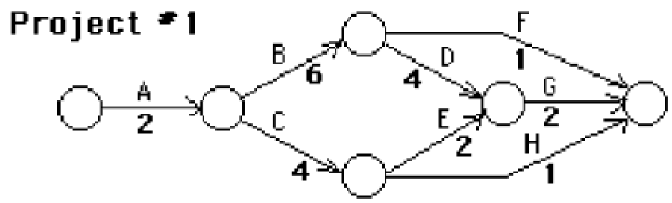
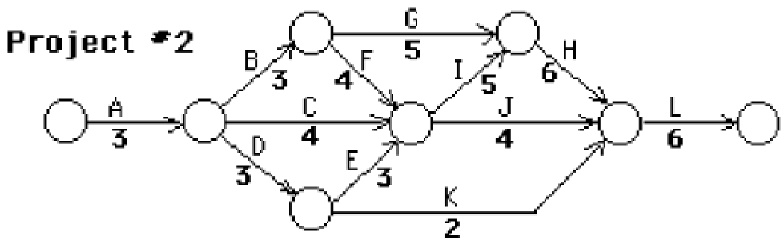
2. (*Problems #16 & 17, page 449-450 of text by Winston, with modifications*) Consider two project networks as below:





For each project network above:

a. Label the nodes of the AOA network, so that $i < j$ if there is an activity with node i as its start and node j as its end node. Using the forward pass & backward pass procedures, compute the early time (ET) and late time (LT) for each node.

b. Find the critical path. What is the earliest completion time for each project?

c. Draw the corresponding A-O-N (activity on node) network for each project.

In the case of project #2, assume that the durations specified on the network are the *expected* values (in days), but that the actual durations are random variables. Also assume that the standard deviations of all of the activities are 25% of the expected values. (E.g., the duration of activity L has mean 6 days and standard deviation 1.5 days.)

d. Assuming (as does PERT) that the critical path found in (c) is always critical, what is the expected length and the standard deviation of the length of the critical path

e. Assuming (as does PERT) that the length of the critical path is normally distributed, what is the probability that project #2 is completed within 30 days?

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Part II: Dynamic Programming

1. (*Problem #2, page 988 of text by Winston, with numbers modified*) Suppose that a new car costs $12,000 and that the annual operating cost and resale value of the car are as shown in the table below:

Age of Car (years)	Resale Value	Operating Cost
1	$9000	$400 (year 1)
2	$8000	$600 (year 2)
3	$6000	$900 (year 3)
4	$4000	$1200 (year 4)

5	$3000	$1600 (year 5)
6	$2000	$2200 (year 6)

If I have a new car now, determine a replacement policy that minimizes the net cost of owning and operating a car for the next six years. Use the dynamic programming to solve this problem (3 points).

2. Consider a production/inventory system with the following characteristics:
   - Maximum inventory level is 8
   - Storage costs are $1/week per unit in inventory at the beginning of the week
   - Initially, the inventory contains 2 units.
   - Maximum production level is 6/week
   - Setup cost for production is $10 in each week in which production is scheduled
   - Marginal production costs (costs in excess of setup cost) are $2 per unit
   - Demand in each of the next 8 weeks is assumed to be known and must be satisfied They are (where t =stage="weeks remaining", i.e., D[8] = first week demand, ... D[1] = 8th week demand):

Demands

t	8	7	6	5	4	3	2	1
D[t]	5	1	4	4	3	2	3	1

   - Anything produced during a certain week (plus anything in inventory at the beginning of the week) may be used to satisfy demand during that week, while anything in excess of the maximum inventory level (8) at the end of the week, after demand is satisfied, is discarded)
   - At the end of the 8 weeks, a salvage value of $3 per unit remaining in inventory is recovered.

