3.1 Exercises

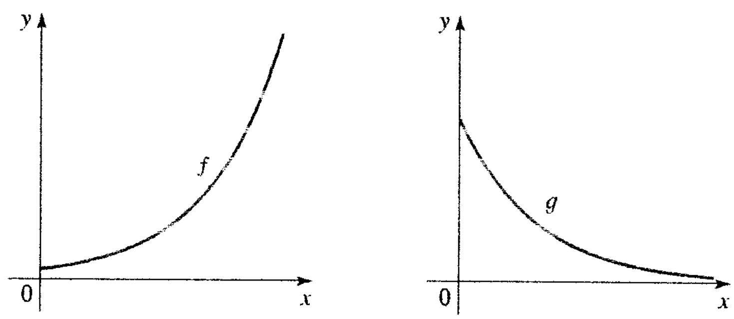
CONCEPTS

Fundamentals

1. The number of bacteria in a culture at time t (in hours) is modeled by $N(t) = 200(1.8)^t$.
   (a) The initial bacteria population is ________.
   (b) The growth factor is ________.
   (c) The growth rate is ________ (in decimal form).
   (d) The growth rate is ________ (in percentage form).

2. The amount (in grams) of a radioactive substance remaining at time t (years) is modeled by $f(t) = 100(0.4)^t$.
   (a) The initial amount is ________.
   (b) The decay factor is ________.
   (c) The decay rate is ________ (in decimal form).
   (d) The decay rate is ________ (in percentage form).

3. Graphs of exponential functions f and g are shown.
   (a) The function f models exponential ________ (growth/decay).
   (b) The function g models exponential ________ (growth/decay).



4. (a) The function $f(x) = 50\cdot(0.25)^x$ models exponential ________ (growth/decay).
   (b) The function $f(x) = 200\cdot(1.1)^x$ models exponential ________ (growth/decay).

Think About It

5. Population A increases by 10% each year, and Population B is multiplied by 1.10 each year. Compare the growth factors and growth rates of each population. Are they the same? Explain why or why not.

6. Population A is modeled by $P(x) = 200\cdot\left(\frac{1}{3}\right)^x$, and Population B is modeled by $Q(x) = 200\cdot 3^{-x}$. Explain why both populations decay exponentially. What are the decay factor and the decay rate for each population?

7. Which of the following are not models of exponential growth?
   (a) $f(x) = 100\cdot 7^x$ (b) $P(x) = x^{10}$ (c) $h(t) = 100x$ (d) $Q(x) = 4(0.5)^x$

8. Which of the following are not models of exponential decay?
   (a) $f(x) = 100\cdot\left(\frac{1}{5}\right)^x$ (b) $h(x) = 100 - x$ (c) $Q(x) = 10(0.3)^x$ (d) $P(x) = (0.5x)^4$

9–16 ■ An exponential growth or decay model is given.
   (a) Determine whether the model represents growth or decay.
   (b) Find the growth or decay factor.
   (c) Find the growth or decay rate.

9. $f(x) = 50\cdot(1.2)^x$ **10.** $q(t) = 20\cdot(1.9)^t$

11. $y = \left(\frac{1}{3}\right)^x$ **12.** $h(T) = (0.15)^T$

13. $R(x) = 11(0.25)^x$ **14.** $Q(x) = 8\cdot(2.5)^x$

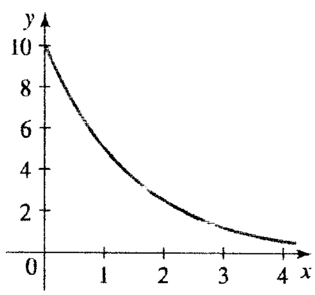
15. $n(x) = 39\cdot(2.8)^x$ **16.** $m(t) = 3.5(0.7)^t$

17–20 ■ Match the exponential function with its graph.

17. $f(x) = 10\cdot 2^x$ **18.** $f(x) = 10\cdot(0.5)^x$

19. $f(x) = 5\cdot\left(\frac{1}{4}\right)^x$ **20.** $f(x) = 5(1.4)^x$

I



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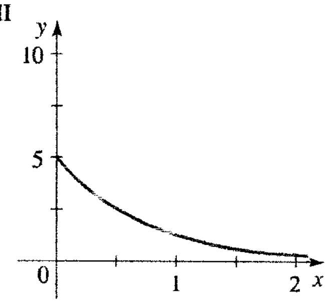

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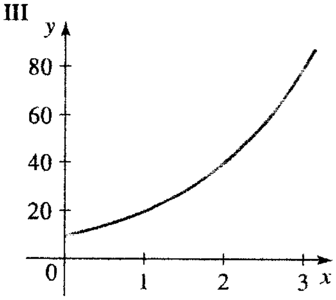

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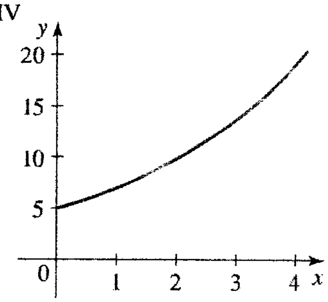


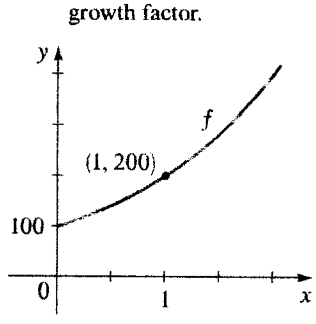
21–22 ■ A population P is initially 3750 and six hours later reaches the given number. Find the six-hour growth or decay factor.

