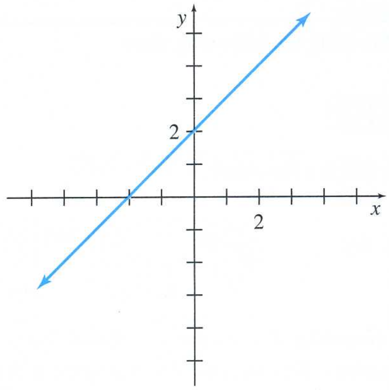
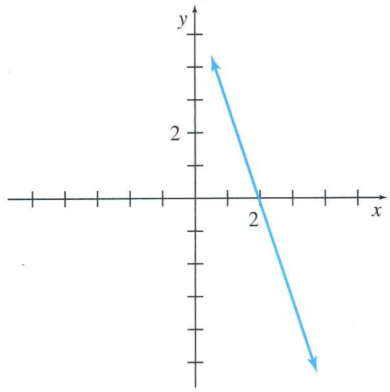
**For the lines in Exercises 39 and 40, which of the following is closest to the slope of the line? (a) 1 (b) 2 (c) 3 (d) 21 (e) 22 (f) −3**

**39.**

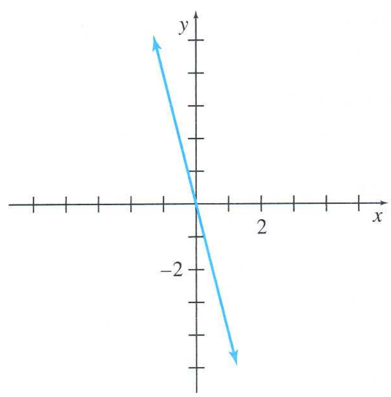



**40.**

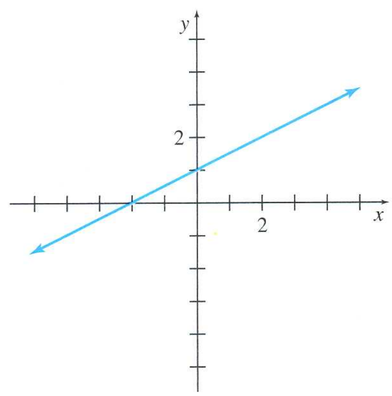



**In Exercises 41 and 42, estimate the slope of the lines.**

**41.**




**42.**



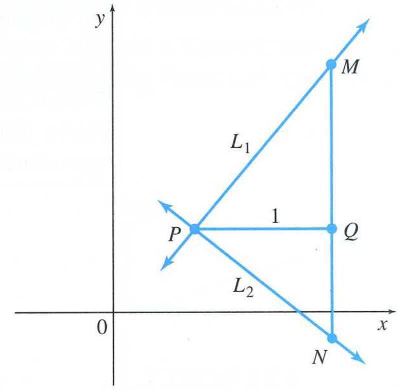

**43.** To show that two perpendicular lines, neither of which is vertical, have slopes with a product of −1, go through the following steps. Let line $L_1$ have equation $y = m_1x + b_1$, and let $L_2$ have equation $y = m_2x + b_2$, with $m_1 > 0$ and $m_2 < 0$. Assume that $L_1$ and $L_2$ are perpendicular, and use right triangle *MPN* shown in the figure. Prove each of the following statements.

**a.** *MQ* has length $m_1$.

**b.** *QN* has length $-m_2$.

**c.** Triangles *MPQ* and *PNQ* are similar.

**d.** $m_1/1 = 1/(-m_2)$ and $m_1m_2 = -1$




**44.** Consider the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

**a.** Show that this equation represents a line by writing it in the form $y = mx + b$.

**b.** Find the $x$- and $y$-intercepts of this line.

**c.** Explain in your own words why the equation in this exercise is known as the intercept form of a line.

**Graph each equation.**

| | |
|---|---|
| **45.** $y = x - 1$ | **46.** $y = 4x + 5$ |
| **47.** $y = -4x + 9$ | **48.** $y = -6x + 12$ |
| **49.** $2x - 3y = 12$ | **50.** $3x - y = -9$ |
| **51.** $3y - 7x = -21$ | **52.** $5y + 6x = 11$ |
| **53.** $y = -2$ | **54.** $x = 4$ |
| **55.** $x + 5 = 0$ | **56.** $y + 8 = 0$ |
| **57.** $y = 2x$ | **58.** $y = -5x$ |
| **59.** $x + 4y = 0$ | **60.** $3x - 5y = 0$ |

