

In Problems 25–36, two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle at all. Solve any triangle(s) that results.

25. $a = 3$, $b = 2$, $A = 50^\circ$

26. $b = 4$, $c = 3$, $B = 40^\circ$

27. $b = 5$, $c = 3$, $B = 100^\circ$

28. $a = 2$, $c = 1$, $A = 120^\circ$

29. $a = 4$, $b = 5$, $A = 60^\circ$

30. $b = 2$, $c = 3$, $B = 40^\circ$

31. $b = 4$, $c = 6$, $B = 20^\circ$

32. $a = 3$, $b = 7$, $A = 70^\circ$

33. $a = 2$, $c = 1$, $C = 100^\circ$

34. $b = 4$, $c = 5$, $B = 95^\circ$

35. $a = 2$, $c = 1$, $C = 25^\circ$

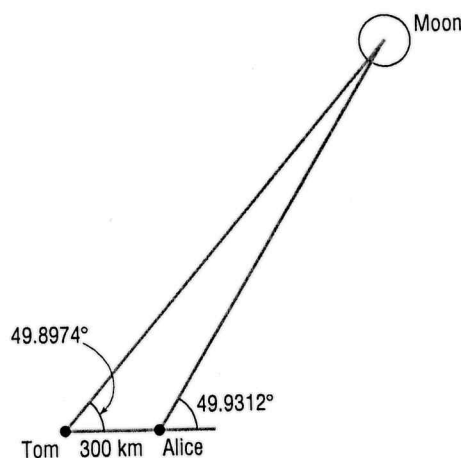
36. $b = 4$, $c = 5$, $B = 40^\circ$

Applications and Extensions

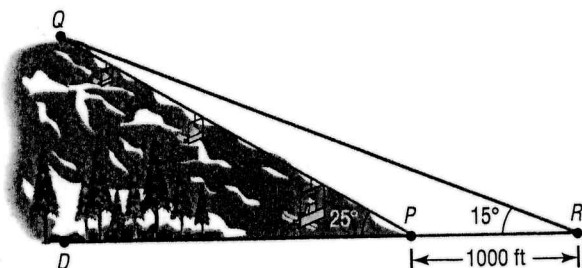
37. **Rescue at Sea** Coast Guard Station Able is located 150 miles due south of Station Baker. A ship at sea sends an SOS call that is received by each station. The call to Station Able indicates that the ship is located $N55^\circ E$; the call to Station Baker indicates that the ship is located $S60^\circ E$.

- (a) How far is each station from the ship?
 (b) If a helicopter capable of flying 200 miles per hour is dispatched from the station nearest the ship, how long will it take to reach the ship?

38. **Distance to the Moon** At exactly the same time, Tom and Alice measured the angle of elevation to the moon while standing exactly 300 km apart. The angle of elevation to the moon for Tom was 49.8974° and the angle of elevation to the moon for Alice was 49.9312° . See the figure. To the nearest 1000 km, how far was the moon from Earth when the measurement was obtained?

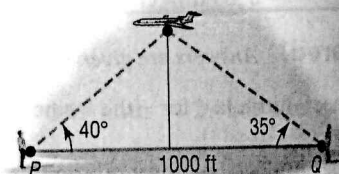


39. **Finding the Length of a Ski Lift** Consult the figure. To find the length of the span of a proposed ski lift from P to Q , a surveyor measures $\angle DPQ$ to be 25° and then walks off a distance of 1000 feet to R and measures $\angle PRQ$ to be 15° . What is the distance from P to Q ?



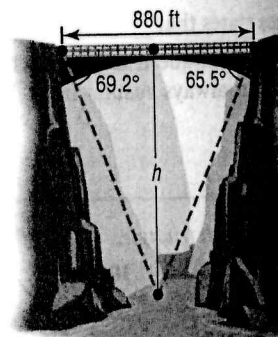
40. **Finding the Height of a Mountain** Use the illustration to find the height QD of the mountain.

41. **Finding the Height of an Airplane** An aircraft is seen by two observers who are 1000 feet apart. As the airplane passes over the line joining them, each observer takes a reading of the angle of elevation to the plane, as indicated in the figure. How high is the airplane?



42. **Finding the Height of the Bridge over the Royal Gorge** The highest bridge in the world is the bridge over the Gorge of the Arkansas River in Colorado. Sight is taken from the same point at water level directly under the bridge from each side of the 880-foot-long bridge, as shown in the figure. How high is the bridge?

Source: Guinness Book of World Records



43. **Landscaping** Pat needs to determine the height of a tree before cutting it down to be sure that it will not fall on a nearby fence. The angle of elevation of the top of the tree from a position on a flat path from the tree is 30° , and from a position 40 feet farther along this path it is 20° . How high is the tree?