21. (a) 7250 (b) 5000 (c) 2500 (d) 750

22. (a) 5800 (b) 9600 (c) 1875 (d) 1250

23–24 ■ The graph of a function that models exponential growth or decay is shown, where x represents one-year time periods. Find the initial population and the one-year growth factor.

23.



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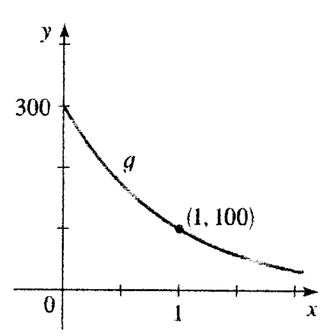


25–34 ■ A population P is initially 350. Find an exponential growth model in terms of the number of time periods x if in each time period the population P

25. quadruples.

26. doubles.

27. decreases by $\frac{1}{2}$.

28. decreases by $\frac{1}{3}$.

29. decreases by 35%.

30. increases by 100%.

31. increases by 300%.

32. decreases by 2%.

33. is multiplied by 1.7.

34. is multiplied by 2.3.

CONTEXTS

Laurence Gough/Shutterstock.com 2009

35. Bacterial Infection The bacteria *Streptococcus pneumoniae* (*S. pneumoniae*) is the cause of many human diseases, the most common being pneumonia. A culture of these bacteria initially has 10 bacteria and increases at a rate of 26% per hour.

(a) Find the hourly growth factor a, and find an exponential model $f(t) = Ca^t$ for the bacteria count in the sample, where t is measured in hours.

(b) Use the model you found to predict the number of bacteria in the sample after 5 hours.

36. Bacterial Infection The bacteria *R. sphaeroides* is the cause of the disease chlamydia. A culture of these bacteria initially has 25 bacteria and increases at a rate of 28% per hour.

(a) Find the hourly growth factor a, and find an exponential model $f(t) = Ca^t$ for the bacteria count in the sample, where t is measured in hours.

(b) Use the model you found to predict the number of bacteria in the sample after 7 hours.

37. Cost-of-Living Increase Mariel teaches science at a community college in Arizona. Her starting salary in 2003 was \$38,900, and each year her salary increases by 2%.

(a) Complete the table of Mariel's salary.

Year	Years since 2003	Salary ($)
2003	0	38,900
2004		
2005		
2006		
2007		

(b) Find an exponential growth model for Mariel's salary t years after 2003. What is the growth factor?

(c) Use the model found in part (b) to predict Mariel's salary in 2010, assuming that her salary continues to increase at the same rate.

(d) Use a graphing calculator to graph the model you found in part (b) together with a scatter plot of the data in the table.

38. Investment Value Kerri invested \$2000 on January 1, 2005, in a mutual fund that for the past 5 years had a yearly increase of 5%. Assume that the mutual fund continues to grow at 5% each year.

(a) Complete the following table for the investment value at the beginning of each year.

Year	Years since 2005	Investment value ($)
2005	0	2000
2006		
2007		
2008		

(b) Find an exponential growth model for Kerri's investment t years since 2005. What is the growth factor?

(c) Use the model found in part (b) to predict Kerri's investment value in 2010.

(d) Use a graphing calculator to graph the model you found in part (b) together with a scatter plot of the data in the table.

39. **Health-Care Spending** The Centers for Medicare and Medicaid Services report that health-care expenditures per capita were $2813 in 1990 and $3329 in 1993. Assume that this rate of growth continues.

(a) Find the three-year growth factor a, and find an exponential growth model $E(x) = Ca^x$ for the annual health-care expenditures per capita, where x is the number of three-year time periods since 1990.

(b) Use the model found in part (a) to predict health-care expenditures per capita in 1996 and in 2005.

R (c) Search the Internet to find the actual health-care expenditures for 1996 and 2005. Were your predictions reasonable?

40. **Profit Increase** In 2007, Mieko's software company made a profit of $5,165,000. Mieko projects that her company will have a profit of 6 million dollars in 2011. Assume that her predictions are correct and that this rate of growth continues.

(a) Find the four-year growth factor a, and find an exponential growth model $P(x) = Ca^x$ for the annual profit, where x is the number of four-year time periods since 2007.

(b) Use the model found in part (a) to predict the profit for Mieko's company in 2015.

41. **Drug Absorption** A patient is administered 100 mg of a therapeutic drug. It is known that 25% of the drug is expelled from the body each hour.

(a) Find an exponential decay model for the amount of drug remaining in the patient's body after t hours.

(b) Use the model to predict the amount of the drug that remains in the patient's body after 6 hours.