## APPLICATIONS

### Business and Economics

**61. Sales** The sales of a small company were \$27,000 in its second year of operation and \$63,000 in its fifth year. Let $y$ represent sales in the $x$th year of operation. Assume that the data can be approximated by a straight line.

**a.** Find the slope of the sales line, and give an equation for the line in the form $y = mx + b$.

**b.** Use your answer from part a to find out how many years must pass before the sales surpass \$100,000.

Find the quantity demanded for the strawberries at each price.

**d.** \$4.50 **e.** \$3.25 **f.** \$2.40

**g.** Graph $p = 5 - 0.25q$.

Suppose the price and supply of strawberries are related by

$$p = S(q) = 0.25q,$$

where $p$ is the price (in dollars) and $q$ is the quantity supplied (in hundreds of quarts) of strawberries. Find the quantity supplied at each price.

**h.** \$0 **i.** \$2 **j.** \$4.50

**k.** Graph $p = 0.75q$ on the same axis used for part g.

**l.** Find the equilibrium quantity and the equilibrium price.

**29. Supply and Demand** Let the supply and demand functions for butter pecan ice cream be given by

$$p = S(q) = \frac{2}{5}q \quad \text{and} \quad p = D(q) = 100 - \frac{2}{5}q,$$

where $p$ is the price in dollars and $q$ is the number of 10-gallon tubs.

**a.** Graph these on the same axes.

**b.** Find the equilibrium quantity and the equilibrium price. (*Hint:* The way to divide by a fraction is to multiply by its reciprocal.)

**30. Supply and Demand** Let the supply and demand functions for sugar be given by

$$p = S(q) = 1.4q - 0.6 \quad \text{and}$$
$$p = D(q) = -2q + 3.2,$$

where $p$ is the price per pound and $q$ is the quantity in thousands of pounds.

**a.** Graph these on the same axes.

**b.** Find the equilibrium quantity and the equilibrium price.

**31. Supply and Demand** Suppose that the supply function for honey is $p = S(q) = 0.3q + 2.7$, where $p$ is the price in dollars for an 8-oz container and $q$ is the quantity in barrels. Suppose also that the equilibrium price is \$4.50 and the demand is 2 barrels when the price is \$6.10. Find an equation for the demand function, assuming it is linear.

**32. Supply and Demand** Suppose that the supply function for walnuts is $p = S(q) = 0.25q + 3.6$, where $p$ is the price in dollars per pound and $q$ is the quantity in bushels. Suppose also that the equilibrium price is \$5.85, and the demand is 4 bushels when the price is \$7.60. Find an equation for the demand function, assuming it is linear.

**33. T-Shirt Cost** Joanne Wendelken sells silk-screened T-shirts at community festivals and crafts fairs. Her marginal cost to produce one T-shirt is \$3.50. Her total cost to produce 60 T-shirts is \$300, and she sells them for \$9 each.

**a.** Find the linear cost function for Joanne's T-shirt production.

**b.** How many T-shirts must she produce and sell in order to break even?

**c.** How many T-shirts must she produce and sell to make a profit of \$500?

**34. Publishing Costs** Alfred Juarez owns a small publishing house specializing in Latin American poetry. His fixed cost to produce a typical poetry volume is \$525, and his total cost to produce 1000 copies of the book is \$2675. His books sell for \$4.95 each.

