The year 2030 is many years in the future, however. Many factors could affect the tuition, and the actual figure for 2030 could turn out to be very different from our prediction.

TECHNOLOGY NOTE

You can plot data with a TI-84 Plus graphing calculator using the following steps.

1. Store the data in lists.
2. Define the stat plot.
3. Turn off $Y =$ functions (unless you also want to graph a function).
4. Turn on the plot you want to display.
5. Define the viewing window.
6. Display the graph.

Consult the calculator's instruction booklet or the *Graphing Calculator and Excel Spreadsheet Manual*, available with this book, for specific instructions. See the calculator-generated graph in Figure 10(b), which includes the points and line from Example 14. Notice how the line closely approximates the data.

1.1 EXERCISES

Find the slope of each line.

1. Through $(4, 5)$ and $(-1, 2)$
2. Through $(5, -4)$ and $(1, 3)$
3. Through $(8, 4)$ and $(8, -7)$
4. Through $(1, 5)$ and $(-2, 5)$
5. $y = x$
6. $y = 3x - 2$
7. $5x - 9y = 11$
8. $4x + 7y = 1$
9. $x = 5$
10. The x-axis
11. $y = 8$
12. $y = -6$
13. A line parallel to $6x - 3y = 12$
14. A line perpendicular to $8x = 2y - 5$

In Exercises 15–24, find an equation in slope-intercept form for each line.

15. Through $(1, 3)$, $m = -2$
16. Through $(2, 4)$, $m = -1$
17. Through $(-5, -7)$, $m = 0$
18. Through $(-8, 1)$, with undefined slope
19. Through $(4, 2)$ and $(1, 3)$
20. Through $(8, -1)$ and $(4, 3)$
21. Through $(2/3, 1/2)$ and $(1/4, -2)$
22. Through $(-2, 3/4)$ and $(2/3, 5/2)$
23. Through $(-8, 4)$ and $(-8, 6)$
24. Through $(-1, 3)$ and $(0, 3)$

In Exercises 25–34, find an equation for each line in the form $ax + by = c$, where a, b, and c are integers with no factor common to all three and $a \geq 0$.

25. x-intercept -6, y-intercept -3
26. x-intercept -2, y-intercept 4
27. Vertical, through $(-6, 5)$
28. Horizontal, through $(8, 7)$
29. Through $(-4, 6)$, parallel to $3x + 2y = 13$
30. Through $(2, -5)$, parallel to $2x - y = -4$
31. Through $(3, -4)$, perpendicular to $x + y = 4$
32. Through $(-2, 6)$, perpendicular to $2x - 3y = 5$
33. The line with y-intercept 4 and perpendicular to $x + 5y = 7$
34. The line with x-intercept $-2/3$ and perpendicular to $2x - y = 4$
35. Do the points $(4, 3)$, $(2, 0)$, and $(-18, -12)$ lie on the same line? Explain why or why not. (*Hint:* Find the slopes between the points.)
36. Find k so that the line through $(4, -1)$ and $(k, 2)$ is
    a. parallel to $2x + 3y = 6$,
    b. perpendicular to $5x - 2y = -1$.
37. Use slopes to show that the quadrilateral with vertices at $(1, 3)$, $(-5/2, 2)$, $(-7/2, 4)$, and $(2, 1)$ is a parallelogram.
38. Use slopes to show that the square with vertices at $(-2, 5)$, $(4, 5)$, $(4, -1)$, and $(-2, -1)$ has diagonals that are perpendicular.

The F-intercept of the graph is 32, so by the slope-intercept form, the equation of the line is

$$F = \frac{9}{5}C + 32.$$

With simple algebra this equation can be rewritten to give C in terms of F:

$$C = \frac{5}{9}(F - 32).$$

1.2 EXERCISES

For Exercises 1–10, let $f(x) = 7 - 5x$ and $g(x) = 2x - 3$. Find the following.

1. $f(2)$
2. $f(4)$
3. $f(-3)$
4. $f(-1)$
5. $g(1.5)$
6. $g(2.5)$
7. $g(-1/2)$
8. $g(-3/4)$
9. $f(t)$
10. $g(k^2)$

In Exercises 11–14, decide whether the statement is true or false.

11. To find the x-intercept of the graph of a linear function, we solve $y = f(x) = 0$, and to find the y-intercept, we evaluate $f(0)$.
12. The graph of $f(x) = -5$ is a vertical line.
13. The slope of the graph of a linear function cannot be undefined.
14. The graph of $f(x) = ax$ is a straight line that passes through the origin.
15. Describe what fixed costs and marginal costs mean to a company.
16. In a few sentences, explain why the price of a commodity not already at its equilibrium price should move in that direction.
17. Explain why a linear function may not be adequate for describing the supply and demand functions.
18. In your own words, describe the break-even quantity, how to find it, and what it indicates.

Write a linear cost function for each situation. Identify all variables used.

19. A Lake Tahoe resort charges a snowboard rental fee of \$10 plus \$2.25 per hour.
20. An Internet site for downloading music charges a \$10 registration fee plus 99 cents per downloaded song.
21. A parking garage charges 2 dollars plus 75 cents per half-hour.
22. For a one-day rental, a car rental firm charges \$44 plus 28 cents per mile.

Assume that each situation can be expressed as a linear cost function. Find the cost function in each case.

23. Fixed cost: \$100; 50 items cost \$1600 to produce.
24. Fixed cost: \$35; 8 items cost \$395 to produce.
25. Marginal cost: \$75; 50 items cost \$4300 to produce.
26. Marginal cost: \$120; 700 items cost \$96,500 to produce.

APPLICATIONS

Business and Economics

27. **Supply and Demand** Suppose that the demand and price for a certain model of a youth wristwatch are related by

$$p = D(q) = 16 - 1.25q,$$

where p is the price (in dollars) and q is the quantity demanded (in hundreds). Find the price at each level of demand.

a. 0 watches **b.** 400 watches **c.** 800 watches

Find the quantity demanded for the watch at each price.

d. \$8 **e.** \$10 **f.** \$12

g. Graph $p = 16 - 1.25q$.

Suppose the price and supply of the watch are related by

$$p = S(q) = 0.75q,$$

where p is the price (in dollars) and q is the quantity supplied (in hundreds) of watches. Find the quantity supplied at each price.

h. \$0 **i.** \$10 **j.** \$20

k. Graph $p = 0.75q$ on the same axis used for part g.

l. Find the equilibrium quantity and the equilibrium price.

28. **Supply and Demand** Suppose that the demand and price for strawberries are related by

$$p = D(q) = 5 - 0.25q,$$

where p is the price (in dollars) and q is the quantity demanded (in hundreds of quarts). Find the price at each level of demand.

a. 0 quarts **b.** 400 quarts **c.** 840 quarts

SOLUTION For w in the interval $(0, 2]$, the shipping cost is $y = 25$. For w in $(2, 3]$, the shipping cost is $y = 25 + 3 = 28$. For w in $(3, 4]$, the shipping cost is $y = 28 + 3 = 31$, and so on. The graph is shown in Figure 11.

The function discussed in Example 7 is called a **step function**. Many real-life situations are best modeled by step functions. Additional examples are given in the exercises.

In Chapter 1 you saw several examples of linear models. In Example 8, we use a quadratic equation to model the area of a lot.

