Objectives

Section	You should be able to ...	Example(s)	Review Exercises
8.1	1 Find the exact value of an inverse sine, cosine, or tangent function (p. 605)	1, 2, 6, 7, 9	1–6
	2 Find an approximate value of an inverse sine function (p. 606)	3	121–124
	3 Use properties of inverse functions to find exact values of certain composite functions (p. 607)	4, 5, 8	9–20
	4 Find the inverse function of a trigonometric function (p. 612)	10	33–36
	5 Solve equations involving inverse trigonometric functions (p. 613)	11	133, 134
8.2	1 Find the exact value of expressions involving the inverse sine, cosine, and tangent functions (p. 617)	1–3	21–32
	2 Define the inverse secant, cosecant, and cotangent functions (p. 618)	4	7, 8, 29, 30
	3 Use a calculator to evaluate $\sec^{-1} x$, $\csc^{-1} x$, and $\cot^{-1} x$ (p. 618)	5	125, 126
	4 Write a trigonometric expression as an algebraic expression (p. 619)	6	37–40
8.3	1 Solve equations involving a single trigonometric function (p. 622)	1–5	97–106
	2 Solve trigonometric equations using a calculator (p. 625)	6	107, 108
	3 Solve trigonometric equations quadratic in form (p. 626)	7	113, 114
	4 Solve trigonometric equations using fundamental identities (p. 626)	8, 9	109–112, 115–118
	5 Solve trigonometric equations using a graphing utility (p. 627)	10	127–132
8.4	1 Use algebra to simplify trigonometric expressions (p. 633)	1	41–72
	2 Establish identities (p. 634)	2–8	41–57
8.5	1 Use sum and difference formulas to find exact values (p. 641)	1, 2	73–78, 81–90(a)–(d), 135
	2 Use sum and difference formulas to establish identities (p. 642)	3–8	59–62
	3 Use sum and difference formulas involving inverse trigonometric functions (p. 646)	9, 10	91–94
	4 Solve trigonometric equations linear in sine and cosine (p. 647)	11, 12	119, 120
8.6	1 Use double-angle formulas to find exact values (p. 652)	1	81–90(e), (f), 95, 96
	2 Use double-angle formulas to establish identities (p. 653)	2–5	58, 65–67
	3 Use half-angle formulas to find exact values (p. 656)	6, 7	79–90(g), (h), 135
8.7	1 Express products as sums (p. 662)	1	68
	2 Express sums as products (p. 663)	2	69–72

Do: ① 1-32 ② 81-90 ③ 97-120

Review Exercises

In Problems 1–8, find the exact value of each expression. Do not use a calculator.

1. $\sin^{-1} 1$
2. $\cos^{-1} 0$
3. $\tan^{-1} 1$
4. $\sin^{-1}\left(-\frac{1}{2}\right)$
5. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
6. $\tan^{-1}\left(-\sqrt{3}\right)$
7. $\sec^{-1}\sqrt{2}$
8. $\cot^{-1}(-1)$

In Problems 9–32, find the exact value, if any, of each composite function. If there is no value, say it is "not defined." Do not use a calculator.

9. $\sin^{-1}\left(\sin\frac{3\pi}{8}\right)$
10. $\cos^{-1}\left(\cos\frac{3\pi}{4}\right)$
11. $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$
12. $\sin^{-1}\left[\sin\left(-\frac{\pi}{8}\right)\right]$
13. $\cos^{-1}\left(\cos\frac{15\pi}{7}\right)$
14. $\sin^{-1}\left[\sin\left(-\frac{8\pi}{9}\right)\right]$
15. $\sin(\sin^{-1} 0.9)$
16. $\cos(\cos^{-1} 0.6)$
17. $\cos[\cos^{-1}(-0.3)]$
18. $\tan[\tan^{-1} 5]$
19. $\cos[\cos^{-1}(-1.6)]$
20. $\sin(\sin^{-1} 1.6)$
21. $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$
22. $\cos^{-1}\left(\tan\frac{3\pi}{4}\right)$
23. $\tan^{-1}\left(\tan\frac{7\pi}{4}\right)$
24. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

