## MA 480

## Applications I: The Suspension Bridge

A light cable is loaded uniformally at 5 joints equally spaced horizontally at $(x_1, x_2, x_3, x_4, x_5) = (200, 400, 600, 800, 1{,}000)$ feet. It hangs down $D'$ at its center.

A. Derive a difference equation relating the heights $[y_1, y_2, y_3, y_4, y_5] = \mathbf{y}$ at each joint in terms of those on both sides of it. Use $y_0 = H = y_6$ for your boundary conditions at $X_0 = 0$, $X_6 \approx 1200'$. Hint: Diagram the force-balance (equilibrium) conditions at joint $j$, and diagram the displacement vectors from $j-1$ to $j$, and from $j$ to $j+1$.

B. Express these equilibrium conditions as a matrix-vector equation for the height, $y_j \equiv y(x_j)$, of each joint to get the height vector, $\mathbf{y} = [y_1, y_2, y_3, y_4, y_5]$.

C. Use Gaussian elimination to solve this matrix-vector equation. Hint: How are the differences in slope related at each joint?

D. Model your suspension bridge with a string hanging between two posts, say 12" apart, loaded with equal weights spaced at equal horizontal intervals of $h = 2"$. Measure the heights $[Y_1, Y_2, Y_3, Y_4, Y_5] = \mathbf{Y}$ and compare them to your calculated heights in $B$, $C$. (Hint: you might want to average the differences in slope from point to point to get a "best fit" to your experimental data).

E. Next, write a differential equation for the height, $y(x)$ at point $x$ in the continuum limit where the number of joints goes to $\infty$ and their spacings go to 0 (but the length, 1200', and drop, 400', stay the same.)

Solve this equation for $y(x)$. Now **compare** this "continuum solution", $y(x)$,

a.) to the solution of the *difference equation* (B.), and

b.) to the measured values in (D.)

**Extra Credit**

F. What do you notice about the actual slope $\frac{dy}{dx}(x_j)$ of the continuum solution vs. the discreet approximation, $\frac{y_{j+1} - y_{j-1}}{2h}$. Why should this be true? Prove your findings! Is the same thing true for a *line*?, for a cubic, $y(x) = x^3 + bx^2 + cx + d$?

G. Compare your solution $y(x_j)$ in *C.*, and $y(x)$ in *E.* How are they related, and why?

H. Model a similar bridge of the same span and drop, but with 11 joints instead of 5. Does this give a better aproximation to the continuum limit $y(x)$? Why or why not?