EXAMPLE 8 Area

A fence is to be built against a brick wall to form a rectangular lot, as shown in Figure 12. Only three sides of the fence need to be built, because the wall forms the fourth side. The contractor will use 200 m of fencing. Let the length of the wall be l and the width w, as shown in Figure 12.

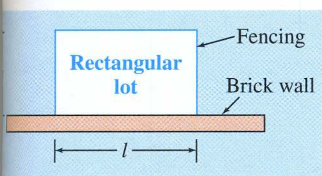


FIGURE 12

(a) Find the area of the lot as a function of the length l.

SOLUTION The area formula for a rectangle is area = length × width, or

$$A = lw.$$

We want the area as a function of the length only, so we must eliminate the width. We use the fact that the total amount of fencing is the sum of the three sections, one length and two widths, so $200 = l + 2w$. Solve this for w:

$$200 = l + 2w$$
$$200 - l = 2w \qquad \text{Subtract } l \text{ from both sides.}$$
$$100 - l/2 = w. \qquad \text{Divide both sides by 2.}$$

Substituting this into the formula for area gives

$$A = l(100 - l/2).$$

(b) Find the domain of the function in part (a).

SOLUTION The length cannot be negative, so $l \geq 0$. Similarly, the width cannot be negative, so $100 - l/2 \geq 0$, from which we find $l \leq 200$. Therefore, the domain is $[0, 200]$.

(c) Sketch a graph of the function in part (a).

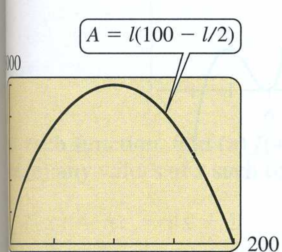


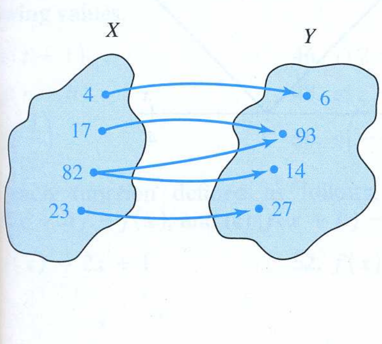
FIGURE 13

SOLUTION The result from a graphing calculator is shown in Figure 13. Notice that at the endpoints of the domain, when $l = 0$ and $l = 200$, the area is 0. This makes sense: If the length or width is 0, the area will be 0 as well. In between, as the length increases from 0 to 100 m, the area gets larger, and seems to reach a peak of 5000 m^2 when $l = 100$ m. After that, the area gets smaller as the length continues to increase because the width is becoming smaller.

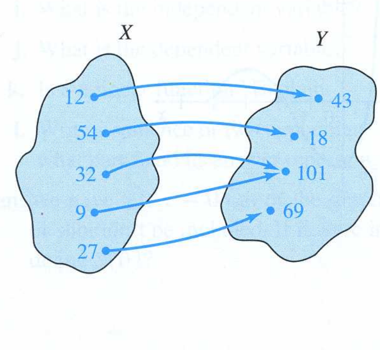
In the next section, we will study this type of function in more detail and determine exactly where the maximum occurs.

0.1 EXERCISES

hich of the following rules define y as a function of x?



2.



72. Rental Car Cost The cost to rent a mid-size car is \$54 per day or fraction of a day. If the car is picked up in Pittsburgh and dropped off in Cleveland, there is a fixed \$44 drop-off charge. Let $C(x)$ represent the cost of renting the car for x days, taking it from Pittsburgh to Cleveland. Find the following.

a. $C(3/4)$ **b.** $C(9/10)$ **c.** $C(1)$ **d.** $C\left(1\frac{5}{8}\right)$

e. Find the cost of renting the car for 2.4 days.

f. Graph $y = C(x)$.

g. Is C a function? Explain.

h. Is C a linear function? Explain.

i. What is the independent variable?

j. What is the dependent variable?

73. Attorney Fees According to Massachusetts state law, the maximum amount of a jury award that attorneys can receive is:

40% of the first \$150,000,

33.3% of the next \$150,000,

30% of the next \$200,000, and

24% of anything over \$500,000.

Let $f(x)$ represent the maximum amount of money that an attorney in Massachusetts can receive for a jury award of size x. Find each of the following, and describe in a sentence what the answer tells you. ***Source: The New Yorker.***

a. $f(250{,}000)$ **b.** $f(350{,}000)$ **c.** $f(550{,}000)$

d. Sketch a graph of $f(x)$.

74. Tax Rates In New York state in 2010, the income tax rates for a single person were as follows:

4% of the first \$8000 earned,

4.5% of the next \$3000 earned,

5.25% of the next \$2000 earned,

5.9% of the next \$7000 earned,

6.85% of the next \$180,000 earned,

7.85% of the next \$300,000 earned, and

8.97% of any amount earned over \$500,000.

Let $f(x)$ represent the amount of tax owed on an income of x dollars. Find each of the following, and explain in a sentence what the answer tells you. ***Source: New York State.***

a. $f(10{,}000)$

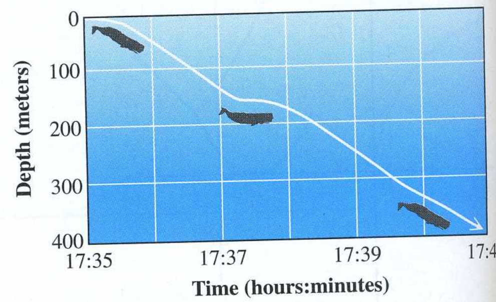
b. $f(12{,}000)$

c. $f(18{,}000)$

d. Sketch a graph of $f(x)$.

Life Sciences

75. Whales Diving The figure in the next column shows the depth of a diving sperm whale as a function of time, as recorded by researchers at the Woods Hole Oceanographic Institution in Massachusetts. ***Source: Peter Tyack, Woods Hole Oceanographic Institution.***



Find the depth of the whale at the following times.

a. 17 hours and 37 minutes

b. 17 hours and 39 minutes

76. Metabolic Rate The basal metabolic rate (in kcal/day) for large anteaters is given by

$$y = f(x) = 19.7x^{0.753},$$

where x is the anteater's weight in kilograms.* ***Source: Wildlife Feeding and Nutrition.***

a. Find the basal metabolic rate for anteaters with the following weights.

i. 5 kg **ii.** 25 kg

b. Suppose the anteater's weight is given in pounds rather than kilograms. Given that 1 lb = 0.454 kg, find a function $x = g(z)$ giving the anteater's weight in kilograms if z is the animal's weight in pounds.

c. Write the basal metabolic rate as a function of the weight in pounds in the form $y = az^b$ by calculating $f(g(z))$.

77. Swimming Energy The energy expenditure (in kcal/km) for animals swimming at the surface of the water is given by

$$y = f(x) = 0.01x^{0.88},$$

where x is the animal's weight in grams. ***Source: Wildlife Feeding and Nutrition.***

a. Find the energy for the following animals swimming at the surface of the water.

i. A muskrat weighing 800 g

ii. A sea otter weighing 20,000 g

b. Suppose the animal's weight is given in kilograms rather than grams. Given that 1 kg = 1000 g, find a function $x = g(z)$ giving the animal's weight in grams if z is the animal's weight in kilograms.

c. Write the energy expenditure as a function of the weight in kilograms in the form $y = az^b$ by calculating $f(g(z))$.

