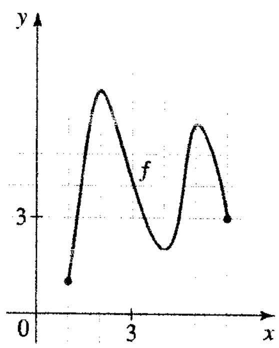
1.7 – 1.9

1.7 Exercises

CONCEPTS



Fundamentals

1–4 ▪ These exercises refer to the graph of the function f shown at the left.

1. To find a function value $f(a)$ from the graph of f, we find the height of the graph above the x-axis at $x =$ ________. From the graph of f we see that $f(3) =$ ________.

2. The domain of the function f is all the ________-values of the points on its graph, and the range is all the corresponding ________-values. From the graph we see that the domain of f is the interval ________ and the range of f is the interval ________.

3. (a) If f is increasing on an interval, then the y-values of the points on the graph ________ (increase/decrease) as the x-values increase. From the graph we see that f is increasing on the intervals ________ and ________.

(b) If f is decreasing on an interval, then the y-values of the points on the graph ________ (increase/decrease) as the x-values increase. From the graph we see that f is decreasing on the intervals ________ and ________.

4. (a) A function value $f(a)$ is a local maximum value of f if $f(a)$ is the ________ value of f on some interval containing a. From the graph we see that one local maximum value of f is ________ and that this value occurs when x is ________.

(b) A function value $f(a)$ is a local minimum value of f if $f(a)$ is the ________ value of f on some interval containing a. From the graph we see that one local minimum value of f is ________ and that this value occurs when x is ________.

Think About It

5. In Example 7 we saw a real-world situation in which it is important to find the maximum value of a function. Name several other everyday situations in which a maximum or minimum is important.

6. Draw a graph of a function f that is defined for all real numbers and that satisfies the following conditions: f is always decreasing and $f(x) > 0$ for all x.

SKILLS

7. The graph of a function h is given.

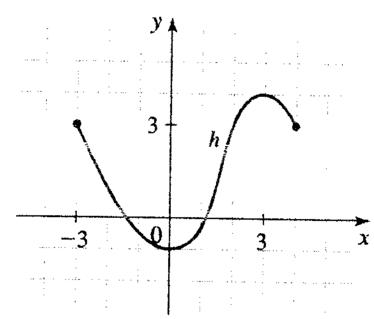
(a) Find $h(-2)$, $h(0)$, $h(2)$, and $h(3)$.

(b) Find the domain and range of h.

(c) Find the values of x for which $h(x) = 3$.

(d) Find the values of x for which $h(x) \le 3$.

(e) Find the net change in the value of h when x changes from -2 to 4.



8. The graph of a function g is given.

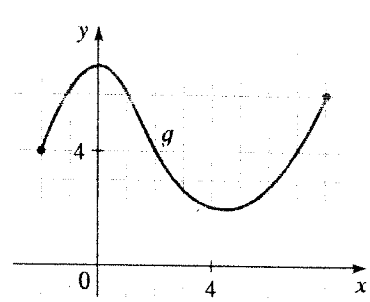
(a) Find $g(-2)$, $g(0)$, and $g(7)$.

(b) Find the domain and range of g.

(c) Find the values of x for which $g(x) = 4$.

(d) Find the values of x for which $g(x) > 4$.

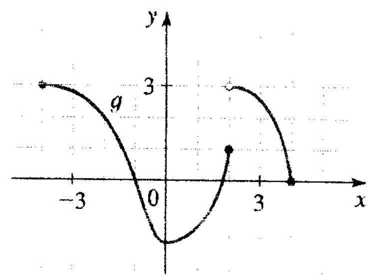
(e) Find the net change in the value of g when x changes from 2 to 7.



9. The graph of a function g is given.

(a) Find $g(-4)$, $g(-2)$, $g(0)$, $g(2)$, and $g(4)$.

(b) Find the domain and range of g.

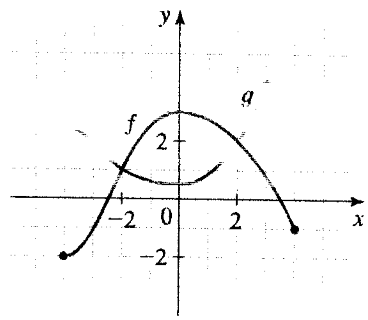


10. Graphs of the functions f and g are given.

(a) Which is larger, $f(0)$ or $g(0)$?

(b) Which is larger, $f(-3)$ or $g(-3)$?

(c) For which values of x is $f(x) = g(x)$?



11–14 ■ Graph the functions f and g with a graphing calculator. Use the graphs to find the indicated values or intervals; state your answer correct to two decimal places.

(a) Find the value(s) of x for which $f(x) = g(x)$.

(b) Find the values of x for which $f(x) \geq g(x)$.

(c) Find the values of x for which $f(x) < g(x)$.

11. $f(x) = x^2 - 5x + 1, \quad g(x) = -3x + 4$

12. $f(x) = -2x^2 + 3x - 1, \quad g(x) = 3x - 9$

13. $f(x) = 2x^2 + 3, \quad g(x) = -x^2 + 3x + 5$

14. $f(x) = 1 - x^2, \quad g(x) = x^2 - 2x - 1$

15–22 ■ A function f is given.

