

QN 1 (Non-linear budget constraints, demands and welfare: 40 marks; all parts give 10 marks)

Consumers have preferences over bread (x) and water (y) given by

$$U(x, y) = xy$$

with marginal utilities $U_x = y$ and $U_y = x$. The price of bread is p_x and the price of water is p_y .

- Write down the conditions for an interior optimum. Express the (uncompensated) demand for x and y as a function of income I , and the prices, p_x and p_y .
- Income is £10. The price of bread is £1 per loaf. (Consumers can buy parts of loaves.) Water costs £1 per litre for the first 6 litres, then 50p per litre for all subsequent litres. Draw this budget constraint with water on the horizontal axis, marking all intercepts.
- (Hard) Using part (a) or otherwise compute the optimal consumption for each consumer. (Hint: notice the budget constraint is non-linear. In this case there are two bundles which satisfy the necessary conditions for an interior optimum.)
- (Hard) There are 10 consumers in this economy. They realize they can save some money if they buy together. Why is that? Work out how much each person will consume when they co-operate in this way.