*Technically, kilograms are a measure of mass, not weight. Weight is a measure of the force of gravity, which varies with the distance from the center of Earth. For objects on the surface of Earth, weight and mass are often used interchangeably, and we will do so in this text.

10.4 EXERCISES

A ream of 20-lb paper contains 500 sheets and is about 2 in. high. Suppose you take one sheet, fold it in half, then fold it in half again, continuing in this way as long as possible. *Source: The AMATYC Review.*

1. Complete the table.

Number of Folds	1	2	3	4	5	…	10	…	50
Layers of Paper									

2. After folding 50 times (if this were possible), what would be the height (in miles) of the folded paper?

For Exercises 3–11, match the correct graph A–F to the function without using your calculator. Notice that there are more functions than graphs; some of the functions are equivalent. After you have answered all of them, use a graphing calculator to check your answers. Each graph in this group is plotted on the window $[-2, 2]$ by $[-4, 4]$.

3. $y = 3^x$

4. $y = 3^{-x}$

5. $y = \left(\frac{1}{3}\right)^{1-x}$

6. $y = 3^{x+1}$

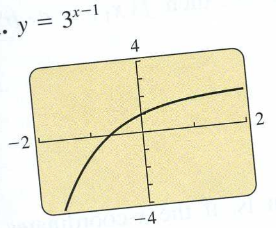
7. $y = 3(3)^x$

8. $y = \left(\frac{1}{3}\right)^x$

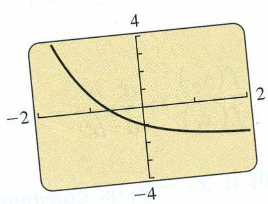
9. $y = 2 - 3^{-x}$

10. $y = -2 + 3^{-x}$

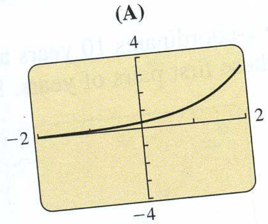
11. $y = 3^{x-1}$



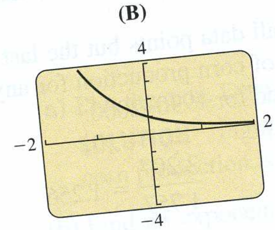
(A)



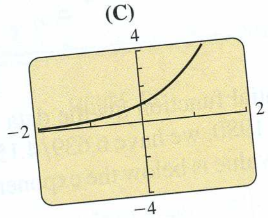
(B)



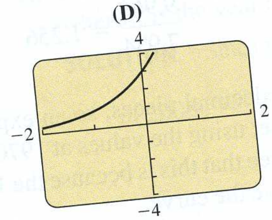
(C)



(D)



(E)



(F)

12. In Exercises 3–11, there were more formulas for functions than there were graphs. Explain how this is possible.

Solve each equation.

13. $2^x = 32$

14. $4^x = 64$

15. $3^x = \frac{1}{81}$

16. $e^x = \frac{1}{e^5}$

17. $4^x = 8^{x+1}$

18. $25^x = 125^{x+2}$

19. $16^{x+3} = 64^{2x-5}$

20. $(e^3)^{-2x} = e^{-x+5}$

21. $e^{-x} = (e^4)^{x+3}$

22. $2^{|x|} = 8$

23. $5^{-|x|} = \frac{1}{25}$

24. $2^{x^2-4x} = \left(\frac{1}{16}\right)^{x-4}$

25. $5^{x^2+x} = 1$

26. $8^{x^2} = 2^{x+4}$

27. $27^x = 9^{x^2+x}$

28. $e^{x^2+5x+6} = 1$

Graph each of the following.

29. $y = 5e^x + 2$

30. $y = -2e^x - 3$

31. $y = -3e^{-2x} + 2$

32. $y = 4e^{-x/2} - 1$

33. In our definition of exponential function, we ruled out negative values of a. The author of a textbook on mathematical economics, however, obtained a "graph" of $y = (-2)^x$ by plotting the following points and drawing a smooth curve through them.

x	−4	−3	−2	−1	0	1	2
y	1/16	−1/8	1/4	−1/2	1	−2	4

The graph oscillates very neatly from positive to negative values of y. Comment on this approach. (This exercise shows the dangers of point plotting when drawing graphs.)

34. Explain why the exponential equation $4^x = 6$ cannot be solved using the method described in Example 3.

35. Explain why $3^x > e^x > 2^x$ when $x > 0$, but $3^x <$ … when $x < 0$.

36. A friend claims that as x becomes large, the expression … gets closer and closer to 1, and 1 raised to any power … Therefore, $f(x) = (1 + 1/x)^x$ gets closer and closer … gets larger. Use a graphing calculator to graph … $0.1 \le x \le 50$. How might you use this graph to expla… friend why $f(x)$ does not approach 1 as x becom… What does it approach?

APPLICATIONS

Business and Economics

37. Interest Find the interest earned on $10,000 invested … at 4% interest compounded as follows.

a. Annually **b.** Semiannually (twice a year)

c. Quarterly **d.** Monthly **e.** Continuously

38. Interest Suppose $26,000 is borrowed for 4 years a… est. Find the interest paid over this period if the inter… pounded as follows.

a. Annually **b.** Semiannually

c. Quarterly **d.** Monthly **e.** Continuously

39. Interest Ron Hampton needs to choose between two i… One pays 6% compounded annually, and the other … compounded monthly. If he plans to invest $18,000 … which investment should he choose? How much extra … he earn by making the better choice?

40. Interest Find the interest rate re… $5000 to grow to $7500 in 5 years … follows.

a. Annually **b.** Quarter…

41. Inflation Assuming continuous … cost to buy a $10 item in 3 yea… rates?

a. 3% **b.** 4%

42. Interest Ali Williams invests a … paying 5.5% per year compounde… the amount on deposit after each…

a. 1 year **b.** 5 year…

43. Interest Leigh Jacks plans to i… ket account. Find the interest rat… to grow to $1200 in 14 years … quarterly.

44. Interest Kristi Perez puts $10… money to buy a car in 12 yea… dreams to cost $30,000 by then. … essay if the interest is compute…

a. Compounded quarterly **b.** …

45. Inflation If money loses value … value of $1 in t years is given b…

$y = (1 - 0.08$…

a. Use a calculator to help com…

t	0	1	2	3	4
y	1				0…

b. Graph $y = (0.92)^t$.

c. Suppose a house costs $1… part a to estimate the cost …

d. Find the cost of a $50 text…

. Interest On January 1, 200… X to earn interest at the rate … annually. On January 1, 20… Bank Y to earn interest at th… quarterly. On January 1, 2… $1990.76. If Jack could hav… annum compounded quarte… January 1, 2008, his bala… Which of the following r… *Society of Actuaries.*

a. 1.25 **b.** 1.30 **c.** 1.…

…e Sciences

Population Growth Since … (in millions) closely fits th…

$A(t) =$ …

where t is the number of … *Nations.*

a. World population was … closely does the functio…

40. **Interest** Find the interest rate required for an investment of \$5000 to grow to \$7500 in 5 years if interest is compounded as follows.

a. Annually **b.** Quarterly

41. **Inflation** Assuming continuous compounding, what will it cost to buy a \$10 item in 3 years at the following inflation rates?

a. 3% **b.** 4% **c.** 5%

42. **Interest** Ali Williams invests a \$25,000 inheritance in a fund paying 5.5% per year compounded continuously. What will be the amount on deposit after each time period?

a. 1 year **b.** 5 years **c.** 10 years

43. **Interest** Leigh Jacks plans to invest \$500 into a money market account. Find the interest rate that is needed for the money to grow to \$1200 in 14 years if the interest is compounded quarterly.

