0$)
- d) $\log_{\frac{2}{5}} (3x-1) = 3 \Rightarrow \left(\frac{2}{5}\right)^3 = 3x-1 \Rightarrow \frac{8}{125} + 1 = 3x$
 $\frac{133}{125} = 3x \Rightarrow x = \frac{133}{375}$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad \text{When } n \rightarrow \infty \quad A = Pe^{rt}$$

5. Suppose \$13500 is invested at an annual rate of 3.75 % for 6 years, find the future value A if interest is compounded as follows:

- a) Annually $13500 \left(1 + \frac{0.0375}{1}\right)^{6 \times 1} = 16836.91$
- b) Semi-Annually $13500 \left(1 + \frac{0.0375}{2}\right)^{6 \times 2} = 16871.17$
- c) Quarterly $13500 \left(1 + \frac{0.0375}{4}\right)^{6 \times 4} = 16888.65$
- d) Monthly $13500 \left(1 + \frac{0.0375}{12}\right)^{6 \times 12} = 16900.43$
- e) Daily (365 Days) $13500 \left(1 + \frac{0.0375}{365}\right)^{6 \times 365} = 16906.16$
- f) Continuously $13500 e^{0.0375 \times 6} = 16906.36$

(Use your calculator properly)
 you should be able to see from a) to f)
 A increase