(a) Use a graphing calculator to draw the graph of f.

(b) Find the domain and range of f from the graph.

15. $f(x) = x - 1$

16. $f(x) = 4$

17. $f(x) = -x^2$

18. $f(x) = 4 - x^2$

19. $f(x) = \sqrt{16 - x^2}$

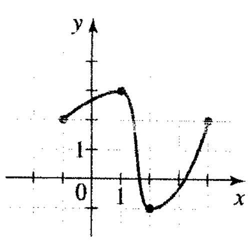
20. $f(x) = -\sqrt{25 - x^2}$

21. $f(x) = \sqrt{x - 1}$

22. $f(x) = \sqrt{x + 2}$

23–26 ■ The graph of a function is given. Determine the intervals on which the function is **(a)** increasing and **(b)** decreasing.

23.



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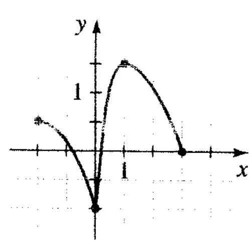

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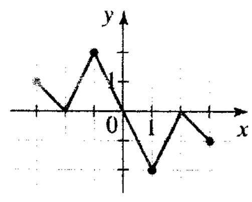

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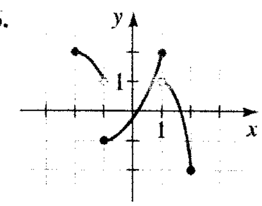


27–32 ■ A function f is given.

(a) Use a graphing device to draw the graph of f.

(b) State approximately the intervals on which f is increasing and on which f is decreasing.

27. $f(x) = x^2 - 5x$

28. $f(x) = x^3 - 4x$

29. $f(x) = 2x^3 - 3x^2 - 12x$

30. $f(x) = x^4 - 16x^2$

31. $f(x) = x^3 + 2x^2 - x - 2$

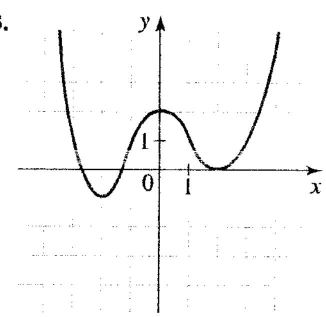
32. $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$

33–36 ■ The graph of a function is given.

(a) Find all the local maximum and minimum values of the function and the value of x at which each occurs.

(b) Find the intervals on which the function is increasing and on which the function is decreasing.

33.



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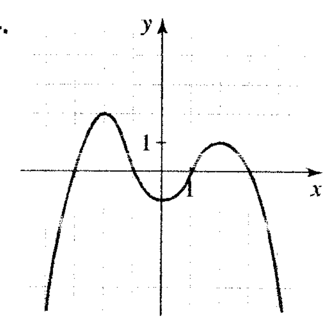

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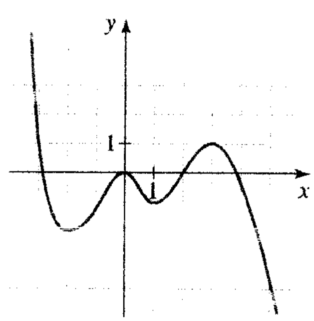

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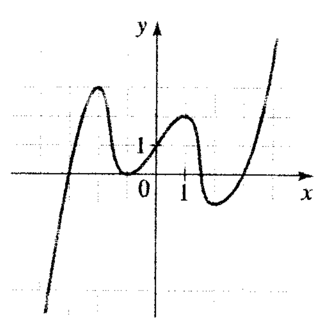


37–40 ■ A function is given. Use a graphing calculator to draw a graph of the function.

(a) Find all the local maximum and minimum values of the function and the value of x at which each occurs. State each answer correct to two decimals.

(b) Find the intervals on which the function is increasing and on which the function is decreasing. State each answer correct to two decimal places.

37. $f(x) = x^3 - x$

38. $g(x) = 3 + x + x^2 - x^3$

39. $F(x) = x\sqrt{6 - x}$

40. $G(x) = x\sqrt{x - x^2}$

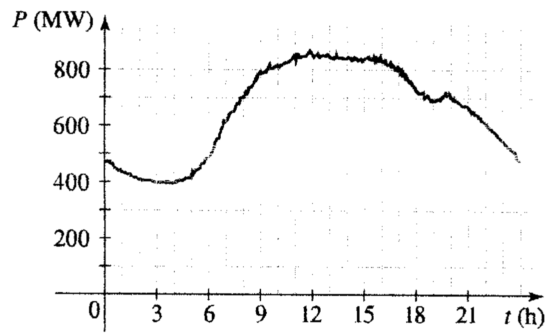
41. Power Consumption The figure shows the power consumption in San Francisco for September 19, 1996. (P is measured in megawatts; t is measured in hours starting at midnight.)

(a) What was the power consumption at 6:00 A.M.? At 6:00 P.M.?

(b) When was the power consumption a maximum?

(c) When was the power consumption a minimum?

(d) What is the net change in the values of P as the value of x changes from 0 to 12?



Source: Pacific Gas & Electric.

