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Student Signature: ______________________________ ID Number: ______________________________

Final Exam, Math 763 A

Take Home Exam, Due at noon on Dec 10, 2013

1. (30pts) Problem 8.2.6, page 332.

a. Run a Hartley test, use $\alpha = 0.05$.

b. If you accept H_o in a, run an ANOVA, use $\alpha = 0.05$.

c. If you reject H_o in b, do the Tukey test, use $\alpha = 0.05$.

2. (10pts) Problem 12.4.6, page 629.

3. (5pts) If the result of an ANOVA test is the rejection of H_o, then you want to estimate μ_j $(1 \le j \le k)$. In the lecture, I said the confidence interval for μ_j is the following form:

$$\bar{x}_j - ?\frac{\sqrt{MSW}}{n} \le \mu_j \le \bar{x}_j + ?\frac{\sqrt{MSW}}{n},$$

where $MSW = \frac{SS_w}{k(n-1)} = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{k(n-1)}$. This, in fact, is the consequence of the fact that $\frac{n(\bar{x}_j-\mu_j)}{\sqrt{MSW}}$ is a t-distribution. Please verify this fact and find its degree of freedom.

4. (5pts) When testing $H_o: \mu_1 = \mu_2 \leftrightarrow H_a: \mu_1 \ne \mu_2$, in case σ_1, σ_2 are unknown but $\sigma_1 = \sigma_2$, we use the following formula

$$\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = t_{n_1+n_2-2}.$$

In case σ_1, σ_2 are unknown but $\sigma_1 = 3\sigma_2$, find the right formula.

5. (5pts) Let $x_1, \ldots, x_{16}$ be an independent sample from a normal distribution $N(\mu, 5)$. For the following test problem: $H_o: \mu = 6.5 \leftrightarrow H_a: \mu \ne 6.5$, let the rejection region be $W = \{|\bar{x} - 6.5| \ge c\}$ *(Note, in the lecture, I often call the rejection region as "the wired region")*. Find C to make the significant level, namely α, of this test be 0.05.

6. (25pts) There are seven types of artificial man-made fibers, 4 pieces were taken from each type to test their strengths. The summery of the date is given as the following:

Types 1	Type 2	Type 3	Type 4	Type 5	Type 6	Type 7
$\bar{x}_1 = 6.3$	$\bar{x}_2 = 6.2$	$\bar{x}_3 = 6.7$	$\bar{x}_4 = 6.8$	$\bar{x}_5 = 6.5$	$\bar{x}_6 = 7.0$	$\bar{x}_7 = 7.1$
$s_1 = .81$	$s_2 = 0.92$	$s_3 = 1.22$	$s_4 = .74$	$s_5 = .88$	$s_6 = .58$	$s_7 = 1.05$

a. With $\alpha = 5\%$, run Hartley test to test $H_o: \sigma_1 = \sigma_2 = \cdots = \sigma_7$

b. If you accept H_o in a, with $\alpha = 5\%$, run ANOVA to test $H_o: \mu_1 = \mu_2 = \cdots = \mu_7$ (assume all dates are from normal distributions)

c. If you accept H_o in b (that means all date from the same population), please find a 95% confidence interval for μ ($\mu = \mu_1 = \mu_2 = \cdots = \mu_7$); if you reject H_o, please run the follow up Tukey text with $\alpha = 5\%$.

7. (5pts) Suppose that two drugs are under study. The researcher is willing to assume that response times, following administration of two drugs, are normally distributed with equal variance of 68. As part of the evaluation of two drugs, drug A was administered to 16 subjects and drug B to administered to 19 subjects. If the difference of sample mean, namely $\bar{x}_A - \bar{x}_B$, is 3.6, can you find a number k such that $P(\mu_A - \mu_B > k) = 90\%$? If so, find this number k.

8. (5pts) Let $x_1, \ldots, x_n$ be an independent sample from a normal distribution $N(\mu, \sigma)$. Let $S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ *(Note, this is different from* $s_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, *but related!)*. Find the smallest n such that

$$P\{|S_n^2 - \sigma^2| \le \frac{1}{2}\sigma^2\} \ge 0.8.$$

9. (5pts) Suppose there are 1 black ball and 2 white balls in box 1, and there are 3 white balls in box 2. You randomly pick up one ball from each box, then put the ball you choose from box 1 to box 2 and ball you choose from box 2 to box 1 (namely you do the exchange). Repeat this procedure n times, find the probability that black ball is still in box 1. Let p_n be this probability, show that $\lim_{n\to\infty} p_n = \frac{1}{2}$.

10. (5pts) Let X be a random variable that only takes natural numbers $1, 2, 3, \ldots n, \ldots$ as possible values, for which we know that $P(X = 1) = 0.2$. If for any n and m, we have $P(X > n + m | X > m) = P(X > n)$, find $P(X = 15)$.