25. $\tan\left[\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ **26.** $\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]$ **27.** $\sec\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$ **28.** $\csc\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$

29. $\sin\left(\cot^{-1}\frac{3}{4}\right)$ **30.** $\cos\left(\csc^{-1}\frac{5}{3}\right)$ **31.** $\tan\left[\sin^{-1}\left(-\frac{4}{5}\right)\right]$ **32.** $\tan\left[\cos^{-1}\left(-\frac{3}{5}\right)\right]$

In Problems 33–36, find the inverse function f^{-1} of each function f. Find the range of f and the domain and range of f^{-1}.

33. $f(x) = 2\sin(3x)$, $-\frac{\pi}{6} \le x \le \frac{\pi}{6}$ **34.** $f(x) = \tan(2x+3) - 1$, $-\frac{3}{2} - \frac{\pi}{4} < x < -\frac{3}{2} + \frac{\pi}{4}$ **35.** $f(x) = -\cos x + 3$, $0 \le x \le \pi$ **36.** $f(x) = 2\sin(-x+$, $1 - \frac{\pi}{2} \le x \le 1+$

In Problems 37–40, write each trigonometric expression as an algebraic expression in u.

37. $\cos(\sin^{-1} u)$ **38.** $\cos(\csc^{-1} u)$ **39.** $\sin(\csc^{-1} u)$ **40.** $\tan(\csc^{-1} u)$

In Problems 41–72, establish each identity.

41. $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$ **42.** $\sin\theta\csc\theta - \sin^2\theta = \cos^2\theta$ **43.** $\sin^2\theta(1+\cot^2\theta) = 1$

44. $(1-\sin^2\theta)(1+\tan^2\theta) = 1$ **45.** $5\cos^2\theta + 3\sin^2\theta = 3 + 2\cos^2\theta$ **46.** $4\sin^2\theta + 2\cos^2\theta = 4 - 2\cos$

47. $\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} = 2\csc\theta$ **48.** $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\csc\theta$ **49.** $\frac{\cos\theta}{\cos\theta - \sin\theta} = \frac{1}{1-\tan\theta}$

50. $1 - \frac{\sin^2\theta}{1+\cos\theta} = \cos\theta$ **51.** $\frac{\csc\theta}{1+\csc\theta} = \frac{1-\sin\theta}{\cos^2\theta}$ **52.** $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$

53. $\csc\theta - \sin\theta = \cos\theta\cot\theta$ **54.** $\frac{\csc\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin^3\theta}$ **55.** $\frac{1-\sin\theta}{\sec\theta} = \frac{\cos^3\theta}{1+\sin\theta}$

56. $\frac{1-\cos\theta}{1+\cos\theta} = (\csc\theta - \cot\theta)^2$ **57.** $\frac{1-2\sin^2\theta}{\sin\theta\cos\theta} = \cot\theta - \tan\theta$ **58.** $\frac{(2\sin^2\theta - 1)^2}{\sin^4\theta - \cos^4\theta} = 1 - 2\cos^2\theta$

59. $\frac{\cos(\alpha+\beta)}{\cos\alpha\sin\beta} = \cot\beta - \tan\alpha$ **60.** $\frac{\sin(\alpha-\beta)}{\sin\alpha\cos\beta} = 1 - \cot\alpha\tan\beta$ **61.** $\frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta} = 1 +$ an $\alpha\tan$

62. $\frac{\cos(\alpha+\beta)}{\sin\alpha\cos\beta} = \cot\alpha - \tan\beta$ **63.** $(1+\cos\theta)\tan\frac{\theta}{2} = \sin\theta$ **64.** $\sin\theta\tan\frac{\theta}{2} = 1 - \cos\theta$

65. $2\cot\theta\cot(2\theta) = \cot^2\theta - 1$ **66.** $2\sin(2\theta)(1 - 2\sin^2\theta) = \sin(4\theta)$ **67.** $1 - 8\sin^2\theta\cos^2\theta = \cos(4\theta)$