**a.** Find the linear cost function for Alfred's book production.

**b.** How many poetry books must he produce and sell in order to break even?

**c.** How many books must he produce and sell to make a profit of \$1000?

**35. Marginal Cost of Coffee** The manager of a restaurant found that the cost to produce 100 cups of coffee is \$11.02, while the cost to produce 400 cups is \$40.12. Assume the cost $C(x)$ is a linear function of $x$, the number of cups produced.

**a.** Find a formula for $C(x)$.

**b.** What is the fixed cost?

**c.** Find the total cost of producing 1000 cups.

**d.** Find the total cost of producing 1001 cups.

**e.** Find the marginal cost of the 1001st cup.

**f.** What is the marginal cost of *any* cup and what does this mean to the manager?

**36. Marginal Cost of a New Plant** In deciding whether to set up a new manufacturing plant, company analysts have decided that a linear function is a reasonable estimation for the total cost $C(x)$ in dollars to produce $x$ items. They estimate the cost to produce 10,000 items as \$547,500, and the cost to produce 50,000 items as \$737,500.

**a.** Find a formula for $C(x)$.

**b.** Find the fixed cost.

**c.** Find the total cost to produce 100,000 items.

**d.** Find the marginal cost of the items to be produced in this plant and what does this mean to the manager?

**37. Break-Even Analysis** Producing $x$ units of tacos costs $C(x) = 5x + 20$; revenue is $R(x) = 15x$, where $C(x)$ and $R(x)$ are in dollars.

**a.** What is the break-even quantity?

**b.** What is the profit from 100 units?

**c.** How many units will produce a profit of \$500?

**38. Break-Even Analysis** To produce $x$ units of a religious medal costs $C(x) = 12x + 39$. The revenue is $R(x) = 25x$. Both $C(x)$ and $R(x)$ are in dollars.

**a.** Find the break-even quantity.

**b.** Find the profit from 250 units.

**c.** Find the number of units that must be produced for a profit of \$130.

**Break-Even Analysis** **You are the manager of a firm. You are considering the manufacture of a new product, so you ask the accounting department for cost estimates and the sales department for sales estimates. After you receive the data, you must decide whether to go ahead with production of the new product. Analyze the data in Exercises 39–42 (find a break-even**

**Approximate each expression in the form $a^x$ without using $e$.**

67. $e^{3x}$

68. $e^{-4x}$

**Find the domain of each function.**

69. $f(x) = \log(5 - x)$

70. $f(x) = \ln(x^2 - 9)$

71. Lucky Larry was faced with solving

$$\log(2x + 1) - \log(3x - 1) = 0.$$

Larry just dropped the logs and proceeded:

$$\begin{aligned}(2x + 1) - (3x - 1) &= 0\\ -x + 2 &= 0\\ x &= 2.\end{aligned}$$

Although Lucky Larry is wrong in dropping the logs, his procedure will always give the correct answer to an equation of the form

$$\log A - \log B = 0,$$

where $A$ and $B$ are any two expressions in $x$. Prove that this last equation leads to the equation $A - B = 0$, which is what you get when you drop the logs. ***Source: The AMATYC Review.***

72. Find all errors in the following calculation.

$$\begin{aligned}(\log (x + 2))^2 &= 2 \log (x + 2)\\ &= 2(\log x + \log 2)\\ &= 2(\log x + 100)\end{aligned}$$

73. Prove: $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$.

74. Prove: $\log_a x^r = r \log_a x$.

## APPLICATIONS

### Business and Economics

75. **Inflation** Assuming annual compounding, find the time it would take for the general level of prices in the economy to double at the following annual inflation rates.

**a.** 3% **b.** 6% **c.** 8%

**d.** Check your answers using either the rule of 70 or the rule of 72, whichever applies.

76. **Interest** Mary Klingman invests \$15,000 in an account paying 7% per year compounded annually.

**a.** How many years are required for the compound amount to at least double? (Note that interest is only paid at the end of each year.)

**b.** In how many years will the amount at least triple?

**c.** Check your answer to part a using either the rule of 70 or the rule of 72, whichever applies.

77. **Interest** Leigh Jacks plans to invest \$500 into a money market account. Find the interest rate that is needed for the money to grow to \$1200 in 14 years if the interest is compounded continuously. (Compare with Exercise 43 in the previous section.)

