When Can We Assume Normality?

If σ is known and the population is normal, then we can safely use formula 8.6 to construct the confidence interval for μ. If σ is known but we do not know whether the population is normal, a common rule of thumb is that $n \geq 30$ is sufficient to assume a normal distribution for $\overline{X}$ (by the CLT) as long as the population is reasonably symmetric and has no outliers. However, a larger n may be needed to assume normality if you are sampling from a strongly skewed population or one with outliers. When σ is *unknown*, a different approach is used, as described in the next section.

Is σ Ever Known?

Yes, but not very often. In quality control applications with ongoing manufacturing processes, it may be reasonable to assume that σ stays the same over time. The type of confidence interval just seen is therefore important because it is used to construct *control charts* to track the mean of a process (such as bottle filling) over time. However, the case of unknown σ is more typical, and will be examined in the next section.

SECTION EXERCISES

connect

8.9 Find a confidence interval for μ assuming that each sample is from a normal population.

a. $\bar{x} = 14$, $\sigma = 4$, $n = 5$, 90 percent confidence
b. $\bar{x} = 37$, $\sigma = 5$, $n = 15$, 99 percent confidence
c. $\bar{x} = 121$, $\sigma = 15$, $n = 25$, 95 percent confidence

8.10 Calculate a confidence interval for μ assuming that each sample is from a normal population.

a. $\bar{x} = 24$, $\sigma = 3$, $n = 10$, 90 percent confidence
b. $\bar{x} = 125$, $\sigma = 8$, $n = 25$, 99 percent confidence
c. $\bar{x} = 12.5$, $\sigma = 1.2$, $n = 50$, 95 percent confidence

8.11 Use the sample information $\bar{x} = 2.4$, $\sigma = 0.15$, $n = 9$ to calculate the following confidence intervals for μ assuming the sample is from a normal population: (a) 90 percent confidence; (b) 95 percent confidence; (c) 99 percent confidence. (d) Describe how the intervals change as you increase the confidence level.

8.12 Use the sample information $\bar{x} = 37$, $\sigma = 5$, $n = 15$ to calculate the following confidence intervals for μ assuming the sample is from a normal population: (a) 90 percent confidence; (b) 95 percent confidence; (c) 99 percent confidence. (d) Describe how the intervals change as you increase the confidence level.

8.13 A random sample of 25 items is drawn from a population whose standard deviation is known to be $\sigma = 40$. The sample mean is $\bar{x} = 270$.

a. Construct an interval estimate for μ with 95 percent confidence.
b. Repeat part a assuming that $n = 50$.
c. Repeat part a assuming that $n = 100$.
d. Describe how the confidence interval changes as n increases.

8.14 A random sample of 100 items is drawn from a population whose standard deviation is known to be $\sigma = 50$. The sample mean is $\bar{x} = 850$.

a. Construct an interval estimate for μ with 95 percent confidence.
b. Repeat part a assuming that $\sigma = 100$.
c. Repeat part a assuming that $\sigma = 200$.
d. Describe how the confidence interval changes as σ increases.

8.15 The fuel economy of a 2011 Lexus RX 350 2WD 6 cylinder 3.5 L automatic 5-speed using premium fuel is normally distributed with a known standard deviation of 1.25 MPG. If a random sample of 10 tanks of gas yields a mean of 21.0 MPG, find the 95 percent confidence interval for the true mean MPG. (Source: www.fueleconomy.gov.)

8.16 Guest ages at a Vail Resorts ski mountain typically have a right-skewed distribution. Assume the standard deviation (σ) of *age* is 14.5 years. (a) Even though the population distribution of age is right-skewed, what will be the shape of the distribution of $\overline{X}$, the average age, in a random sample of 40 guests? (b) From a random sample of 40 guests, the sample mean is 36.4 years. Calculate a 99 percent confidence interval for μ, the true mean age of Vail Resorts ski mountain guests.

VAIL RESORTS

8.17 The Ball Corporation's beverage can manufacturing plant in Fort Atkinson, Wisconsin, uses a metal supplier that provides metal with a known thickness standard deviation $\sigma = .000959$ mm. If a random sample of 58 sheets of metal resulted in an $\bar{x} = 0.2731$ mm, calculate the 99 percent confidence interval for the true mean metal thickness.