44. **Interest** Kristi Perez puts \$10,500 into an account to save money to buy a car in 12 years. She expects the car of her dreams to cost \$30,000 by then. Find the interest rate that is necessary if the interest is computed using the following methods.

a. Compounded quarterly **b.** Compounded continuously

45. **Inflation** If money loses value at the rate of 8% per year, the value of \$1 in t years is given by

$$y = (1 - 0.08)^t = (0.92)^t.$$

a. Use a calculator to help complete the following table.

t	0	1	2	3	4	5	6	7	8	9	10
y	1					0.66					0.43

b. Graph $y = (0.92)^t$.

c. Suppose a house costs \$165,000 today. Use the results of part a to estimate the cost of a similar house in 10 years.

d. Find the cost of a \$50 textbook in 8 years.

46. **Interest** On January 1, 2000, Jack deposited \$1000 into Bank X to earn interest at the rate of j per annum compounded semi-annually. On January 1, 2005, he transferred his account to Bank Y to earn interest at the rate of k per annum compounded quarterly. On January 1, 2008, the balance at Bank Y was \$1990.76. If Jack could have earned interest at the rate of k per annum compounded quarterly from January 1, 2000, through January 1, 2008, his balance would have been \$2203.76. Which of the following represents the ratio k/j? ***Source: Society of Actuaries.***

a. 1.25 **b.** 1.30 **c.** 1.35 **d.** 1.40 **e.** 1.45

Life Sciences

47. **Population Growth** Since 1960, the growth in world population (in millions) closely fits the exponential function defined by

$$A(t) = 3100e^{0.0166t},$$

where t is the number of years since 1960. ***Source: United Nations.***

a. World population was about 3686 million in 1970. How closely does the function approximate this value?

b. Use the function to approximate world population in 2000. (The actual 2000 population was about 6115 million.)

c. Estimate world population in the year 2015.

48. Growth of Bacteria Salmonella bacteria, found on almost all chicken and eggs, grow rapidly in a nice warm place. If just a few hundred bacteria are left on the cutting board when a chicken is cut up, and they get into the potato salad, the population begins compounding. Suppose the number present in the potato salad after x hours is given by

$$f(x) = 500 \cdot 2^{3x}.$$

a. If the potato salad is left out on the table, how many bacteria are present 1 hour later?

b. How many were present initially?

c. How often do the bacteria double?

d. How quickly will the number of bacteria increase to 32,000?

49. Minority Population According to the U.S. Census Bureau, the United States is becoming more diverse. Based on U.S. Census population projections for 2000 to 2050, the projected Hispanic population (in millions) can be modeled by the exponential function

$$h(t) = 37.79(1.021)^t,$$

where $t = 0$ corresponds to 2000 and $0 \le t \le 50$. ***Source: U.S. Census Bureau.***

a. Find the projected Hispanic population for 2005. Compare this to the actual value of 42.69 million.

b. The U.S. Asian population is also growing exponentially, and the projected Asian population (in millions) can be modeled by the exponential function

$$a(t) = 11.14(1.023)^t,$$

where $t = 0$ corresponds to 2000 and $0 \le t \le 50$. Find the projected Asian population for 2005, and compare this to the actual value of 12.69 million.

c. Determine the expected annual percentage increase for Hispanics and for Asians. Which minority population, Hispanic or Asian, is growing at a faster rate?

d. The U.S. black population is growing at a linear rate, and the projected black population (in millions) can be modeled by the linear function

$$b(t) = 0.5116t + 35.43,$$

where $t = 0$ corresponds to 2000 and $0 \le t \le 50$. Find the projected black population for 2005 and compare this projection to the actual value of 37.91 million.

e. Graph the projected population function for Hispanics and estimate when the Hispanic population will be double its actual value for 2005. Then do the same for the Asian and black populations. Comment on the accuracy of these numbers.

50. Physician Demand The demand for physicians is expected to increase in the future, as shown in the table on the following page. ***Source: Association of American Medical Colleges.***

EXAMPLE 10 Index of Diversity

One measure of the diversity of the species in an ecological community is given by t **index of diversity** H, where

$$H = -[P_1 \ln P_1 + P_2 \ln P_2 + \cdots + P_n \ln P_n],$$

and $P_1, P_2, \ldots, P_n$ are the proportions of a sample belonging to each of n species found in sample. ***Source: Statistical Ecology.*** For example, in a community with two species, where th are 90 of one species and 10 of the other, $P_1 = 90/100 = 0.9$ and $P_2 = 10/100 = 0.1$, wi

$$H = -[0.9 \ln 0.9 + 0.1 \ln 0.1] \approx 0.325.$$

Verify that $H \approx 0.673$ if there are 60 of one species and 40 of the other. As the prop tions of n species get closer to $1/n$ each, the index of diversity increases to a maximum $\ln n$.

10.5 EXERCISES

Write each exponential equation in logarithmic form.

1. $5^3 = 125$

2. $7^2 = 49$

3. $3^4 = 81$

4. $2^7 = 128$

5. $3^{-2} = \dfrac{1}{9}$

6. $\left(\dfrac{5}{4}\right)^{-2} = \dfrac{16}{25}$

Write each logarithmic equation in exponential form.

7. $\log_2 32 = 5$

8. $\log_3 81 = 4$

9. $\ln \dfrac{1}{e} = -1$

10. $\log_2 \dfrac{1}{8} = -3$

11. $\log 100{,}000 = 5$

12. $\log 0.001 = -3$

Evaluate each logarithm without using a calculator.

13. $\log_8 64$

14. $\log_9 81$

15. $\log_4 64$

16. $\log_3 27$

17. $\log_2 \dfrac{1}{16}$

18. $\log_3 \dfrac{1}{81}$

19. $\log_2 \sqrt[3]{\dfrac{1}{4}}$

20. $\log_8 \sqrt[4]{\dfrac{1}{2}}$

21. $\ln e$

22. $\ln e^3$

23. $\ln e^{5/3}$

24. $\ln 1$

25. Is the "logarithm to the base 3 of 4" written as $\log_4 3$ or $\log_3 4$?

26. Write a few sentences describing the relationship between e^x and $\ln x$.

Use the properties of logarithms to write each expression as a sum, difference, or product of simpler logarithms. For example, $\log_2(\sqrt{3}x) = \frac{1}{2}\log_2 3 + \log_2 x$.

27. $\log_5(3k)$

28. $\log_9(4m)$

29. $\log_3 \dfrac{3p}{5k}$

30. $\log_7 \dfrac{15p}{7y}$

31. $\ln \dfrac{3\sqrt{5}}{\sqrt[3]{6}}$

32. $\ln \dfrac{9\sqrt[3]{5}}{\sqrt[4]{3}}$

Suppose $\log_b 2 = a$ and $\log_b 3 = c$. Use the properties of lo rithms to find the following.