78. **Rule of 72** Complete the following table, and use the results to discuss when the rule of 70 gives a better approximation for the doubling time, and when the rule of 72 gives a better approximation.

| $r$ | 0.001 | 0.02 | 0.05 | 0.08 | 0.12 |
|---|---|---|---|---|---|
| $(\ln 2)/\ln(1 + r)$ | | | | | |
| $70/100r$ | | | | | |
| $72/100r$ | | | | | |

79. **Pay Increases** You are offered two jobs starting July 1, 2013. Humongous Enterprises offers you \$45,000 a year to start, with a raise of 4% every July 1. At Crabapple Inc. you start at \$30,000, with an annual increase of 6% every July 1. On July 1 of what year would the job at Crabapple Inc. pay more than the job at Humongous Enterprises? Use the algebra of logarithms to solve this problem, and support your answer by using a graphing calculator to see where the two salary functions intersect.

### Life Sciences

80. **Insect Species** An article in *Science* stated that the number of insect species of a given mass is proportional to $m^{-0.6}$, where $m$ is the mass in grams. ***Source: Science.*** A graph accompanying the article shows the common logarithm of the mass on the horizontal axis and the common logarithm of the number of species on the vertical axis. Explain why the graph is a straight line. What is the slope of the line?

**Index of Diversity** For Exercises 81–83, refer to Example 10.

81. Suppose a sample of a small community shows two species with 50 individuals each.

**a.** Find the index of diversity $H$.

**b.** What is the maximum value of the index of diversity for two species?

**c.** Does your answer for part a equal ln 2? Explain why.

82. A virgin forest in northwestern Pennsylvania has 4 species of large trees with the following proportions of each: hemlock, 0.521; beech, 0.324; birch, 0.081; maple, 0.074. Find the index of diversity $H$.

83. Find the value of the index of diversity for populations with $n$ species and $1/n$ of each if

**a.** $n = 3$; **b.** $n = 4$.

**c.** Verify that your answers for parts a and b equal ln 3 and ln 4, respectively.

84. **Allometric Growth** The allometric formula is used to describe a wide variety of growth patterns. It says that $y = nx^m$, where $x$ and $y$ are variables, and $n$ and $m$ are constants. For example, the famous biologist J. S. Huxley used this formula to relate the weight of the large claw of the fiddler crab to the weight of the body without the claw. ***Source: Problems of Relative Growth.*** Show that if $x$ and $y$ are given by the allometric formula, then

17. If $k(x) = \dfrac{x^3 - 2x - 4}{x - 2}$, find $\lim_{x\to 2} k(x)$.

| $x$ | 1.9 | 1.99 | 1.999 | 2.001 | 2.01 | 2.1 |
|---|---|---|---|---|---|---|
| $k(x)$ | | | | | | |

18. If $f(x) = \dfrac{2x^3 + 3x^2 - 4x - 5}{x + 1}$, find $\lim_{x\to -1} f(x)$.

| $x$ | −1.1 | −1.01 | −1.001 | −0.999 | −0.99 | −0.9 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

19. If $h(x) = \dfrac{\sqrt{x} - 2}{x - 1}$, find $\lim_{x\to 1} h(x)$.

| $x$ | 0.9 | 0.99 | 0.999 | 1.001 | 1.01 | 1.1 |
|---|---|---|---|---|---|---|
| $h(x)$ | | | | | | |

20. If $f(x) = \dfrac{\sqrt{x} - 3}{x - 3}$, find $\lim_{x\to 3} f(x)$.

| $x$ | 2.9 | 2.99 | 2.999 | 3.001 | 3.01 | 3.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