68. $\frac{\sin(3\theta)\cos\theta - \sin\theta\cos(3\theta)}{\sin(2\theta)} = 1$ **69.** $\frac{\sin(2\theta) + \sin(4\theta)}{\cos(2\theta) + \cos(4\theta)} = \tan(3\theta)$ **70.** $\frac{\sin(2\theta) + \sin(4\theta)}{\sin(2\theta) - \sin(4\theta)} + \frac{\tan(3\theta)}{\tan\theta}$

71. $\frac{\cos(2\theta) - \cos(4\theta)}{\cos(2\theta) + \cos(4\theta)} - \tan\theta\tan(3\theta) = 0$ **72.** $\cos(2\theta) - \cos(10\theta) = \tan(4\theta)[\sin(2\theta) + \sin(10\theta)]$

In Problems 73–80, find the exact value of each expression.

73. $\sin 165°$ **74.** $\tan 105°$ **75.** $\cos\frac{5\pi}{12}$ **76.** $\sin\left(-\frac{\pi}{12}\right)$

77. $\cos 80°\cos 20° + \sin 80°\sin 20°$ **78.** $\sin 70°\cos 40° - \cos 70°\sin 40°$

79. $\tan\frac{\pi}{8}$ **80.** $\sin\frac{5\pi}{8}$

In Problems 81–90, use the information given about the angles α and β to find the exact value of:

(a) $\sin(\alpha+\beta)$ (b) $\cos(\alpha+\beta)$ (c) $\sin(\alpha-\beta)$ (d) $\tan(\alpha+\beta)$

(e) $\sin(2\alpha)$ (f) $\cos(2\beta)$ (g) $\sin\frac{\beta}{2}$ (h) $\cos\frac{\alpha}{2}$

81. $\sin\alpha = \frac{4}{5}, 0 < \alpha < \frac{\pi}{2}; \sin\beta = \frac{5}{13}, \frac{\pi}{2} < \beta < \pi$ **82.** $\cos\alpha = \frac{4}{5}, 0 < \alpha < \frac{\pi}{2}; \cos\beta = \frac{5}{13}, -\frac{\pi}{2} < \beta < 0$

83. $\sin\alpha = -\frac{3}{5}, \pi < \alpha < \frac{3\pi}{2}; \cos\beta = \frac{12}{13}, \frac{3\pi}{2} < \beta < 2\pi$ **84.** $\sin\alpha = -\frac{4}{5}, -\frac{\pi}{2} < \alpha < 0; \cos\beta = -\frac{5}{13}, \frac{\pi}{2} <$

85. $\tan\alpha = \frac{3}{4}, \pi < \alpha < \frac{3\pi}{2}; \tan\beta = \frac{12}{5}, 0 < \beta < \frac{\pi}{2}$

86. $\tan\alpha = -\frac{4}{3}, \frac{\pi}{2} < \alpha < \pi; \cot\beta = \frac{12}{5}, \pi < \beta < \frac{3\pi}{2}$

87. $\sec\alpha = 2, -\frac{\pi}{2} < \alpha < 0; \sec\beta = 3, \frac{3\pi}{2} < \beta < 2\pi$

88. $\csc\alpha = 2, \frac{\pi}{2} < \alpha < \pi; \sec\beta = -3, \frac{\pi}{2} < \beta < \pi$

89. $\sin\alpha = -\frac{2}{3}, \pi < \alpha < \frac{3\pi}{2}; \cos\beta = -\frac{2}{3}, \pi < \beta < \frac{3\pi}{2}$

90. $\tan\alpha = -2, \frac{\pi}{2} < \alpha < \pi; \cot\beta = -2, \frac{\pi}{2} < \beta < \pi$

In Problems 91–96, find the exact value of each expression.

91. $\cos\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{1}{2}\right)$

92. $\sin\left(\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5}\right)$

93. $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}\frac{3}{4}\right]$

94. $\cos\left[\tan^{-1}(-1) + \cos^{-1}\left(-\frac{4}{5}\right)\right]$

95. $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

96. $\cos\left(2\tan^{-1}\frac{4}{3}\right)$

In Problems 97–120, solve each equation on the interval $0 \le \theta < 2\pi$.