33. $\log_b 32$

34. $\log_b 18$

35. $\log_b(72b)$

36. $\log_b(9b^2)$

Use natural logarithms to evaluate each logarithm to the ne est thousandth.

37. $\log_5 30$

38. $\log_{12} 210$

39. $\log_{1.2} 0.95$

40. $\log_{2.8} 0.12$

Solve each equation in Exercises 41–64. Round decimal ans to four decimal places.

41. $\log_x 36 = -2$

42. $\log_9 27 = m$

43. $\log_8 16 = z$

44. $\log_y 8 = \dfrac{3}{4}$

45. $\log_r 5 = \dfrac{1}{2}$

46. $\log_4(5x + 1) = 2$

47. $\log_5(9x - 4) = 1$

48. $\log_4 x - \log_4(x + 3) = -1$

49. $\log_9 m - \log_9(m - 4) = -2$

50. $\log(x + 5) + \log(x + 2) = 1$

51. $\log_3(x - 2) + \log_3(x + 6) = 2$

52. $\log_3(x^2 + 17) - \log_3(x + 5) = 1$

53. $\log_2(x^2 - 1) - \log_2(x + 1) = 2$

54. $\ln(5x + 4) = 2$

55. $\ln x + \ln 3x = -1$

56. $\ln(x + 1) - \ln x =$

57. $2^x = 6$

58. $5^x = 12$

59. $e^{k-1} = 6$

60. $e^{2y} = 15$

61. $3^{x+1} = 5^x$

62. $2^{x+1} = 6^{x-1}$

63. $5(0.10)^x = 4(0.12)^x$

64. $1.5(1.05)^x = 2(1.01)^x$

Write each expression using base e rather than base 10.

65. 10^{x+1}

66. 10^{x^2}

Approximate each expression in the form a^x without using e.

67. e^{3x} **68.** e^{-4x}

Find the domain of each function.

69. $f(x) = \log(5 - x)$ **70.** $f(x) = \ln(x^2 - 9)$

71. Lucky Larry was faced with solving

$$\log(2x + 1) - \log(3x - 1) = 0.$$

Larry just dropped the logs and proceeded:

$$\begin{aligned}(2x + 1) - (3x - 1) &= 0\\ -x + 2 &= 0\\ x &= 2.\end{aligned}$$

Although Lucky Larry is wrong in dropping the logs, his procedure will always give the correct answer to an equation of the form

$$\log A - \log B = 0,$$

where A and B are any two expressions in x. Prove that this last equation leads to the equation $A - B = 0$, which is what you get when you drop the logs. ***Source: The AMATYC Review.***

72. Find all errors in the following calculation.

$$\begin{aligned}(\log (x + 2))^2 &= 2 \log (x + 2)\\ &= 2(\log x + \log 2)\\ &= 2(\log x + 100)\end{aligned}$$

73. Prove: $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$.

74. Prove: $\log_a x^r = r \log_a x$.

APPLICATIONS

Business and Economics

75. **Inflation** Assuming annual compounding, find the time it would take for the general level of prices in the economy to double at the following annual inflation rates.

a. 3% **b.** 6% **c.** 8%

d. Check your answers using either the rule of 70 or the rule of 72, whichever applies.

76. **Interest** Mary Klingman invests $15,000 in an account paying 7% per year compounded annually.

a. How many years are required for the compound amount to at least double? (Note that interest is only paid at the end of each year.)

b. In how many years will the amount at least triple?

c. Check your answer to part a using either the rule of 70 or the rule of 72, whichever applies.

77. **Interest** Leigh Jacks plans to invest $500 into a money market account. Find the interest rate that is needed for the money to grow to $1200 in 14 years if the interest is compounded continuously. (Compare with Exercise 43 in the previous section.)

78. Rule of 72 Complete the following table, and use the results to discuss when the rule of 70 gives a better approximation for the doubling time, and when the rule of 72 gives a better approximation.

r	0.001	0.02	0.05	0.08	0.12
$(\ln 2)/\ln(1 + r)$					
$70/100r$					
$72/100r$					

79. Pay Increases You are offered two jobs starting July 1, 2013. Humongous Enterprises offers you $45,000 a year to start, with a raise of 4% every July 1. At Crabapple Inc. you start at $30,000, with an annual increase of 6% every July 1. On July 1 of what year would the job at Crabapple Inc. pay more than the job at Humongous Enterprises? Use the algebra of logarithms to solve this problem, and support your answer by using a graphing calculator to see where the two salary functions intersect.

Life Sciences

80. Insect Species An article in *Science* stated that the number of insect species of a given mass is proportional to $m^{-0.6}$, where m is the mass in grams. ***Source: Science.*** A graph accompanying the article shows the common logarithm of the mass on the horizontal axis and the common logarithm of the number of species on the vertical axis. Explain why the graph is a straight line. What is the slope of the line?

Index of Diversity For Exercises 81–83, refer to Example 10.

81. Suppose a sample of a small community shows two species with 50 individuals each.

a. Find the index of diversity H.

b. What is the maximum value of the index of diversity for two species?

c. Does your answer for part a equal ln 2? Explain why.

82. A virgin forest in northwestern Pennsylvania has 4 species of large trees with the following proportions of each: hemlock, 0.521; beech, 0.324; birch, 0.081; maple, 0.074. Find the index of diversity H.

83. Find the value of the index of diversity for populations with n species and $1/n$ of each if

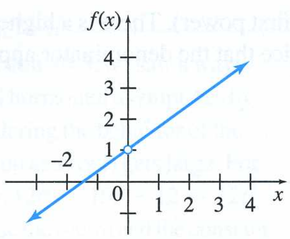
a. $n = 3$; **b.** $n = 4$.

c. Verify that your answers for parts a and b equal ln 3 and ln 4, respectively.

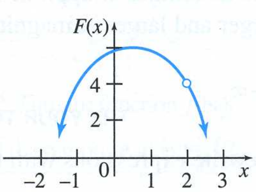
84. Allometric Growth The allometric formula is used to describe a wide variety of growth patterns. It says that $y = nx^m$, where x and y are variables, and n and m are constants. For example, the famous biologist J. S. Huxley used this formula to relate the weight of the large claw of the fiddler crab to the weight of the body without the claw. ***Source: Problems of Relative Growth.*** Show that if x and y are given by the allometric formula, then

Decide whether each limit exists. If a limit exists, estimate its value.