Stage 8:

State	Optimal Values	Optimal Decisions	Resulting State
0	96.00	6	1
1	95.00	5	1
2	94.00	4	1
3	93.00	3	1
4	92.00	2	1
5	85.00	0	0
6	80.00	0	1
7	81.00	0	2
8	80.00	0	3

Stage 7:

State	Optimal Values	Optimal Decisions	Resulting State
0	80.00	6	5
1	74.00	0	0
2	74.00	0	1
3	72.00	0	2
4	72.00	0	3
5	68.00	0	4
6	64.00	0	5
7	65.00	0	6
8	66.00	0	7

Stage 6:

State	Optimal Values	Optimal Decisions	Resulting State
0	73.00	5	1
1	72.00	4	1
2	69.00	6	4
3	68.00	5	4
4	63.00	0	0
5	58.00	0	1
6	58.00	0	2
7	58.00	0	3
8	53.00	0	4

Stage 5:

State	Optimal Values	Optimal Decisions	Resulting State
0	59.00	4	0
1	53.00	6	3
2	52.00	5	3
3	51.00	4	3
		6	5
4	45.00	0	0
5	45.00	0	1
6	45.00	0	2
7	37.00	0	3
8	38.00	0	4

Stage 4:

State	Optimal Values	Optimal Decisions	Resulting State
0	41.00	5	2
1	40.00	4	2
2	39.00	3	2
		6	5
3	30.00	0	0
4	30.00	0	1
5	26.00	0	2
6	27.00	0	3
7	28.00	0	4
8	23.00	0	5

Stage 3:

State	Optimal Values	Optimal Decisions	Resulting State
0	27.00	6	4
1	26.00	5	4
2	21.00	0	0
3	21.00	0	1
4	21.00	0	2
5	15.00	0	3
6	11.00	0	4
7	11.00	0	5
8	11.00	0	6

Stage 2:

State	Optimal Values	Optimal Decisions	Resulting State
0	19.00	4	1
		5	2
		6	3
1	18.00	3	1
		4	2
		5	3
		6	4
2	17.00	2	1
		3	2
		4	3
		5	4
		6	5
3	10.00	0	0
4	5.00	0	1
5	4.00	0	2
6	3.00	0	3
7	2.00	0	4
8	1.00	0	5

Stage 1:

State	Optimal Values	Optimal Decisions	Resulting State
0	7.00	6	5
1	1.00	0	0
2	¯1.00	0	1
3	¯3.00	0	2
4	¯5.00	0	3
5	¯7.00	0	4
6	¯9.00	0	5
7	¯11.00	0	6
8	¯13.00	0	7

a. Suppose that, upon recounting the initial inventory, you find that you had previously overlooked three units, so that you actually have **5** units in stock. What is now your optimal total cost for the 8-week period?

b. If you have 5 units in stock initially, what is now your optimal production schedule? That is, in which weeks should you produce, and how much?

c. Suppose that, in the third week (i.e. stage #6), you have an unexpected cancellation of an order for one unit. Does this change your production schedule for the remaining 6 weeks? If so, what is the new production schedule?

Hint: To answer all of these questions, use the original tables above. (No re-computation of these tables is required.)

3. Consider the following zero-one knapsack problem, with a capacity of 12 units of weight:

Item #	Weight	Value
1	3	4
2	9	5
3	6	6
4	4	3
5	3	2

Dynamic programming output for this problem is given below:

STAGE 1

s \ x:	0	1
0	0.00	¯999999.00
1	0.00	¯999999.00
2	0.00	¯999999.00
3	0.00	4.00
4	0.00	4.00
5	0.00	4.00
6	0.00	4.00
7	0.00	4.00
8	0.00	4.00
9	0.00	4.00
10	0.00	4.00
11	0.00	4.00
12	0.00	4.00

STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE
0	0.00	0	0
1	0.00	0	1
2	0.00	0	2
3	4.00	1	0
4	4.00	1	1
5	4.00	1	2
6	4.00	1	3
7	4.00	1	4
8	4.00	1	5
9	4.00	1	6
10	4.00	1	7
11	4.00	1	8
12	4.00	1	9

STAGE 2

s \ x:	0	1
0	0.00	¯999999.00
1	0.00	¯999999.00
2	0.00	¯999999.00
3	4.00	¯999999.00
4	4.00	¯999999.00
5	4.00	¯999999.00
6	4.00	¯999999.00
7	4.00	¯999999.00
8	4.00	¯999999.00
9	4.00	5.00
10	4.00	5.00
11	4.00	5.00
12	4.00	9.00

STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE
0	0.00	0	0
1	0.00	0	1
2	0.00	0	2
3	4.00	0	3
4	4.00	0	4
5	4.00	0	5
6	4.00	0	6
7	4.00	0	7
8	4.00	0	8
9	5.00	1	0
10	5.00	1	1
11	5.00	1	2
12	9.00	1	3

STAGE 3

s \ x:	0	1
0	0.00	¯999999.00
1	0.00	¯999999.00
2	0.00	¯999999.00
3	4.00	¯999999.00
4	4.00	¯999999.00
5	4.00	¯999999.00
6	4.00	6.00
7	4.00	6.00
8	4.00	6.00
9	5.00	10.00
10	5.00	10.00
11	5.00	10.00
12	9.00	10.00

STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE
0	0.00	0	0
1	0.00	0	1
2	0.00	0	2
3	4.00	0	3
4	4.00	0	4
5	4.00	0	5
6	6.00	1	0
7	6.00	1	1
8	6.00	1	2
9	10.00	1	3
10	10.00	1	4
11	10.00	1	5
12	10.00	1	6

STAGE 4

s \ x:	0	1
0	0.00	¯999999.00
1	0.00	¯999999.00
2	0.00	¯999999.00
3	4.00	¯999999.00
4	4.00	3.00
5	4.00	3.00
6	6.00	3.00
7	6.00	7.00
8		
9		
10	10.00	9.00
11	10.00	9.00
12	10.00	9.00

STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE
0	0.00	0	0
1	0.00	0	1
2	0.00	0	2
3	4.00	0	3
4	4.00	0	4
5	4.00	0	5
6	6.00	0	6
7	7.00	1	3
8			
9			
10	10.00	0	10
11	10.00	0	11
12	10.00	0	12

STAGE 5

s \ x:	0	1
0	0.00	¯999999.00
1	0.00	¯999999.00
2	0.00	¯999999.00
3	4.00	2.00
4	4.00	2.00
5	4.00	2.00
6	6.00	6.00
7	7.00	6.00
8	7.00	6.00
9	10.00	8.00
10	10.00	9.00
11	10.00	9.00
12	10.00	12.00

STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE
0	0.00	0	0
1	0.00	0	1
2	0.00	0	2
3	4.00	0	3
4	4.00	0	4
5	4.00	0	5
6	6.00	0	6
		1	3
7	7.00	0	7
8	7.00	0	8
9	10.00	0	9
10	10.00	0	10
11	10.00	0	11
12	12.00	1	9

a. Do the computations to complete the five blank boxes in stage 4 of the DP output above (2 points).

b. What is the optimal solution of this knapsack problem?

c. Suppose that the capacity of the knapsack is only 10 pounds. What items should be included in this case?

Part I: Project Scheduling

1. Christine Phillips is in charge of planning and coordinating next spring's sales management training program for her company. Christine has listed the following activity information for this project. (Durations are given in weeks.)

Activity	Description	Immediate predecessors	Expected Duration	Estimated variance
A	Select location	--	2	1
B	Obtain keynote speaker	--	1	0.2
C	Obtain other speakers	B	2	1
D	Make travel plans for keynote speaker	A,B	2	1
E	Make travel plans for other speakers	A, C	3	1
F	Make food arrangements	A	2	1
G	Negotiate hotel rates	A	1	0.2
H	Prepare brochure	C, G	1	0.2
I	Mail brochure	H	1	0.2
J	Take reservations	I	3	1
K	Prepare handouts	C, F	4	4

a. Draw the AON (activity-on-node) project network.

b. Draw the AOA (activity-on-arrow) project network. And then label the nodes (events) of the AOA network so that if there is an arrow $i \rightarrow j$ then $i < j$.

In parts (c), assume that the expected durations are the actual durations.

c. Which activities are critical?

Now assume that the durations are random with the variance shown above.

d. What is the expected completion time of the project?

e. Assuming that the project completion time has normal distribution, what is the probability that the project will be completed no later than two weeks after the expected completion time? (Use the table below)

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298