97. $\cos\theta = \frac{1}{2}$

98. $\sin\theta = -\frac{\sqrt{3}}{2}$

99. $2\cos\theta + \sqrt{2} = 0$

100. $\tan\theta + \sqrt{3} = 0$

101. $\sin(2\theta) + 1 = 0$

102. $\cos(2\theta) = 0$

103. $\tan(2\theta) = 0$

104. $\sin(3\theta) = 1$

105. $\sec^2\theta = 4$

106. $\csc^2\theta = 1$

107. $0.2\sin\theta = 0.05$

108. $0.9\cos(2\theta) = 0.7$

109. $\sin\theta + \sin(2\theta) = 0$

110. $\cos(2\theta) = \sin\theta$

111. $\sin(2\theta) - \cos\theta - 2\sin\theta + 1 = 0$

112. $\sin(2\theta) - \sin\theta - 2\cos\theta + 1 = 0$

113. $2\sin^2\theta - 3\sin\theta + 1 = 0$

114. $2\cos^2\theta + \cos\theta - 1 = 0$

115. $4\sin^2\theta = 1 + 4\cos\theta$

116. $8 - 12\sin^2\theta = 4\cos^2\theta$

117. $\sin(2\theta) = \sqrt{2}\cos\theta$

118. $1 + \sqrt{3}\cos\theta + \cos(2\theta) = 0$

119. $\sin\theta - \cos\theta = 1$

120. $\sin\theta - \sqrt{3}\cos\theta = 2$

In Problems 121–126, use a calculator to find an approximate value for each expression, rounded to two decimal places.

121. $\sin^{-1}0.7$

122. $\cos^{-1}\frac{4}{5}$

123. $\tan^{-1}(-2)$

124. $\cos^{-1}(-0.2)$

125. $\sec^{-1}3$

126. $\cot^{-1}(-4)$

In Problems 127–132, use a graphing utility to solve each equation on the interval $0 \le x \le 2\pi$. Approximate any solutions rounded to two decimal places.

127. $2x = 5\cos x$

128. $2x = 5\sin x$

129. $2\sin x + 3\cos x = 4x$

130. $3\cos x + x = \sin x$

131. $\sin x = \ln x$

132. $\sin x = e^{-x}$

In Problems 133 and 134, find the exact solution of each equation.

133. $-3\sin^{-1}x = \pi$

134. $2\cos^{-1}x + \pi = 4\cos^{-1}x$

135. Use a half-angle formula to find the exact value of $\sin 15°$. Then use a difference formula to find the exact value of $\sin 15°$. Show that the answers found are the same.

136. If you are given the value of $\cos\theta$ and want the exact value of $\cos(2\theta)$, what form of the double-angle formula for $\cos(2\theta)$ is most efficient to use?

Do 1-42

In Problems 5–24, find the remaining angle(s) and side(s) of each triangle, if it (they) exists. If no triangle exists, say "No triangle."

5. $A = 50°$, $B = 30°$, $a = 1$

6. $A = 10°$, $C = 40°$, $c = 2$

7. $A = 100°$, $a = 5$, $c = 2$

8. $a = 2$, $c = 5$, $A = 60°$

9. $a = 3$, $c = 1$, $C = 110°$

10. $a = 3$, $c = 1$, $C = 20°$

11. $a = 3$, $c = 1$, $B = 100°$

12. $a = 3$, $b = 5$, $B = 80°$

13. $a = 2$, $b = 3$, $c = 1$

14. $a = 10$, $b = 7$, $c = 8$

15. $a = 1$, $b = 3$, $C = 40°$

16. $a = 4$, $b = 1$, $C = 100°$

17. $a = 5$, $b = 3$, $A = 80°$

18. $a = 2$, $b = 3$, $A = 20°$

19. $a = 1$, $b = \frac{1}{2}$, $c = \frac{4}{3}$

20. $a = 3$, $b = 2$, $c = 2$

21. $a = 3$, $A = 10°$, $b = 4$

22. $a = 4$, $A = 20°$, $B = 100°$

23. $c = 5$, $b = 4$, $A = 70°$

24. $a = 1$, $b = 2$, $C = 60°$

In Problems 25–34, find the area of each triangle.