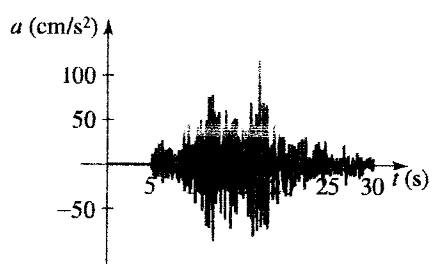
42. Earthquake The graph shows the vertical acceleration of the ground from the 1994 Northridge earthquake in Los Angeles, as measured by a seismograph. (Here t represents the time in seconds.)

(a) At what time t did the earthquake first make noticeable movements of the earth?

(b) At what time t did the earthquake seem to end?

(c) At what time t was the maximum intensity of the earthquake reached?

(d) What is the approximate net change in the intensity of the earthquake as the value of t changes from 5 to 30?



Source: California Department of Mines and Geology.

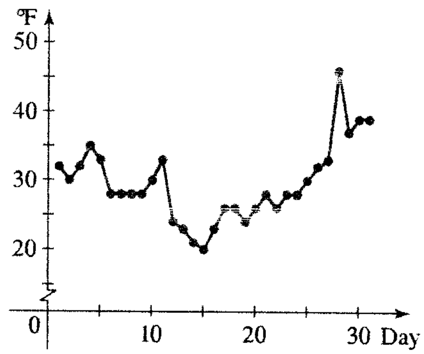
43. Low Temperatures In January 2007 the state of California experienced remarkably cold weather. Many crops that usually thrive in California were lost because of the frost. Orange crops just ripening in Tulare County, California, were frozen on the trees. The table and graph on the next page show the daily low temperatures T in Tulare County for the month of January 2007.

(a) Find $T(1)$ and $T(15)$.

(b) Which is larger, $T(15)$ or $T(19)$?

(c) On what day(s) was the daily low temperature below 32°F? On what day was it the lowest?

(d) Find the net change in the daily low temperatures from January 1 to January 31.



Day	Daily low temperature (°F)	Day	Daily low temperature (°F)	Day	Daily low temperature (°F)
1	32	12	24	23	28
2	30	13	23	24	28
3	32	14	21	25	30
4	35	15	20	26	32
5	33	16	23	27	33
6	28	17	26	28	46
7	28	18	26	29	37
8	28	19	24	30	39
9	28	20	26	31	39
10	30	21	28		
11	33	22	26		

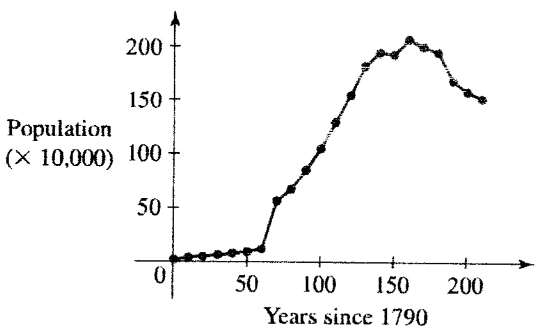
44. Population of Philadelphia The table and graph below show the history of the population P of the city of Philadelphia from 1790 to 2000.

(a) Find $P(100)$ and $P(200)$.

(b) Which is larger, $P(160)$ or $P(210)$?

(c) In what year did Philadelphia have its largest population? In what year(s) did Philadelphia have a population over 1.8 million?

(d) Find the net change in the population of Philadelphia from 1950 to 2000.

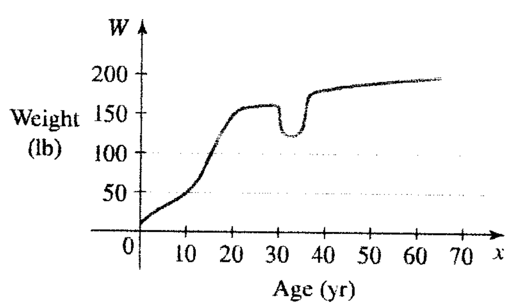


Years since 1790	Population (× 10,000)	Years since 1790	Population (× 10,000)
0	2.85	110	129.37
10	4.12	120	154.90
20	5.37	130	182.34
30	6.38	140	195.10
40	8.05	150	193.13
50	9.37	160	207.16
60	12.14	170	200.25
70	56.55	180	194.86
80	67.40	190	168.82
90	84.72	200	158.56
100	104.70	210	151.76

45. Weight Function The graph gives the weight W of a person at age x.

(a) Determine the intervals on which the function W is increasing and on which it is decreasing.

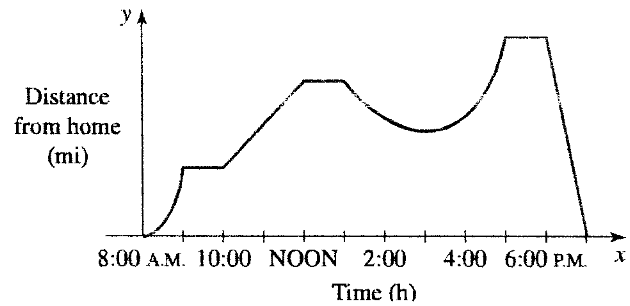
(b) What do you think happened when this person was 30 years old?