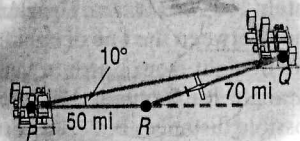
44. **Construction** A loading ramp 10 feet long that makes an angle of 18° with the horizontal is to be replaced by a new ramp that makes an angle of 12° with the horizontal. How long is the new ramp?

45. **Commercial Navigation** Adam must fly home from a business meeting in Oklahoma City. One day he is flying on a course that is 10° north of west. He is 100 miles from home when he receives a radio message from his wife that she is at home. How far is he from home?

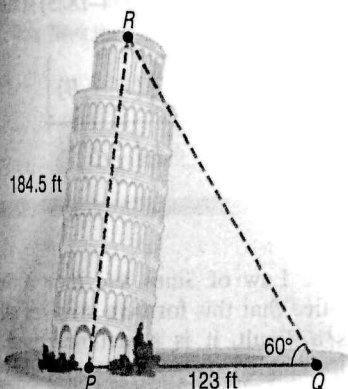
directly to St. Louis, a distance of about 461.1 miles. A flight option flies first to Kansas City and then to St. Louis. The bearing from Oklahoma City to Kansas City is $N29.6^\circ E$, and the bearing from Oklahoma City to St. Louis is $N57.7^\circ E$. The bearing from St. Louis to Kansas City is $S57.7^\circ W$, and the bearing from St. Louis to Kansas City is $N79.4^\circ W$. How many more frequent flyer miles will Adam receive if he takes the connecting flight than the direct flight?

www.landings.com

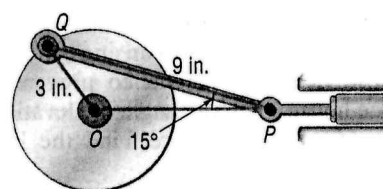
Lost due to a Navigation Error In attempting to fly from city P to city Q , an aircraft followed a course that was in error, as indicated in the figure. After flying a distance of 50 miles, the pilot corrected the course by turning at point R and flying 70 miles farther. If the constant speed of the aircraft was 250 miles per hour, how much time was lost due to the error?



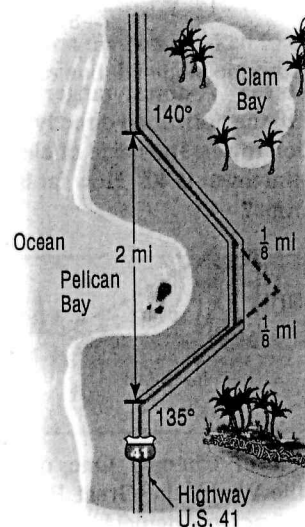
Measuring the Lean of the Leaning Tower of Pisa The famous Leaning Tower of Pisa was originally 184.5 feet high.* At a distance of 123 feet from the base of the tower, the angle of elevation to the top of the tower is found to be 60° . Find the distance PQ indicated in the figure. Also, find the perpendicular distance from R to PQ .



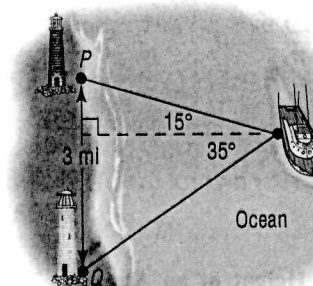
Crankshafts on Cars On a certain automobile, the crankshaft is 3 inches long and the connecting rod is 9 inches long (see the figure). At the time when $\angle OPQ$ is 15° , how far is the piston (P) from the center (O) of the crankshaft?



- 49. Constructing a Highway** U.S. 41, a highway whose primary directions are north-south, is being constructed along the west coast of Florida. Near Naples, a bay obstructs the straight path of the road. Since the cost of a bridge is prohibitive, engineers decide to go around the bay. The illustration shows the path that they decide on and the measurements taken. What is the length of highway needed to go around the bay?



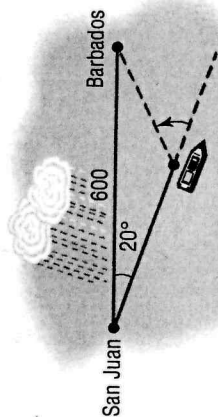
- 50. Calculating Distances at Sea** The navigator of a ship at sea spots two lighthouses that she knows to be 3 miles apart along a straight seashore. She determines that the angles formed between two line-of-sight observations of the lighthouses and the line from the ship directly to shore are 15° and 35° . See the illustration.
- How far is the ship from lighthouse P ?
 - How far is the ship from lighthouse Q ?
 - How far is the ship from shore?