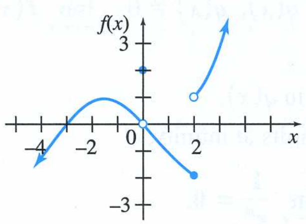
5. a. $\lim_{x\to 3} f(x)$ **b.** $\lim_{x\to 0} f(x)$



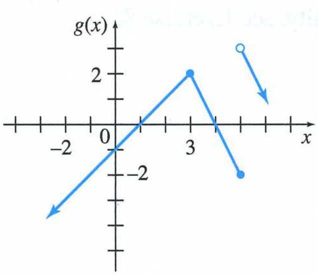
6. a. $\lim_{x\to 2} F(x)$ **b.** $\lim_{x\to -1} F(x)$



7. a. $\lim_{x\to 0} f(x)$ **b.** $\lim_{x\to 2} f(x)$

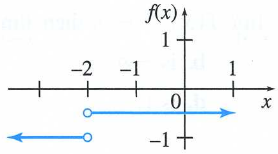


8. a. $\lim_{x\to 3} g(x)$ **b.** $\lim_{x\to 5} g(x)$

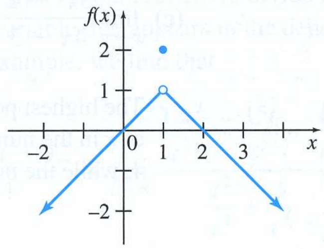


In Exercises 9 and 10, use the graph to find (i) $\lim_{x\to a^-} f(x)$, **(ii)** $\lim_{x\to a^+} f(x)$, **(iii)** $\lim_{x\to a} f(x)$, **and (iv)** $f(a)$ **if it exists.**

9. a. $a = -2$ **b.** $a = -1$

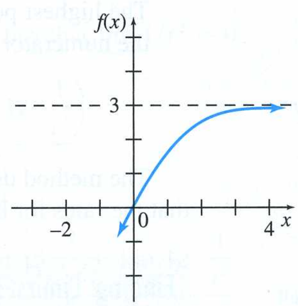


10. a. $a = 1$ **b.** $a = 2$

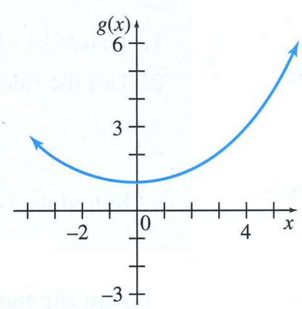


Decide whether each limit exists. If a limit exists, find its valu

11. $\lim_{x\to\infty} f(x)$



12. $\lim_{x\to -\infty} g(x)$



13. Explain why $\lim_{x\to 2} F(x)$ in Exercise 6 exists, but $\lim_{x\to -2} f(x)$ i
Exercise 9 does not.

14. In Exercise 10, why does $\lim_{x\to 1} f(x) = 1$, even thoug
$f(1) = 2$?

15. Use the table of values to estimate $\lim_{x\to 1} f(x)$.

x	0.9	0.99	0.999	0.9999	1.0001	1.001	1.01	1.1
$f(x)$	3.9	3.99	3.999	3.9999	4.0001	4.001	4.01	4.1

Complete the tables and use the results to find the indicat limits.

16. If $f(x) = 2x^2 - 4x + 7$, find $\lim_{x\to 1} f(x)$.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$			5.000002	5.000002		

17. If $k(x) = \dfrac{x^3 - 2x - 4}{x - 2}$, find $\lim_{x \to 2} k(x)$.

x	1.9	1.99	1.999	2.001	2.01	2.1
$k(x)$						

18. If $f(x) = \dfrac{2x^3 + 3x^2 - 4x - 5}{x + 1}$, find $\lim_{x \to -1} f(x)$.

x	−1.1	−1.01	−1.001	−0.999	−0.99	−0.9
$f(x)$						

19. If $h(x) = \dfrac{\sqrt{x} - 2}{x - 1}$, find $\lim_{x \to 1} h(x)$.

x	0.9	0.99	0.999	1.001	1.01	1.1
$h(x)$						

20. If $f(x) = \dfrac{\sqrt{x} - 3}{x - 3}$, find $\lim_{x \to 3} f(x)$.

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$						

Let $\lim_{x \to 4} f(x) = 9$ and $\lim_{x \to 4} g(x) = 27$. Use the limit rules to find each limit.

21. $\lim_{x \to 4} [f(x) - g(x)]$

22. $\lim_{x \to 4} [g(x) \cdot f(x)]$

23. $\lim_{x \to 4} \dfrac{f(x)}{g(x)}$

24. $\lim_{x \to 4} \log_3 f(x)$

25. $\lim_{x \to 4} \sqrt{f(x)}$

26. $\lim_{x \to 4} \sqrt[3]{g(x)}$

27. $\lim_{x \to 4} 2^{f(x)}$

28. $\lim_{x \to 4} [1 + f(x)]^2$

29. $\lim_{x \to 4} \dfrac{f(x) + g(x)}{2g(x)}$

30. $\lim_{x \to 4} \dfrac{5g(x) + 2}{1 - f(x)}$

Use the properties of limits to help decide whether each limit exists. If a limit exists, find its value.

31. $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

32. $\lim_{x \to -2} \dfrac{x^2 - 4}{x + 2}$

33. $\lim_{x \to 1} \dfrac{5x^2 - 7x + 2}{x^2 - 1}$

34. $\lim_{x \to -3} \dfrac{x^2 - 9}{x^2 + x - 6}$

35. $\lim_{x \to -2} \dfrac{x^2 - x - 6}{x + 2}$

36. $\lim_{x \to 5} \dfrac{x^2 - 3x - 10}{x - 5}$

37. $\lim_{x \to 0} \dfrac{1/(x + 3) - 1/3}{x}$

38. $\lim_{x \to 0} \dfrac{-1/(x + 2) + 1/2}{x}$

39. $\lim_{x \to 25} \dfrac{\sqrt{x} - 5}{x - 25}$

40. $\lim_{x \to 36} \dfrac{\sqrt{x} - 6}{x - 36}$

41. $\lim_{h \to 0} \dfrac{(x + h)^2 - x^2}{h}$

42. $\lim_{h \to 0} \dfrac{(x + h)^3 - x^3}{h}$

43. $\lim_{x \to \infty} \dfrac{3x}{7x - 1}$

44. $\lim_{x \to -\infty} \dfrac{8x + 2}{4x - 5}$

45. $\lim_{x \to -\infty} \dfrac{3x^2 + 2x}{2x^2 - 2x + 1}$

46. $\lim_{x \to \infty} \dfrac{x^2 + 2x - 5}{3x^2 + 2}$

47. $\lim_{x \to \infty} \dfrac{3x^3 + 2x - 1}{2x^4 - 3x^3 - 2}$

48. $\lim_{x \to \infty} \dfrac{2x^2 - 1}{3x^4 + 2}$

49. $\lim_{x \to \infty} \dfrac{2x^3 - x - 3}{6x^2 - x - 1}$

50. $\lim_{x \to \infty} \dfrac{x^4 - x^3 - 3x}{7x^2 + 9}$

51. $\lim_{x \to \infty} \dfrac{2x^2 - 7x^4}{9x^2 + 5x - 6}$

52. $\lim_{x \to \infty} \dfrac{-5x^3 - 4x^2 + 8}{6x^2 + 3x + 2}$

53. Let $f(x) = \begin{cases} x^3 + 2 & \text{if } x \neq -1 \\ 5 & \text{if } x = -1 \end{cases}$. Find $\lim_{x \to -1} f(x)$.

54. Let $g(x) = \begin{cases} 0 & \text{if } x = -2 \\ \frac{1}{2}x^2 - 3 & \text{if } x \neq -2 \end{cases}$. Find $\lim_{x \to -2} g(x)$.

55. Let $f(x) = \begin{cases} x - 1 & \text{if } x < 3 \\ 2 & \text{if } 3 \le x \le 5 \\ x + 3 & \text{if } x > 5 \end{cases}$.

a. Find $\lim_{x \to 3} f(x)$. **b.** Find $\lim_{x \to 5} f(x)$.

