1. Which of the following is a negation for "Jim is inside and Jan is at the pool."

   (a) Jim is inside or Jan is not at the pool.
   (b) Jim is inside or Jan is at the pool.
   (c) Jim is not inside or Jan is at the pool.
   (d) Jim is not inside and Jan is not at the pool.
   (e) Jim is not inside or Jan is not at the pool.

8. Write the converse, inverse, and contrapositive of "If Ann is Jan's mother, then Jose is Jan's cousin."

13. Consider the argument form:

$$\begin{array}{ll} & p \to \sim q \\ & q \to \sim p \\ \therefore & p \vee q \end{array}$$

Use the truth table below to determine whether this form of argument is valid or invalid. Include a truth table and a few words explaining how the truth table supports your answer.

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$q \to \sim p$	$p \vee q$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

18. Write the form of the following argument. Is the argument valid or invalid? Justify your answer.

If Ann has the flu, then Ann has a fever.
Ann has a fever.
Therefore, Ann has the flu.

21. Write 110101_2 in decimal form.

22. Write 75 in binary notation.

2. Consider the statement "The square of any odd integer is odd."

   (a) Rewrite the statement in the form $\forall$______ n, ______. (Do not use the words "if" or "then.")

   (b) Rewrite the statement in the form $\forall$______ n, if ______ then ______. (Make sure you use the variable n when you fill in each of the second two blanks.)

   (c) Write a negation for the statement.

5. Write a negation for each of the following statements:

   (a) For all integers n, if n is prime then n is odd.

   (b) $\forall$ real numbers x, if $x < 1$ then $\dfrac{1}{x} > 1$.

12. Are the following two statements logically equivalent? Justify your answer.

   (a) A real number is less than 1 only if its reciprocal is greater than 1.

   (b) Having a reciprocal greater than 1 is a sufficient condition for a real number to be less than 1.

14. For each of the following statements, (1) write the statement informally without using variables or the symbols $\forall$ or $\exists$, and (2) indicate whether the statement is true or false and briefly justify your answer.

   (a) $\forall$ real numbers x, $\exists$ a real number y such that $x < y$.

   (b) $\exists$ a real number y such that $\forall$ real numbers x, $x < y$.

3. Consider the following statement:

   Statement A: $\forall$ integers m and n, if $2m + n$ is odd then m and n are both odd.

   (a) Write a negation for Statement A.

   (b) Disprove Statement A. That is, show that Statement A is false.

7. Is 605.83 a rational number? Justify your answer.

22. Consider the following statement: For all real numbers r, if r^3 is irrational then r is irrational.

    (a) Prove the statement either by contradiction or by contraposition. Clearly indicate which method you are using.

    (b) If you used proof by contradiction in part (a), write what you would "suppose" and what you would "show" to prove the statement by contraposition. If you used proof by contraposition in part (a), write what you would "suppose" and what you would "show" to prove the statement by contradiction.

27. Use the Euclidean algorithm to find the greatest common divisor of 284 and 168. Show your work.

2. Compute $\sum_{k=1}^{4} k^2$.

3. Use summation notation to rewrite the following: $1^3 - 2^3 + 3^3 - 4^3 + 5^3$.

6. Transform the following summation by making the change of variable $i = k + 1$:

$$\sum_{k=0}^{n} \frac{k^2}{k+n}.$$

12. Use mathematical induction to prove that for all integers $n \geq 1$,

$$4 + 8 + 12 + \cdots + 4n = 2n^2 + 2n.$$

20. A sequence $s_1, s_2, s_3, \ldots$ is defined recursively as follows:

$$\begin{aligned} s_k &= 5s_{k-1} + (s_{k-2})^2 \quad \text{for all integers } k \geq 3 \\ s_1 &= 4 \\ s_2 &= 8 \end{aligned}$$

Use (strong) mathematical induction to prove that s_n is divisible by 4 for all integers $n \geq 1$.