**Let $\lim_{x\to 4} f(x) = 9$ and $\lim_{x\to 4} g(x) = 27$. Use the limit rules to find each limit.**

21. $\lim_{x\to 4}[f(x) - g(x)]$

22. $\lim_{x\to 4}[g(x) \cdot f(x)]$

23. $\lim_{x\to 4} \dfrac{f(x)}{g(x)}$

24. $\lim_{x\to 4} \log_3 f(x)$

25. $\lim_{x\to 4} \sqrt{f(x)}$

26. $\lim_{x\to 4} \sqrt[3]{g(x)}$

27. $\lim_{x\to 4} 2^{f(x)}$

28. $\lim_{x\to 4}[1 + f(x)]^2$

29. $\lim_{x\to 4} \dfrac{f(x) + g(x)}{2g(x)}$

30. $\lim_{x\to 4} \dfrac{5g(x) + 2}{1 - f(x)}$

**Use the properties of limits to help decide whether each limit exists. If a limit exists, find its value.**

31. $\lim_{x\to 3} \dfrac{x^2 - 9}{x - 3}$

32. $\lim_{x\to -2} \dfrac{x^2 - 4}{x + 2}$

33. $\lim_{x\to 1} \dfrac{5x^2 - 7x + 2}{x^2 - 1}$

34. $\lim_{x\to -3} \dfrac{x^2 - 9}{x^2 + x - 6}$

35. $\lim_{x\to -2} \dfrac{x^2 - x - 6}{x + 2}$

36. $\lim_{x\to 5} \dfrac{x^2 - 3x - 10}{x - 5}$

37. $\lim_{x\to 0} \dfrac{1/(x + 3) - 1/3}{x}$

38. $\lim_{x\to 0} \dfrac{-1/(x + 2) + 1/2}{x}$

39. $\lim_{x\to 25} \dfrac{\sqrt{x} - 5}{x - 25}$

40. $\lim_{x\to 36} \dfrac{\sqrt{x} - 6}{x - 36}$

41. $\lim_{h\to 0} \dfrac{(x + h)^2 - x^2}{h}$

42. $\lim_{h\to 0} \dfrac{(x + h)^3 - x^3}{h}$

43. $\lim_{x\to \infty} \dfrac{3x}{7x - 1}$

44. $\lim_{x\to -\infty} \dfrac{8x + 2}{4x - 5}$

45. $\lim_{x\to -\infty} \dfrac{3x^2 + 2x}{2x^2 - 2x + 1}$

46. $\lim_{x\to \infty} \dfrac{x^2 + 2x - 5}{3x^2 + 2}$

47. $\lim_{x\to \infty} \dfrac{3x^3 + 2x - 1}{2x^4 - 3x^3 - 2}$

48. $\lim_{x\to \infty} \dfrac{2x^2 - 1}{3x^4 + 2}$

49. $\lim_{x\to \infty} \dfrac{2x^3 - x - 3}{6x^2 - x - 1}$

50. $\lim_{x\to \infty} \dfrac{x^4 - x^3 - 3x}{7x^2 + 9}$

51. $\lim_{x\to \infty} \dfrac{2x^2 - 7x^4}{9x^2 + 5x - 6}$

52. $\lim_{x\to \infty} \dfrac{-5x^3 - 4x^2 + 8}{6x^2 + 3x + 2}$

53. Let $f(x) = \begin{cases} x^3 + 2 & \text{if } x \neq -1 \\ 5 & \text{if } x = -1 \end{cases}$. Find $\lim_{x\to -1} f(x)$.

54. Let $g(x) = \begin{cases} 0 & \text{if } x = -2 \\ \frac{1}{2}x^2 - 3 & \text{if } x \neq -2 \end{cases}$. Find $\lim_{x\to -2} g(x)$.

55. Let $f(x) = \begin{cases} x - 1 & \text{if } x < 3 \\ 2 & \text{if } 3 \le x \le 5 \\ x + 3 & \text{if } x > 5 \end{cases}$.

**a.** Find $\lim_{x\to 3} f(x)$. **b.** Find $\lim_{x\to 5} f(x)$.

56. Let $g(x) = \begin{cases} 5 & \text{if } x < 0 \\ x^2 - 2 & \text{if } 0 \le x \le 3 \\ 7 & \text{if } x > 3 \end{cases}$.