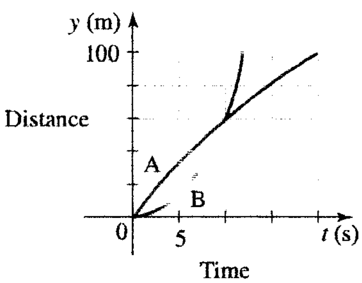


46. Distance Function The graph gives a sales representative's distance from his home as a function of time on a certain day.

(a) Determine the time intervals on which his distance from home was increasing and on which it was decreasing.

(b) Describe in words what the graph indicates about his travels on this day.





47. Running Race Two runners compete in a 100-meter race. The graph in the margin depicts the distance run as a function of time for each runner.

(a) Did each runner finish the race? Who won the race?

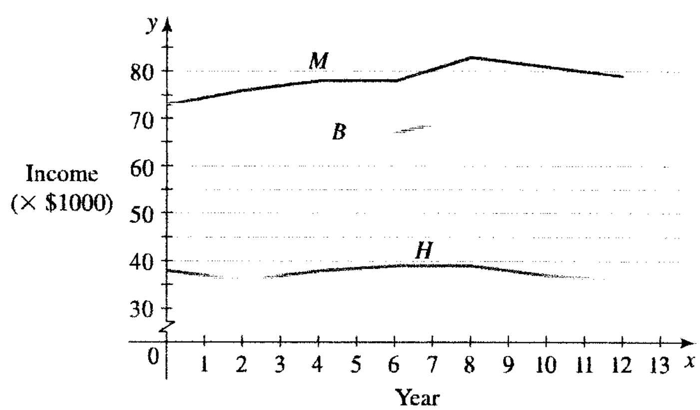
(b) At what time did one runner overtake the other?

(c) On what time interval was Runner A leading?

48. Education and Income The graph shows the yearly median income H for Americans with a high school diploma, the yearly median income B of Americans with a bachelor's degree, and the yearly median income M of Americans with a master's degree, all in the time period 1991 to 2003 (with $x = 0$ corresponding to 1991).

(a) Find the interval on which all three functions are decreasing.

(b) Find the net change of each function from 1995 to 1999. Which function had the most net change in this time period?



49. Migrating Fish Suppose a fish swims at a speed v relative to the water, against a current of 5 mi/h. Using a mathematical model of energy expenditure, it can be shown that the total energy E required to swim a distance of 10 mi is given by

$$E(v) = 2.73v^3 \frac{10}{v - 5} \qquad 5.1 \le v \le 10$$

Biologists believe that migrating fish try to minimize the total energy required to swim a fixed distance.

(a) Graph the function E in the viewing rectangle [5.1, 10] by [4000, 13,000].

(b) Where is the function E increasing and where is the function E decreasing?

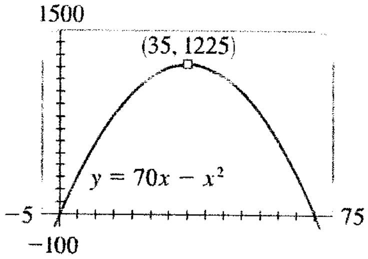


figure **7**

(d) We need to find where the maximum value of the function $A(x) = 70x - x^2$ occurs. The function is graphed in Figure 7. Using the [TRACE] feature on a graphing calculator, we find that the function achieves its maximum value at $x = 35$. So the maximum area that she can fence occurs when the garden's width is 35 ft and its length is $70 - 35 = 35$ ft. Then the maximum area is $35 \times 35 = 1225$ ft^2.

■ **NOW TRY EXERCISE 29** ■

1.8 Exercises

1–6 ■ Find a function that models the quantity described.

1. The number N of days in w weeks.

2. The number N of cents in q quarters.

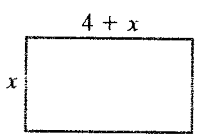
3. The sum S of two consecutive integers, the first integer being n.

4. The sum S of a number n and its square.

5. The product P of a number x and twice that number.

6. The product P of a number y and one and a half times that number.

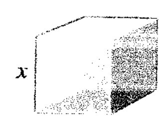
7–12 ■ Find a function that models the quantity described. You may need to consult the formulas for area and volume listed on the inside back cover of this book.



7. The area A of a rectangle whose length is 4 ft more than its width x.

8. The perimeter P of a rectangle whose length is 4 ft more than its width x.

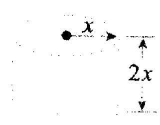
9. The volume V of a cube of side x.



10. The volume B of a box with a square base of side x and height $2x$.

11. The area A of a triangle whose base is twice its height h.

12. The volume V of a cylindrical can whose height is twice its radius, as shown in the figure.



13–16 ■ In these problems you find a function that models a real-life situation and then use the graphing calculator to graph the model and answer questions about the situation. Exercise 13 shows the steps involved in solving these problems.

First number	Second number	Product
1	18	18
2	17	34
3	16	48
⋮	⋮	⋮

13. Consider the following problem: Find two numbers whose sum is 19 and whose product is as large as possible.

(a) Experiment with the problem by making a table like the one in the margin, showing the product of different pairs of numbers that add up to 19. On the basis of the evidence in your table, estimate the answer to the problem.