56. Let $g(x) = \begin{cases} 5 & \text{if } x < 0 \\ x^2 - 2 & \text{if } 0 \le x \le 3 \\ 7 & \text{if } x > 3 \end{cases}$.

a. Find $\lim_{x \to 0} g(x)$. **b.** Find $\lim_{x \to 3} g(x)$.

In Exercises 57–60, calculate the limit in the specified exercise, using a table such as in Exercises 15–20. Verify your answer by using a graphing calculator to zoom in on the point on the graph.

57. Exercise 31

58. Exercise 32

59. Exercise 33

60. Exercise 34

61. Let $F(x) = \dfrac{3x}{(x + 2)^3}$.

a. Find $\lim_{x \to -2} F(x)$.

b. Find the vertical asymptote of the graph of $F(x)$.

c. Compare your answers for parts a and b. What can you conclude?

62. Let $G(x) = \dfrac{-6}{(x - 4)^2}$.

a. Find $\lim_{x \to 4} G(x)$.

b. Find the vertical asymptote of the graph of $G(x)$.

c. Compare your answers for parts a and b. Are they related? How?

63. How can you tell that the graph in Figure 10 is more representative of the function $f(x) = 1/(x - 1)$ than the graph in Figure 11?

64. A friend who is confused about limits wonders why you investigate the value of a function closer and closer to a point, instead of just finding the value of a function at the point. How would you respond?

EXAMPLE 4 **Cost Analysis**

A trailer rental firm charges a flat $8 to rent a hitch. The trailer itself is rented for $2
day or fraction of a day. Let $C(x)$ represent the cost of renting a hitch and trailer for x

(a) Graph C.

SOLUTION The charge for one day is $8 for the hitch and $22 for the trailer, or
In fact, if $0 < x \leq 1$, then $C(x) = 30$. To rent the trailer for more than one day
not more than two days, the charge is $8 + 2 \cdot 22 = 52$ dollars. For any value of
isfying $1 < x \leq 2$, the cost is $C(x) = 52$. Also, if $2 < x \leq 3$, then $C(x)$
These results lead to the graph in Figure 22.

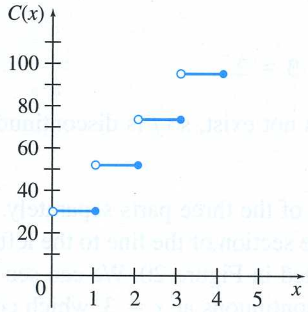


FIGURE 22

(b) Find any values of x where C is discontinuous.

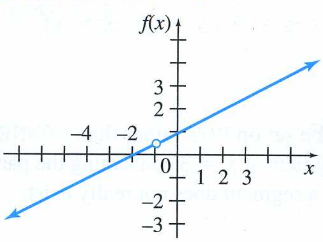
SOLUTION As the graph suggests, C is discontinuous at $x = 1, 2, 3, 4,$ and all
positive integers.

One application of continuity is the **Intermediate Value Theorem**, which says tha
function is continuous on a closed interval $[a, b]$, the function takes on every value betw
$f(a)$ and $f(b)$. For example, if $f(1) = -3$ and $f(2) = 5$, then f must take on every v
between -3 and 5 as x varies over the interval $[1, 2]$. In particular (in this case), there mu
a value of x in the interval $(1, 2)$ such that $f(x) = 0$. If f were discontinuous, however
conclusion would not necessarily be true. This is important because, if we are searchin
a solution to $f(x) = 0$ in $[1, 2]$, we would like to know that a solution exists.

11.2 EXERCISES

In Exercises 1–6, find all values $x = a$ where the function is discontinuous. For each point of discontinuity, give (a) $f(a)$ if it exists, (b) $\lim_{x \to a^-} f(x)$, (c) $\lim_{x \to a^+} f(x)$, (d) $\lim_{x \to a} f(x)$, and (e) identify which conditions for continuity are not met. Be sure to note when the limit doesn't exist.

1.



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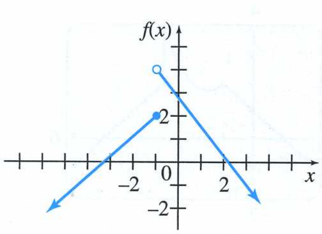

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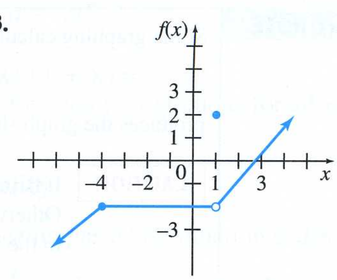

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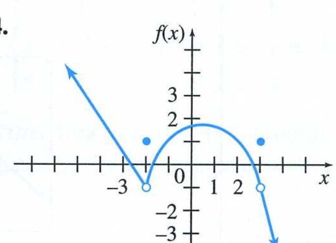

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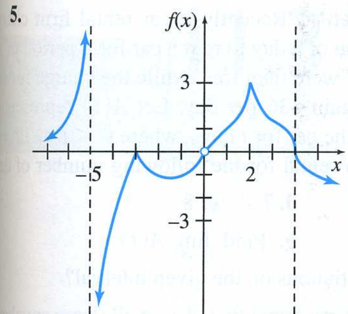
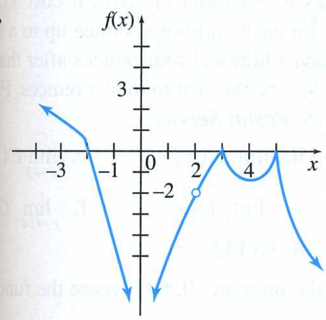


6.



Find all values $x = a$ where the function is discontinuous. For each value of x, give the limit of the function as x approaches a. Be sure to note when the limit doesn't exist.

7. $f(x) = \dfrac{5 + x}{x(x - 2)}$

8. $f(x) = \dfrac{-2x}{(2x + 1)(3x + 6)}$

9. $f(x) = \dfrac{x^2 - 4}{x - 2}$

10. $f(x) = \dfrac{x^2 - 25}{x + 5}$

11. $p(x) = x^2 - 4x + 11$

12. $q(x) = -3x^3 + 2x^2 - 4x + 1$

13. $p(x) = \dfrac{|x + 2|}{x + 2}$

14. $r(x) = \dfrac{|5 - x|}{x - 5}$

15. $k(x) = e^{\sqrt{x-1}}$

16. $j(x) = e^{1/x}$

17. $r(x) = \ln\left|\dfrac{x}{x - 1}\right|$

18. $j(x) = \ln\left|\dfrac{x + 2}{x - 3}\right|$

In Exercises 19–24, (a) graph the given function, (b) find all values of x where the function is discontinuous, and (c) find the limit from the left and from the right at any values of x found in part b.

19. $f(x) = \begin{cases} 1 & \text{if } x < 2 \\ x + 3 & \text{if } 2 \le x \le 4 \\ 7 & \text{if } x > 4 \end{cases}$

20. $f(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ 0 & \text{if } 1 \le x \le 4 \\ x - 2 & \text{if } x > 4 \end{cases}$

21. $g(x) = \begin{cases} 11 & \text{if } x < -1 \\ x^2 + 2 & \text{if } -1 \le x \le 3 \\ 11 & \text{if } x > 3 \end{cases}$

22. $g(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 - 5x & \text{if } 0 \le x \le 5 \\ 5 & \text{if } x > 5 \end{cases}$

23. $h(x) = \begin{cases} 4x + 4 & \text{if } x \le 0 \\ x^2 - 4x + 4 & \text{if } x > 0 \end{cases}$

24. $h(x) = \begin{cases} x^2 + x - 12 & \text{if } x \le 1 \\ 3 - x & \text{if } x > 1 \end{cases}$

In Exercises 25–28, find the value of the constant k that makes the function continuous.