25. $a = 2$, $b = 3$, $C = 40°$

26. $b = 5$, $c = 5$, $A = 20°$

27. $b = 4$, $c = 10$, $A = 70°$

28. $a = 2$, $b = 1$, $C = 100°$

29. $a = 4$, $b = 3$, $c = 5$

30. $a = 10$, $b = 7$, $c = 8$

31. $a = 4$, $b = 2$, $c = 5$

32. $a = 3$, $b = 2$, $c = 2$

33. $A = 50°$, $B = 30°$, $a = 1$

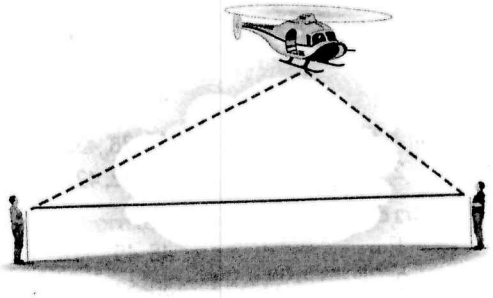
34. $A = 10°$, $C = 40°$, $c = 3$

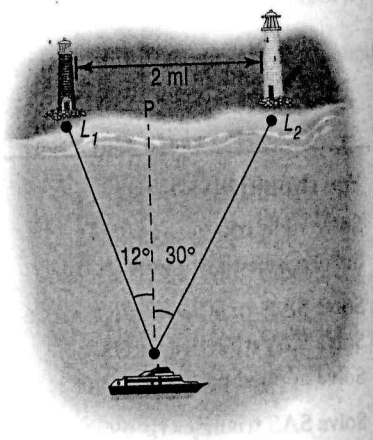
---

35. Finding the Grade of a Mountain Trail A straight trail with a uniform inclination leads from a hotel, elevation 5000 feet, to a lake in a valley, elevation 4100 feet. The length of the trail is 4100 feet. What is the inclination (grade) of the trail?

36. Geometry The hypotenuse of a right triangle is 12 feet. If one leg is 8 feet, find the degree measure of each angle.

37. Finding the Height of a Helicopter Two observers simultaneously measure the angle of elevation of a helicopter. One angle is measured as 25°, the other as 40° (see the figure). If the observers are 100 feet apart and the helicopter lies over the line joining them, how high is the helicopter?

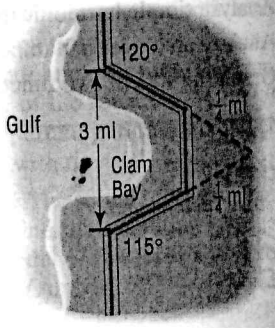




38. Determining Distances at Sea Rebecca, the navigator of a ship at sea, spots two lighthouses that she knows to be 2 miles apart along a straight shoreline. She determines that the angles formed between two line-of-sight observations of the lighthouses and the line from the ship directly to shore are 12° and 30°. See the illustration.

(a) How far is the ship from lighthouse L_1?
(b) How far is the ship from lighthouse L_2?
(c) How far is the ship from shore?

39. Constructing a Highway A highway whose primary direction is north–south is being constructed along the west coast of Florida. Near Naples, a bay obstructs the straight path of the road. Since the cost of a bridge is prohibitive, engineers decide to go around the bay. The illustration shows the path that they decide on and the measurements taken. What is the length of highway needed to go around the bay?

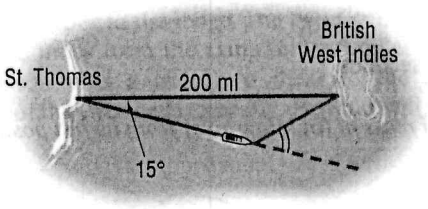


40. Correcting a Navigational Error A sailboat leaves St. Thomas bound for an island in the British West Indies, 200 miles away. Maintaining a constant speed of 18 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds after 4 hours that the sailboat is off course by 15°.