**a.** Find $\lim_{x\to 0} g(x)$. **b.** Find $\lim_{x\to 3} g(x)$.

**In Exercises 57–60, calculate the limit in the specified exercise, using a table such as in Exercises 15–20. Verify your answer by using a graphing calculator to zoom in on the point on the graph.**

57. Exercise 31

58. Exercise 32

59. Exercise 33

60. Exercise 34

61. Let $F(x) = \dfrac{3x}{(x + 2)^3}$.

**a.** Find $\lim_{x\to -2} F(x)$.

**b.** Find the vertical asymptote of the graph of $F(x)$.

**c.** Compare your answers for parts a and b. What can you conclude?

62. Let $G(x) = \dfrac{-6}{(x - 4)^2}$.

**a.** Find $\lim_{x\to 4} G(x)$.

**b.** Find the vertical asymptote of the graph of $G(x)$.

**c.** Compare your answers for parts a and b. Are they related? How?

63. How can you tell that the graph in Figure 10 is more representative of the function $f(x) = 1/(x - 1)$ than the graph in Figure 11?

64. A friend who is confused about limits wonders why you investigate the value of a function closer and closer to a point, instead of just finding the value of a function at the point. How would you respond?

**65.** Use a graph of $f(x) = e^x$ to answer the following questions.

**a.** Find $\lim_{x \to -\infty} e^x$.

**b.** Where does the function $e^x$ have a horizontal asymptote?

**66.** Use a graphing calculator to answer the following questions.

**a.** From a graph of $y = xe^{-x}$, what do you think is the value of $\lim_{x \to \infty} xe^{-x}$? Support this by evaluating the function for several large values of $x$.

**b.** Repeat part a, this time using the graph of $y = x^2 e^{-x}$.

**c.** Based on your results from parts a and b, what do you think is the value of $\lim_{x \to \infty} x^n e^{-x}$, where $n$ is a positive integer? Support this by experimenting with other positive integers $n$.

**67.** Use a graph of $f(x) = \ln x$ to answer the following questions.

**a.** Find $\lim_{x \to 0^+} \ln x$.

**b.** Where does the function $\ln x$ have a vertical asymptote?

**68.** Use a graphing calculator to answer the following questions.

**a.** From a graph of $y = x \ln x$, what do you think is the value of $\lim_{x \to 0^+} x \ln x$? Support this by evaluating the function for several small values of $x$.

**b.** Repeat part a, this time using the graph of $y = x(\ln x)^2$.

**c.** Based on your results from parts a and b, what do you think is the value of $\lim_{x \to 0^+} x(\ln x)^n$, where $n$ is a positive integer? Support this by experimenting with other positive integers $n$.

**69.** Explain in your own words why the rules for limits at infinity should be true.

**70.** Explain in your own words what Rule 4 for limits means.

**Find each of the following limits (a) by investigating values of the function near the $x$-value where the limit is taken, and (b) using a graphing calculator to view the function near that value of $x$.**

**71.** $\lim_{x \to 1} \dfrac{x^4 + 4x^3 - 9x^2 + 7x - 3}{x - 1}$

**72.** $\lim_{x \to 2} \dfrac{x^4 + x - 18}{x^2 - 4}$

**73.** $\lim_{x \to -1} \dfrac{x^{1/3} + 1}{x + 1}$

**74.** $\lim_{x \to 4} \dfrac{x^{3/2} - 8}{x + x^{1/2} - 6}$

**Use a graphing calculator to graph the function. (a) Determine the limit from the graph. (b) Explain how your answer could be determined from the expression for $f(x)$.**