25. $f(x) = \begin{cases} kx^2 & \text{if } x \le 2 \\ x + k & \text{if } x > 2 \end{cases}$

26. $g(x) = \begin{cases} x^3 + k & \text{if } x \le 3 \\ kx - 5 & \text{if } x > 3 \end{cases}$

27. $g(x) = \begin{cases} \dfrac{2x^2 - x - 15}{x - 3} & \text{if } x \ne 3 \\ kx - 1 & \text{if } x = 3 \end{cases}$

28. $h(x) = \begin{cases} \dfrac{3x^2 + 2x - 8}{x + 2} & \text{if } x \ne -2 \\ 3x + k & \text{if } x = -2 \end{cases}$

29. Explain in your own words what the Intermediate Value Theorem says and why it seems plausible.

30. Explain why $\lim_{x \to 2}(3x^2 + 8x)$ can be evaluated by substituting $x = 2$.

In Exercises 31–32, (a) use a graphing calculator to tell where the rational function $P(x)/Q(x)$ is discontinuous, and (b) verify your answer from part (a) by using the graphing calculator to plot $Q(x)$ and determine where $Q(x) = 0$. You will need to choose the viewing window carefully.

31. $f(x) = \dfrac{x^2 + x + 2}{x^3 - 0.9x^2 + 4.14x - 5.4}$

32. $f(x) = \dfrac{x^2 + 3x - 2}{x^3 - 0.9x^2 + 4.14x + 5.4}$

33. Let $g(x) = \dfrac{x + 4}{x^2 + 2x - 8}$. Determine all values of x at which g is discontinuous, and for each of these values of x, define g in such a manner so as to remove the discontinuity, if possible. Choose one of the following. *Source: Society of Actuaries.*

a. g is discontinuous only at -4 and 2.
Define $g(-4) = -\frac{1}{6}$ to make g continuous at -4.
$g(2)$ cannot be defined to make g continuous at 2.

b. g is discontinuous only at -4 and 2.
Define $g(-4) = -\frac{1}{6}$ to make g continuous at -4.
Define $g(2) = 6$ to make g continuous at 2.

c. g is discontinuous only at -4 and 2.
$g(-4)$ cannot be defined to make g continuous at -4.
$g(2)$ cannot be defined to make g continuous at 2.

d. g is discontinuous only at 2.
Define $g(2) = 6$ to make g continuous at 2.

e. g is discontinuous only at 2.
$g(2)$ cannot be defined to make g continuous at 2.

11.3 EXERCISES

Find the average rate of change for each function over the given interval.

1. $y = x^2 + 2x$ between $x = 1$ and $x = 3$
2. $y = -4x^2 - 6$ between $x = 2$ and $x = 6$
3. $y = -3x^3 + 2x^2 - 4x + 1$ between $x = -2$ and $x = 1$
4. $y = 2x^3 - 4x^2 + 6x$ between $x = -1$ and $x = 4$
5. $y = \sqrt{x}$ between $x = 1$ and $x = 4$
6. $y = \sqrt{3x - 2}$ between $x = 1$ and $x = 2$
7. $y = e^x$ between $x = -2$ and $x = 0$
8. $y = \ln x$ between $x = 2$ and $x = 4$

Suppose the position of an object moving in a straight line is given by $s(t) = t^2 + 5t + 2$. Find the instantaneous velocity at each time.

9. $t = 6$
10. $t = 1$

Suppose the position of an object moving in a straight line is given by $s(t) = 5t^2 - 2t - 7$. Find the instantaneous velocity at each time.

11. $t = 2$
12. $t = 3$

Suppose the position of an object moving in a straight line is given by $s(t) = t^3 + 2t + 9$. Find the instantaneous velocity at each time.

13. $t = 1$
14. $t = 4$

Find the instantaneous rate of change for each function at the given value.

15. $f(x) = x^2 + 2x$ at $x = 0$
16. $s(t) = -4t^2 - 6$ at $t = 2$
17. $g(t) = 1 - t^2$ at $t = -1$
18. $F(x) = x^2 + 2$ at $x = 0$

Use the formula for instantaneous rate of change, approximating the limit by using smaller and smaller values of h, to find the instantaneous rate of change for each function at the given value.

19. $f(x) = x^x$ at $x = 2$
20. $f(x) = x^x$ at $x = 3$
21. $f(x) = x^{\ln x}$ at $x = 2$
22. $f(x) = x^{\ln x}$ at $x = 3$
23. Explain the difference between the average rate of change of y as x changes from a to b, and the instantaneous rate of change of y at $x = a$.
24. If the instantaneous rate of change of $f(x)$ with respect to x is positive when $x = 1$, is f increasing or decreasing there?

APPLICATIONS

Business and Economics

25. **Profit** Suppose that the total profit in hundreds of dollars from selling x items is given by

$$P(x) = 2x^2 - 5x + 6.$$

Find the average rate of change of profit for the following chang in x.

a. 2 to 4 **b.** 2 to 3

c. Find and interpret the instantaneous rate of change of pro with respect to the number of items produced when $x =$ (This number is called the *marginal profit* at $x = 2$.)

d. Find the marginal profit at $x = 4$.

26. **Revenue** The revenue (in thousands of dollars) from produ ing x units of an item is

$$R(x) = 10x - 0.002x^2.$$

a. Find the average rate of change of revenue when producti is increased from 1000 to 1001 units.

b. Find and interpret the instantaneous rate of change of re enue with respect to the number of items produced wh 1000 units are produced. (This number is called the *ma ginal revenue* at $x = 1000$.)

c. Find the additional revenue if production is increased fr 1000 to 1001 units.

d. Compare your answers for parts a and c. What do you fi How do these answers compare with your answer to part

27. **Demand** Suppose customers in a hardware store are willing buy $N(p)$ boxes of nails at p dollars per box, as given by

$$N(p) = 80 - 5p^2, \quad 1 \le p \le 4.$$

a. Find the average rate of change of demand for a change price from \$2 to \$3.

b. Find and interpret the instantaneous rate of change demand when the price is \$2.

c. Find the instantaneous rate of change of demand when price is \$3.

d. As the price is increased from \$2 to \$3, how is dem changing? Is the change to be expected? Explain.

28. **Interest** If \$1000 is invested in an account that pays 5% co pounded annually, the total amount, $A(t)$, in the account a t years is

$$A(t) = 1000(1.05)^t.$$

a. Find the average rate of change per year of the total am in the account for the first five years of the investment (fr $t = 0$ to $t = 5$).

b. Find the average rate of change per year of the total am in the account for the second five years of the investm (from $t = 5$ to $t = 10$).

c. Estimate the instantaneous rate of change for $t = 5$.

29. **Interest** If \$1000 is invested in an account that pays 5% co pounded continuously, the total amount, $A(t)$, in the acco after t years is

$$A(t) = 1000e^{0.05t}.$$