(b) Find a function f that models the product $f(x)$ in terms of one of the numbers x.

(c) Use a graphing calculator to graph the model and solve the problem. Compare with your answer to part (a).

14. Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

15. Find two numbers whose sum is -24 and whose product is a maximum.

16. Among all rectangles that have a perimeter of 20 ft, find the dimensions of the one with the largest area.

17. Tee Shirt Cost A tee shirt company makes tee shirts with school logos. The company charges a fixed fee of $200 to set up the machines plus $3.50 per tee shirt.

(a) Find a function C that models the cost of purchasing x tee shirts.

(b) Use the model to find the cost of purchasing 600 tee shirts.

18. Rental Cost A flea market charges vendors a fixed fee of $60 a month plus 75 cents per square foot for renting a space.

(a) Find a function C that models the cost for one month's rental of a space with area x square feet.

(b) Use the model to find the cost of one month's rental of a space with area 150 square feet.

19. Gas Cost The cost of driving a car depends on the number of miles driven and the gas mileage of the car. Kristi owns a Honda Accord that gets 30 miles to the gallon.

(a) Find a function C that models the cost of driving Kristi's car x miles if the cost of gas is $3.20 per gallon.

(b) Use the model to find the cost of driving Kristi's car 500 miles.

(c) Kristi's budget for gas is $250 a month. Use the model to find the number of miles Kristi can drive each month without exceeding her monthly gas budget.

20. Exchange Rate Jason travels from his home in Connecticut to Germany to visit his grandparents. At the time the euro/dollar exchange rate was 1.5532, which means that each euro cost 1.5532 U.S. dollars.

(a) Find a function A that models the number of U.S. dollars required to purchase x euros.

(b) Jason bought a vase in Hamburg for his grandmother for 153.00 euros. Use the model to find the price of the vase in U.S. dollars.

(c) The day before returning home, Jason found that he had 200 U.S. dollars worth of traveler's checks left. He decided to convert these to euros to spend in Germany. Use the model to find how many euros he received for his $200.

21. Cost of Wedding Sherri and Jonathan are getting married. They have a budget of $5000. They are planning the reception and choose a reception hall that costs $700, a DJ that costs $300, a caterer that charges $18.50 a plate, and a wedding cake that costs $1.50 per guest.

(a) Complete the table for the cost of the reception for the given number of guests.

Number of guests	Cost of hall	Cost of DJ	Cost of caterer	Cost of wedding cake	Total cost of reception
10	$700	$300	$185	$15	$1200
20					
30					
40					
50					

(b) Find a function C that models the cost of the reception when x guests attend.

(c) Determine how much the reception would cost if 75 people attend; that is, find the value of $C(75)$.

(d) Determine how many people can attend the reception if Sherri and Jonathan spend their total budget of $5000; that is, find the value of x when $C(x) = 5000$.

22. Cost of Reception A business group is hosting a reception for local dignitaries. The group chooses to hold the event at an exclusive country club that charges a $2000 rental fee. In addition, they choose a caterer that charges $21.00 a plate, gifts that cost $5.00 per guest, and decorations that cost $1500.

(a) Find a function C that models the cost of hosting the reception when x guests attend.

(b) Determine how much the reception would cost if 200 people attend; that is, find the value of $C(200)$.

(c) Determine how many people can attend the reception if the business group's budget for the reception is $10,000.

23. Discounts An art supply store has a sale on picture frames, advertising "Buy One, Get the 2nd for One Penny."

(a) Complete the table for the total cost of purchasing the indicated number of picture frames.

Number of frames	Number that are $10 each	Number that are 1 cent each	Total cost
2	1	1	$10.01
4			
6			
8			
10			

(b) Find a function C that models the cost of purchasing x frames that normally cost $10 each. (Assume that x is an even number.)

(c) Aaron needs 20 picture frames for all his childhood pictures. Use the model to find Aaron's cost of getting these frames, if all the frames he gets normally cost $10 each.

24. Discounts A competitor of the art supply store in Exercise 23 offers a 35% discount on all frames.

(a) Find a function C that models the cost of purchasing x frames that normally cost $10 each.

(b) Use the model to find Aaron's cost of getting 20 frames with the competitor's sale, if all the frames normally cost $10. Is this a better deal than the one in Exercise 23?

25. Volume of Cereal Box A breakfast cereal manufacturer packages cereal in boxes that are 4 inches taller than they are wide and always have a depth of 3 inches.

(a) Complete the table for the dimensions and volume of a cereal box.

Width (in.)	Height (in.)	Depth (in.)	Volume (in^3)
3	7	3	63
4			
5			
6			
7			

(b) Find a function V that models the volume of a cereal box that is x inches wide.

(c) Use the model to find the volume of a cereal box that is 10 in. wide.

(d) The manufacturer makes a box of wheat bran cereal with a volume of 300 in^3. Use a graphing calculator to find the width of this box, as in Example 4.

26. Profit of Fund-Raiser A land conservancy in California organizes several fund-raisers every year. One year, the board of directors for the conservancy suggests raising money by offering tours of their nature preserve for a price of $50 per person. They believe they can attract more people if they offer group discounts of $1 per person. So if two people go on the tour, they will charge $49 per person (for a total of $98); if three people go on the tour, they will charge $48 per person (for a total of $144), and so on.