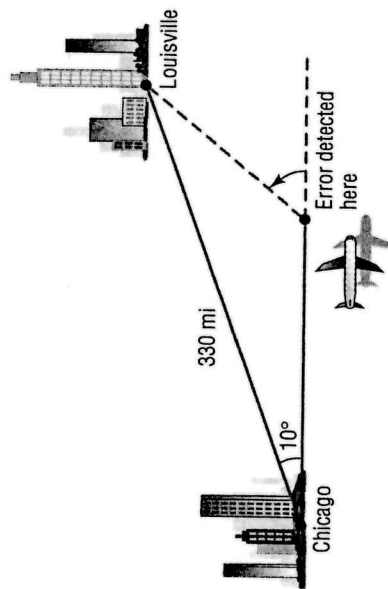
On February 27, 1964, the government of Italy requested aid in preventing the tower from toppling. A multinational task force of engineers, mathematicians, and historians was assigned and met on the Azores islands to discuss stabilization methods. After over two decades of work on the subject, the tower was reopened to the public in January 1990. During the time that the tower was closed, the bells were removed to relieve some weight, and cables were cinched to the third level and anchored several hundred meters away. Apartments and houses in the path of the tower were vacated for safety concerns. After corrective reconstruction and stabilization efforts, the tower was reopened to the public on December 15, 2001. Many methods were proposed for the tower, including the addition of 800 metric tons of lead counterweights to the raised end of the base. The final solution to correcting the lean involved moving 38 cubic meters of soil from underneath the raised end. The tower has been declared stable for at least another 300 years.

http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa

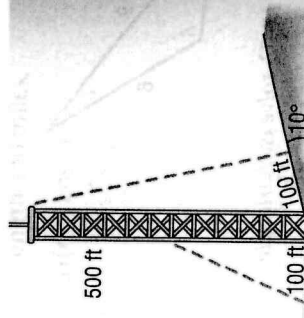
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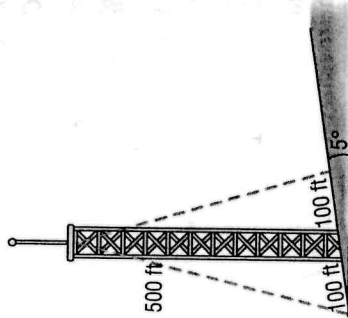
46. **Revising a Flight Plan** In attempting to fly from Chicago to Louisville, a distance of 330 miles, a pilot inadvertently took a course that was 10° in error, as indicated in the figure.
- If the aircraft maintains an average speed of 220 miles per hour and if the error in direction is discovered after 15 minutes, through what angle should the pilot turn to head toward Louisville?
 - What new average speed should the pilot maintain so that the total time of the trip is 90 minutes?



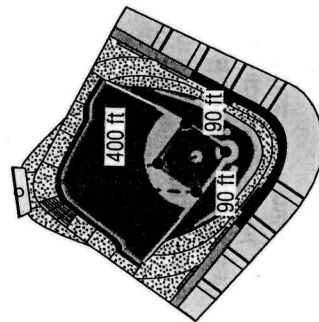
47. **Major League Baseball Field** A Major League baseball diamond is actually a square 90 feet on a side. The pitching rubber is located 60.5 feet from home plate on a line joining home plate and second base.
- How far is it from the pitching rubber to first base?
 - How far is it from the pitching rubber to second base?
 - If a pitcher faces home plate, through what angle does he need to turn to face first base?
48. **Little League Baseball Field** According to Little League baseball official regulations, the diamond is a square 60 feet on a side. The pitching rubber is located 46 feet from home plate on a line joining home plate and second base.
- How far is it from the pitching rubber to first base?
 - How far is it from the pitching rubber to second base?
 - If a pitcher faces home plate, through what angle does he need to turn to face first base?



50. **Finding the Length of a Guy Wire** A radio tower 500 feet high is located on the side of a hill with an inclination to the horizontal of 5° . See the figure. How long should two guy wires be if they are to connect to the top of the tower and be secured at two points 100 feet directly above and directly below the base of the tower?



51. **Wrigley Field, Home of the Chicago Cubs** The distance from home plate to the fence in dead center in Wrigley Field is 400 feet (see the figure). How far is it from the fence in dead center to third base?

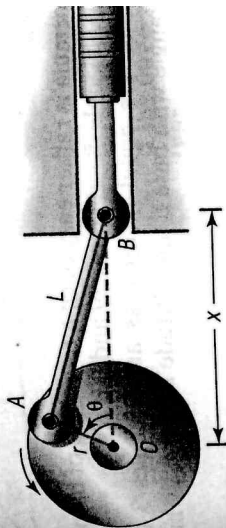


over concrete footings. How far apart should the footings be in each A-frame be?

Rods and Pistons Rod OA rotates about the fixed point O . Point A travels on a circle of radius r . Connected to point A is another rod AB of length $L > 2r$, and point B is connected to a piston. See the figure. Show that the distance between point O and point B is given by

$$x = r \cos \theta + \sqrt{r^2 \cos^2 \theta + L^2 - r^2}$$

where θ is the angle of rotation of rod OA .