**75.** $\lim_{x \to \infty} \dfrac{\sqrt{9x^2 + 5}}{2x}$

**76.** $\lim_{x \to -\infty} \dfrac{\sqrt{9x^2 + 5}}{2x}$

**77.** $\lim_{x \to -\infty} \dfrac{\sqrt{36x^2 + 2x + 7}}{3x}$

**78.** $\lim_{x \to \infty} \dfrac{\sqrt{36x^2 + 2x + 7}}{3x}$

**79.** $\lim_{x \to \infty} \dfrac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5}$

**80.** $\lim_{x \to -\infty} \dfrac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5}$

**81.** Explain why the following rules can be used to find $\lim_{x \to \infty} [p(x)/q(x)]$:

**a.** If the degree of $p(x)$ is less than the degree of $q(x)$, the limit is 0.

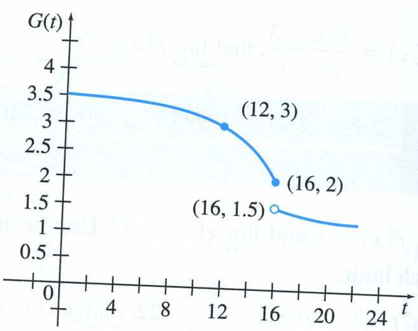
**b.** If the degree of $p(x)$ is equal to the degree of $q(x)$, the limit is $A/B$, where $A$ and $B$ are the leading coefficients of $p(x)$ and $q(x)$, respectively.

**c.** If the degree of $p(x)$ is greater than the degree of $q(x)$, the limit is $\infty$ or $-\infty$.

## APPLICATIONS

### Business and Economics

**82. APPLY IT Consumer Demand** When the price of an essential commodity (such as gasoline) rises rapidly, consumption drops slowly at first. If the price continues to rise, however, a "tipping" point may be reached, at which consumption takes a sudden substantial drop. Suppose the accompanying graph shows the consumption of gasoline, $G(t)$, in millions of gallons, in a certain area. We assume that the price is rising rapidly. Here $t$ is time in months after the price began rising. Use the graph to find the following.




**a.** $\lim_{t \to 12} G(t)$ **b.** $\lim_{t \to 16} G(t)$

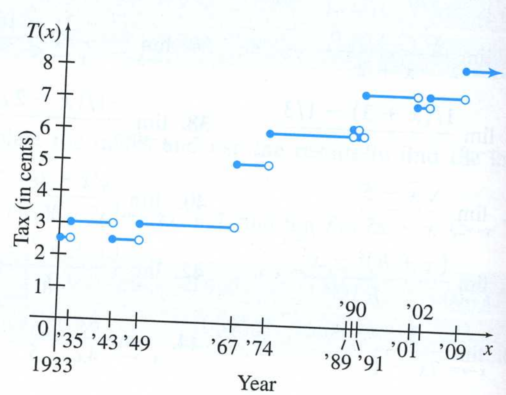
**c.** $G(16)$ **d.** The tipping point (in months)

**83. Sales Tax** Officials in California tend to raise the sales tax in years in which the state faces a budget deficit and then cut the tax when the state has a surplus. The graph below shows the California state sales tax since it was first established in 1933. Let $T(x)$ represent the sales tax per dollar spent in year $x$. Find the following. *Source: California State.*

**a.** $\lim_{x \to 53} T(x)$ **b.** $\lim_{x \to 09^-} T(x)$

**c.** $\lim_{x \to 09^+} T(x)$ **d.** $\lim_{x \to 09} T(x)$

**e.** $T(09)$



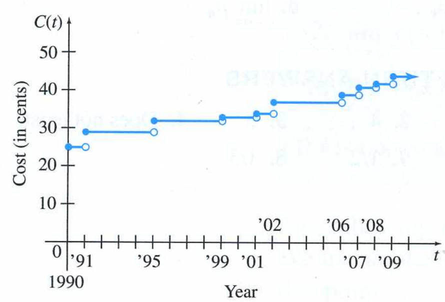
**84. Postage** The graph below shows how the postage required to mail a letter in the United States has changed in recent years. Let $C(t)$ be the cost to mail a letter in the year $t$. Find the following. ***Source: United States Postal Service.***

**a.** $\lim_{t \to 2009^-} C(t)$

**b.** $\lim_{t \to 2009^+} C(t)$

**c.** $\lim_{t \to 2009} C(t)$

**d.** $C(2009)$




**85. Average Cost** The cost (in dollars) for manufacturing a particular DVD is

$$C(x) = 15{,}000 + 6x,$$

where $x$ is the number of DVDs produced. Recall from the previous chapter that the average cost per DVD, denoted by $\overline{C}(x)$, is found by dividing $C(x)$ by $x$. Find and interpret $\lim_{x\to\infty} \overline{C}(x)$.