(a) Complete the table for the revenue from a tour with the given number of people.

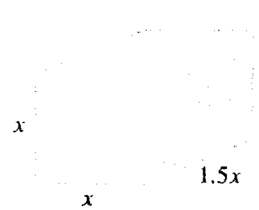
Number of people in tour	Price per person	Revenue
1	$50	$50
2	$49	$98
3	$48	$144
4		
5		
6		

(b) Find a function R that models the revenue when x people take the tour.

(c) Find the revenue if 10 people go on a tour; that is, find the value of $R(10)$.

(d) Use a graphing calculator to find the number of people that must go on the tour in order for the conservancy to raise $650.

27. Volume of a Container A Florida orange grower ships orange juice in rectangular plastic containers that have square ends and are one and a half times as long as they are wide. (See the figure.)

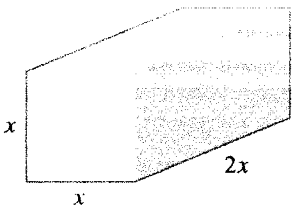


(a) Find a function V that models the volume of a container of width x.

(b) Use the model to find the volume of a container of width 10 in.

(c) Graph the function V. Use the graph to find the width of the plastic container that has a volume of 315 in^3.

(d) Use the graph from part (c) to find the widths for which the container has volume greater than 450 in^3. Express your answer in interval notation.



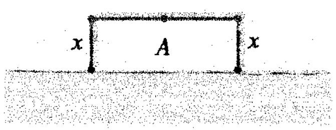
28. Volume of Box A shipping company uses boxes that have square ends and are twice as long as they are wide. (See the figure.)

(a) Find a function S that models the surface area of a box whose square end is x in. wide.

(b) The company uses boxes that are 8 in. wide to ship cans of beans. Use the model to find the area of the material used to make each box.

(c) A box that ships a dozen cans of soup has a surface area of 330 in^2. Graph the function S to find the width of that box.

(d) To ensure that the boxes are strong enough to safely hold their contents, they should have a surface area no larger than 550 in^2. Use the graph from part (c) to find all possible widths for the shipping box. Express your answer in interval notation.



29. Fencing a Field Consider the following problem: A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river (see the figure). What are the dimensions of the field of largest area that he can fence?

(a) Experiment with the problem by drawing several diagrams illustrating the situation. Calculate the area of each configuration, and use your results to estimate the dimensions of the largest possible field.

(b) Find a function A that models the area of a field in terms of one of its sides x.

(c) Use a graphing calculator to find the dimensions of the field of largest area. Compare with your answer to part (a).

30. Dividing a Pen A rancher with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle (see the figure).

(a) Show that the total area of the four pens is modeled by the function

$$A(x) = \frac{x(750 - 5x)}{2}$$

(b) Use a graphing calculator to find the largest possible total area of the four pens.

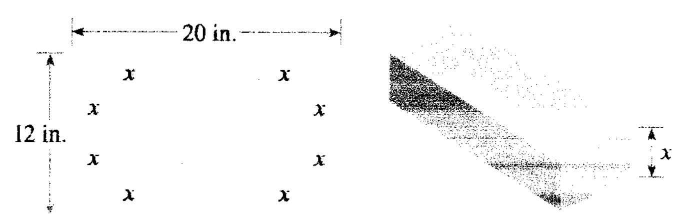
31. Volume of a Box A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides (see the figure).

(a) Show that the volume of the box is modeled by the function

$$V(x) = x(12 - 2x)(20 - 2x)$$

(b) Use a graphing calculator to find the values of x for which the volume is greater than 200 in^3.

(c) Use a graphing calculator to find the largest volume that such a box can have.



32. Area of a Box A box with an open top and a square base is to have a volume of 12 ft^3.

(a) Find a function S that models the surface area of the box.

(b) Use a graphing calculator to find the box dimensions that minimize the amount of material used.

Solution

(a) We equate the two different expressions for F and solve for M:

$$ma = G\frac{mM}{d^2} \qquad \text{Because } F = ma \text{ and } F = G\frac{mM}{d^2}$$

$$a = G\frac{M}{d^2}$$

$$\frac{ad^2}{G} = M \qquad \text{Multiply by } d^2 \text{ and divide by } G$$

So we can write a formula for the mass of the earth:

$$M = \frac{ad^2}{G}$$

This formula allows us to find M if we know a, d, and G.

(b) We use the formula we found in part (a):

$$M = \frac{ad^2}{G}$$

$$= \frac{(9.8)(6.38 \times 10^6)^2}{6.67 \times 10^{-11}} \qquad \text{Substitute the value of each quantity}$$

$$\approx 5.98 \times 10^{24} \qquad \text{Calculator}$$

Scientific notation is studied in Exploration 1, page 312.

So the mass of the earth is just a bit less than 6×10^{24} kg. Written out in full, this is about

6,000,000,000,000,000,000,000,000 kg

or 6 septillion kilograms.

■ **NOW TRY EXERCISE 33** ■

1.9 Exercises

CONCEPTS

Fundamentals

1. The model $L = nS$ gives the total number of legs that S animals have, where each animal has n legs. Using this model, we find that 12 spiders have $L =$ ____ $\times$ ____ $=$ ____ legs, whereas 20 chickens have $L =$ ____ $\times$ ____ $=$ ____ legs.