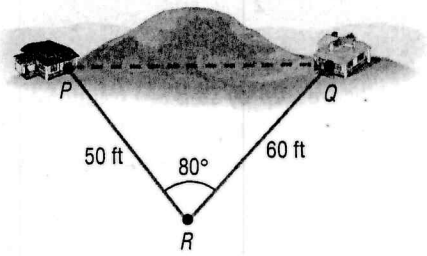
(a) How far is the sailboat from the island at this time?

(b) Through what angle should the sailboat turn to correct its course?

(c) How much time has been added to the trip because of this? (Assume that the speed remains at 18 miles per hour.)

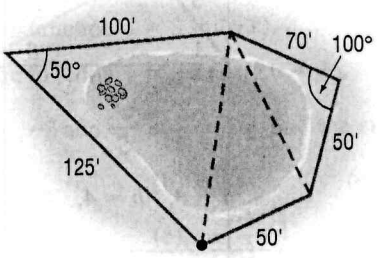


41. Surveying Two homes are located on opposite sides of a small hill. See the illustration. To measure the distance between them, a surveyor walks a distance of 50 feet from house P to point R, uses a transit to measure $\angle PRQ$, which is found to be 80°, and then walks to house Q, a distance of 60 feet. How far apart are the houses?



42. Approximating the Area of a Lake To approximate the area of a lake, Cindy walks around the perimeter of the lake, taking the measurements shown in the illustration. Using this technique, what is the approximate area of the lake?

[**Hint:** Use the Law of Cosines on the three triangles shown and then find the sum of their areas.]

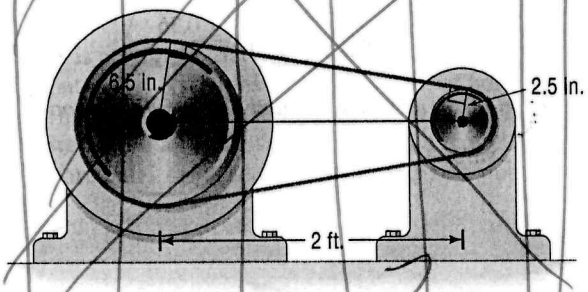


43. Calculating the Cost of Land The irregular parcel of land shown in the figure is being sold for $100 per square foot. What is the cost of this parcel?

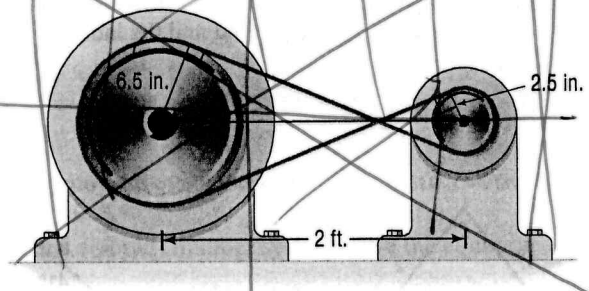
44. Area of a Segment Find the area of the segment of a circle whose radius is 6 inches formed by a central angle of 50°.

45. Finding the Bearing of a Ship The *Majesty* leaves the Port at Boston for Bermuda with a bearing of S80°E at an average speed of 10 knots. After 1 hour, the ship turns 90° toward the southwest. After 2 hours at an average speed of 20 knots, what is the bearing of the ship from Boston?

46. Drive Wheels of an Engine The drive wheel of an engine is 13 inches in diameter, and the pulley on the rotary pump is 5 inches in diameter. If the shafts of the drive wheel and the pulley are 2 feet apart, what length of belt is required to join them as shown in the figure?



47. Rework Problem 46 if the belt is crossed, as shown in the figure.



In Problems 48 and 49, an object attached to a coiled spring is pulled down a distance a from its rest position and then released. Assuming that the motion is simple harmonic with period T, develop a model that relates the displacement d of the object from its rest position after t seconds. Also assume that the positive direction of the motion is up.

48. $a = 3$; $T = 4$ seconds

49. $a = 5$; $T = 6$ seconds