**86. Average Cost** In Chapter 1, we saw that the cost to fly $x$ miles on American Airlines could be approximated by the equation

$$C(x) = 0.0738x + 111.83.$$

Recall from the previous chapter that the average cost per mile, denoted by $\overline{C}(x)$, is found by dividing $C(x)$ by $x$. Find and interpret $\lim_{x\to\infty} \overline{C}(x)$. ***Source: American Airlines.***

**87. Employee Productivity** A company training program has determined that, on the average, a new employee produces $P(s)$ items per day after $s$ days of on-the-job training, where

$$P(s) = \frac{63s}{s + 8}.$$

Find and interpret $\lim_{s\to\infty} P(s)$.

**88. Preferred Stock** In business finance, an annuity is a series of equal payments received at equal intervals for a finite period of time. The *present value* of an $n$-period annuity takes the form

$$P = R\left[\frac{1 - (1 + i)^{-n}}{i}\right],$$

where $R$ is the amount of the periodic payment and $i$ is the fixed interest rate per period. Many corporations raise money by issuing preferred stock. Holders of the preferred stock, called a *perpetuity*, receive payments that take the form of an annuity in that the amount of the payment never changes. However, normally the payments for preferred stock do not end but theoretically continue forever. Find the limit of this present value equation as $n$ approaches infinity to derive a formula for the present value of a share of preferred stock paying a periodic dividend $R$. ***Source: Robert D. Campbell.***

**89. Growing Annuities** For some annuities encountered in business finance, called *growing annuities*, the amount of the periodic payment is not constant but grows at a constant periodic rate. Leases with escalation clauses can be examples of growing annuities. The present value of a growing annuity takes the form

$$P = \frac{R}{i - g}\left[1 - \left(\frac{1 + g}{1 + i}\right)^n\right],$$

where

$R$ = amount of the next annuity payment,

$g$ = expected constant annuity growth rate,

$i$ = required periodic return at the time the annuity is evaluated,

$n$ = number of periodic payments.

A corporation's common stock may be thought of as a claim on a growing annuity where the annuity is the company's annual dividend. However, in the case of common stock, these payments have no contractual end but theoretically continue forever. Compute the limit of the expression above as $n$ approaches infinity to derive the Gordon–Shapiro Dividend Model popularly used to estimate the value of common stock. Make the reasonable assumption that $i > g$. (*Hint:* What happens to $a^n$ as $n \to \infty$ if $0 < a < 1$?) ***Source: Robert D. Campbell.***

## Life Sciences

**90. Alligator Teeth** Researchers have developed a mathematical model that can be used to estimate the number of teeth $N(t)$ at time $t$ (days of incubation) for *Alligator mississippiensis*, where

$$N(t) = 71.8e^{-8.96e^{-0.0685t}}.$$

***Source: Journal of Theoretical Biology.***

**a.** Find $N(65)$, the number of teeth of an alligator that hatched after 65 days.

**b.** Find $\lim_{t\to\infty} N(t)$ and use this value as an estimate of the number of teeth of a newborn alligator. (*Hint:* See Exercise 65.) Does this estimate differ significantly from the estimate of part a?

**91. Sediment** To develop strategies to manage water quality in polluted lakes, biologists must determine the depths of sediments and the rate of sedimentation. It has been determined that the depth of sediment $D(t)$ (in centimeters) with respect to time (in years before 1990) for Lake Coeur d'Alene, Idaho, can be estimated by the equation

$$D(t) = 155(1 - e^{-0.0133t}).$$

***Source: Mathematics Teacher.***

**a.** Find $D(20)$ and interpret.

**b.** Find $\lim_{t\to\infty} D(t)$ and interpret.