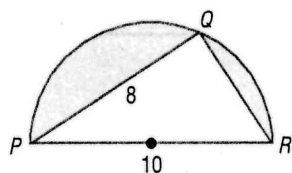


Explaining Concepts: Discussion and Writing

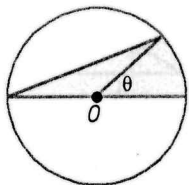
- What do you do first if you are asked to solve a triangle given two sides and the included angle?
- What do you do first if you are asked to solve a triangle given three sides?

'Are You Prepared?' Answers

- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- $\theta =$

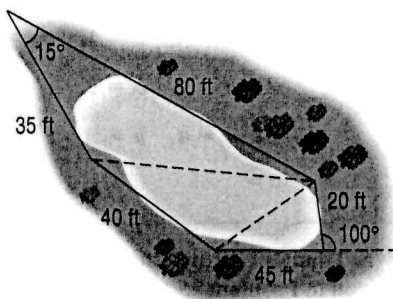


39. **Geometry** Consult the figure, which shows a circle of radius r with center at O . Find the area K of the shaded region as a function of the central angle θ .



40. **Approximating the Area of a Lake** To approximate the area of a lake, a surveyor walks around the perimeter of the lake, taking the measurements shown in the illustration. Using this technique, what is the approximate area of the lake?

[Hint: Use the Law of Cosines on the three triangles shown and then find the sum of their areas.]



41. **The Flatiron Building** Completed in 1902 in New York City, the Flatiron Building is triangular shaped and bounded by 22nd Street, Broadway, and 5th Avenue. The building measures approximately 87 feet on the 22nd Street side, 190 feet on the Broadway side, and 173 feet on the 5th Avenue side. Approximate the ground area covered by the building.

Source: Sarah Bradford Landau and Carl W. Condit, *Rise of the New York Skyscraper: 1865–1913*. New Haven, CT: Yale University Press, 1996



* Suggested by Professor Teddy Koukounas of Suffolk Community College, who learned of it from an old farmer in Virginia. Solution by Professor Kathleen Miranda of SUNY at Old Westbury.

42. **Bermuda Triangle** The Bermuda Triangle is roughly bounded by Hamilton, Bermuda; San Juan, Puerto Rico; and Fort Lauderdale, Florida. The distances from Hamilton to San Juan, Fort Lauderdale to San Juan, and Hamilton to Fort Lauderdale are approximately 1028, 1046, and 1062 miles, respectively. Ignoring the curvature of Earth, find the area of the Bermuda Triangle.

Source: www.worldatlas.com

43. **Geometry** Refer to the figure. If $|OA| = 1$, show that

(a) $\text{Area } \triangle OAC = \frac{1}{2} \sin \alpha \cos \alpha$

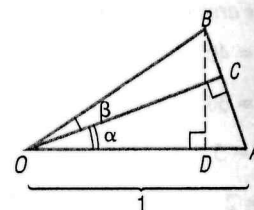
(b) $\text{Area } \triangle OCB = \frac{1}{2} |OB|^2 \sin \beta \cos \beta$

(c) $\text{Area } \triangle OAB = \frac{1}{2} |OB| \sin(\alpha + \beta)$

(d) $|OB| = \frac{\cos \alpha}{\cos \beta}$

(e) $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

[Hint: $\text{Area } \triangle OAB = \text{Area } \triangle OAC + \text{Area } \triangle OCB$]



44. **Geometry** Refer to the figure, in which a circle is drawn. The line segment DB is tangent to the circle at B . The angle AOB is acute.

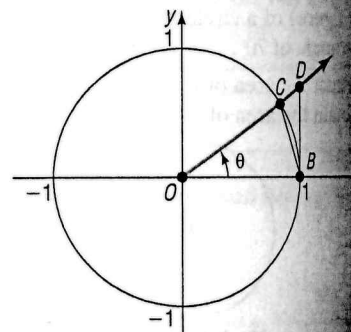
- (a) Express the area of $\triangle OBC$ in terms of $\sin \theta$ and $\cos \theta$.

- (b) Express the area of $\triangle OBD$ in terms of $\sin \theta$ and $\cos \theta$.

- (c) The area of the sector $\widehat{OBC}$ of the circle is $\frac{1}{2} r^2 \theta$, where θ is measured in radians. Use the results of (a) and (b) and the fact that

$\text{Area } \triangle OBC < \text{Area } \widehat{OBC} < \text{Area } \triangle OBD$ to show that

$$1 < \frac{\theta}{\sin \theta} < \frac{1}{\cos \theta}$$



45. **The Cow Problem*** A cow is tethered to a square barn, 10 feet by 10 feet, with a rope 10 feet long. What is the maximum grazing area for the cow? [See the illustration.]