2. The formula $d = rt$ models the distance d in miles that you travel in t hours at a speed of r miles per hour. So the formula that models the time t it takes to go a distance d at a speed r is given by $t = \frac{\square}{\square}$. We use this formula to find how long it takes to go 350 miles at a speed of 55 miles per hour: $t = \frac{\square}{\square} =$ ________.

3–8 ■ Solve the equation to find a formula for the indicated variable.

3. $PV = nRT$; for R

4. $F = G\dfrac{m_1 m_2}{d^2}$; for d

5. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$; for R_1

6. $A = P\left(1 + \dfrac{i}{100}\right)^2$; for i

7. $S = 2lw + 2wh + 2lh$; for w

8. $V = \frac{1}{3}\pi r^2 h$; for r

9–16 ■ Find a formula that models the quantity described.

9. The average A of two numbers a_1 and a_2.

10. The average A of three numbers a_1, a_2, and a_3.

11. The sum S of the squares of n and m.

12. The sum S of the square roots of n and m.

13. The product P of an integer n and two times an integer m.

14. The product P of the squares of n and m.

15. The time t it takes an airplane to travel d miles if its speed is r miles per hour.

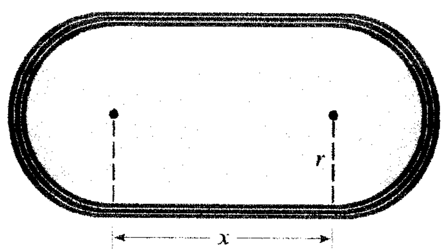
16. The speed r of a boat that travels d miles in t hours.

17–20 ■ Find a formula that models the quantity described. You may need to consult the formulas for area and volume listed on the inside back cover of this book.

17. The surface area A of a box with an open top of dimensions l, w, and h.

18. The surface area A of a cylindrical can with height h and radius r.

19. The length L of the race track shown in the figure.



20. The area A enclosed by the race track in the preceding figure.

21. CO_2 Emissions Many scientists believe that the increase of carbon dioxide (CO_2) in the atmosphere is a major contributor to global warming. The Environmental Protection Agency estimates that one gallon of gasoline produces on average about 19 pounds of CO_2 when it is combusted in a car engine.

(a) Find a formula for the amount A of CO_2 a car produces in terms of the number n of miles driven and the gas mileage G of the car.

(b) Debbie owns an SUV that has a gas mileage of 21 mi/gal. Debbie drives 15,000 miles in one year. Use the formula you found in part (a) to find how much CO_2 Debbie's car produces in one year.

(c) Debbie's friend Lisa owns a hybrid car that has a gas mileage of 55 mi/gal. Lisa also drives 15,000 miles in one year. Use the formula you found in part (a) to find how much CO_2 Lisa's car produces in one year.

22. **Prehistoric Vegetation** Gasoline is refined from crude oil, which was formed from prehistoric organic matter buried under layers of sediment. High pressures and temperatures transformed this material into the hydrocarbons that we call crude oil. Scientists estimate that it takes about 98 tons of prehistoric vegetation to produce one gallon of gasoline. (Today, it takes about 40 acres of farmland to produce 98 tons of vegetation in one season.)
   - **(a)** Find a formula for the amount V of prehistoric vegetation it took to produce the gasoline needed to drive a car in terms of the number n of miles driven and the gas mileage G of the car.
   - **(b)** Sonia owns an SUV that has a gas mileage of 18 mi/gal. Sonia drives 10,000 miles a year. Use the formula you found in part (a) to find the amount of prehistoric vegetation it took to produce the gas that Sonia's SUV uses in one year.

23. **Investing in Stocks** Some investors buy shares of individual stocks and hope to make money by selling the stock when the price increases.
   - **(a)** Find a formula for the profit P an investor makes in terms of the number of shares n she buys, the original price p_0 of a share, and the selling price p_s of a share.
   - **(b)** Silvia plans to make money by buying and selling shares of stock in her favorite retail store. She buys 1000 shares for \$21.50 and waits patiently for many months until the price finally increases to \$25.10. Use the formula found in part (a) to find the profit Silvia will make on her investment if she sells at \$25.10.

24. **Growth of a CD** If you invest in a 24-month CD (certificate of deposit), then the amount A at maturity is given by the formula

$$A = P\left(1 + \frac{r}{100}\right)^2$$

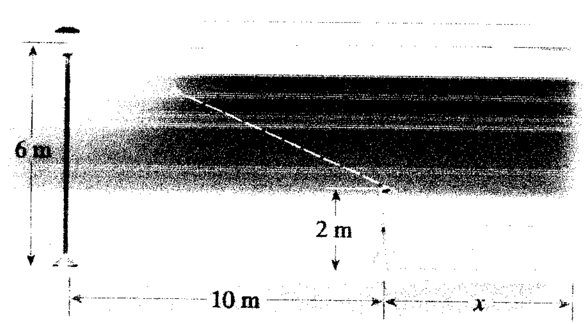
where P is the principal and the interest rate is $r\%$.

Carlos invests \$2000 in a 24-month CD that has a 2.75% interest rate. Use the formula to find how much Carlos' CD is worth at maturity.

25. **Length of Shadow** A man is walking away from a lamppost with a light source 6 m above the ground. If the man is h meters tall and y meters from the lamppost, then the length x of his shadow satisfies the equation

$$\frac{y + x}{6} = \frac{x}{h}$$

   - **(a)** Find a formula for x.
   - **(b)** The figure shows a 2-meter-tall man walking away from the lamppost. Use the formula to find the length of his shadow when he is 10 m from the lamppost.



26. Printing Costs The cost of printing a magazine depends on the number p of pages in the magazine and the number m of copies printed. The cost C is given by the formula

$$C = kpm$$

where k depends on per page printing price.

(a) Find a formula for k.

(b) Find the value of k using the fact that it costs \$12,000 to print 4000 copies of a 120-page magazine.

(c) Use the formula to determine how many copies of a 92-page magazine can be printed if the cost must be no more than \$40,000.

27. Electrical Resistance When two resistors with resistances R_1 and R_2 are connected in parallel, their combined resistance R is given by the formula

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Suppose that an 8-Ω resistor is connected in parallel with a 10-Ω resistor. Use the formula to find their combined resistance R.

28. Temperature of Toaster Wire The resistance R of the heating wire for a toaster depends on temperature. The resistance R at temperature T is given by the formula

$$R = R_0[1 + 0.00045(T - T_0)]$$

where R_0 is the resistance at the initial temperature T_0 (in degrees Celsius). If the initial temperature of a toaster is 20°C and the resistance at that temperature is 147 Ω, find the resistance when the heating wire reaches a temperature of 360°C.

29. The Doppler Effect As a train moves toward an observer (see the figure), the pitch of its whistle sounds higher to the observer than it would if the train were at rest. This phenomenon is called the *Doppler effect*. The observed pitch P_o is given by the formula

$$P_o = \frac{P_s}{1 - \frac{v_s}{s_o}}$$

where P_s is the actual pitch of the whistle at the source, v_s is the speed of the train, and $s_o = 332$ m/s is the speed of sound in air. Suppose the train has a whistle pitched at $P_s = 440$ Hz. Find the pitch of the whistle as perceived by an observer if the speed of the train is 44.7 m/s.

30. Boyle's Law Boyle's Law states that the pressure P in a sample of gas is related to the temperature T and the volume V by the formula

$$P = k\frac{T}{V}$$

where k is a constant.

(a) A certain sample of gas has a volume of 100 L and exerts a pressure of 33.2 kPa at a temperature of 400 K (absolute temperature measured on the Kelvin scale). Use these facts to determine the value of k for this sample.

(b) If the temperature of this sample is increased to 500 K and the volume is decreased to 80 L, use the formula to find the pressure of the gas.

(c) If the volume is quadrupled and the temperature is halved, does the pressure increase or decrease? By what factor?

31. Spread of a Disease The rate r at which a disease spreads in a population of size P is related to the number x of infected people and the number $P - x$ of those who are not infected, by the formula

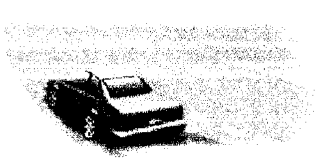
$$r = kx(P - x)$$

where k is a constant that depends on the particular disease. An infection spreads in a town with a population of 5000.

(a) Compare the rate of spread of this infection when 10 people are infected to the rate of spread when 1000 people are infected. Which rate is larger?

(b) Calculate the rate of spread when the entire population is infected. Why does your answer make intuitive sense?

32. Skidding in a Curve A car is traveling on a curve that forms a circular arc. The force F needed to keep the car from skidding is related to weight w of the car and the speed s and the radius r of the curve by the formula



$$F = k\frac{ws^2}{r}$$

where k is a constant that depends on the friction between the tires and the road. A car weighing 1600 lb travels around a curve at 60 mi/h. The next car to round this curve weighs 2500 lb and requires the same force as the first car to keep from skidding. How fast is the second car traveling?

33. Flying Speeds of Migrating Birds Many birds migrate thousands of miles each year between their winter feeding grounds and their summer nesting sites. For instance, the 15-gram blackpoll warbler travels 12,000 miles from western Alaska to South America. The air speed v at which a migrating bird flies depends on its weight w and the surface area S of its wings; these quantities satisfy the equation

$$Sv^2 = 94{,}700w$$

where w is measured in pounds, S in square inches, and v in miles per hour.

(a) Find a formula for v in terms of w and S.

(b) Complete the table to find the ratio w/S and the migrating air speed v for the indicated sea birds.

(c) The ratio w/S is called the *wing loading*; the greater a bird's wing loading, the faster it must fly. A certain bird has a wing loading twice that of the sooty albatross. What is its migrating air speed?

Armin Rose/Shutterstock.com 2009

Bird	w (lb)	S (in^2)	w/S	v (mi/h)
Common tern	0.26	76		
Black-headed gull	0.52	120		
Common gull	0.82	180		
Royal tern	1.1	170		
Herring gull	2.1	280		
Great skua	3.0	330		
Sooty albatross	6.3	530		
Wandering albatross	19.6	960		