42. **Drug Absorption** A patient is administered 250 mg of a therapeutic drug. It is known that 40% of the drug is expelled from the body each hour.

(a) Find an exponential decay model for the amount of drug remaining in the patient's body after t hours.

(b) Use the model to predict the amount of the drug that remains in the patient's body after 10 hours.

43. **Declining Housing Prices** In 2006 the U.S. "housing bubble" was beginning to burst. One news article at the time stated: "The median price of a home sold in the United States in the third quarter of 2006 was $232,300, and forecasters predict that the median price of houses will fall by 8.9% each year." Assume that the median price decreases at the forecasted decay rate of −8.9%.

(a) Find the decay factor a, and find an exponential growth model $E(x) = Ca^x$ for the median price of a home sold in the United States x years since the prediction.

(b) Use the model found in part (a) to predict the median price of homes sold in the third quarter of 2009.

44. Falling Gas Prices A California newspaper published a story on October 17, 2008, stating: "The average price of a gallon of gas has dropped to \$3.60, which is a drop of 10% in just one week. Forecasters predict that this trend will continue and the price of gas will be under \$2.00 a gallon by the end of November." Assume that the price of gas decreases at the forecasted decay rate of -10% per week.

(a) Find the decay factor a, and find an exponential growth model $G(x) = Ca^x$ for the price of gas x weeks since October 17, 2008, when the price was \$3.60 per gallon.

(b) Use the model to predict the price of gas 6 weeks later. Is this what the forecasters predicted?

45. Radioactive Cesium The half-life of cesium-137 is 30 years. Suppose we have a 10-g sample.

(a) Find a function that models the mass $m(x)$ of cesium-137 remaining after x half-lives.

(b) Use the model to predict the amount of cesium-137 remaining after 270 years.

(c) Make a table of values for $m(x)$ with x varying between 0 and 5.

(d) Graph the function $m(x)$ and the entries in the table. What does the graph tell us about how cesium-137 decays?

46. Radioactive Iodine The half-life of iodine-135 is 8 days. Suppose we have a 22-mg sample.

(a) Find a function that models the mass $m(x)$ of iodine-135 remaining after x half-lives.

(b) Use the model to predict the amount of iodine-135 remaining after 32 days.

(c) Make a table of values for $m(x)$ with x varying between 0 and 5.

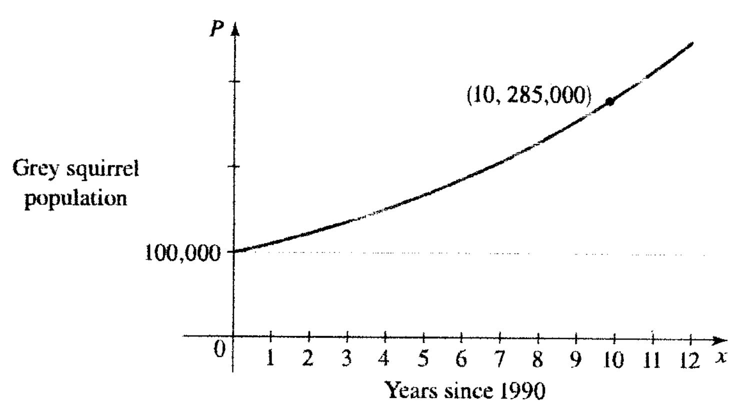
(d) Graph the function $m(x)$ and the entries in the table. What does the graph tell us about how iodine-135 decays?

47. Grey Squirrel Population The American grey squirrel is a species not native to Great Britain that was introduced there in the early twentieth century and has been increasing in numbers ever since. The graph shows the grey squirrel population in a British county between 1990 and 2000. Assume that the population continues to grow exponentially.

(a) What was the grey squirrel population in 1990?

(b) Find the ten-year growth factor a, and find an exponential growth model $P(x) = Ca^x$, where x is the number of ten-year time periods since 1990.

(c) Use the model found in part (b) to predict the grey squirrel population in 2010.

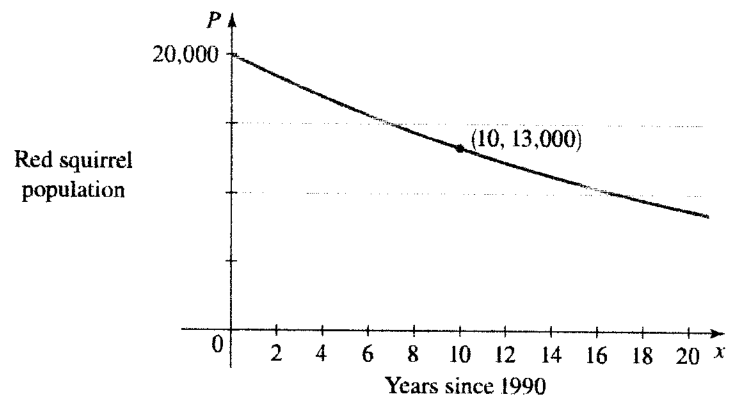


48. Red Squirrel Population The red squirrel is the native squirrel of Great Britain, but its numbers have been declining ever since the American grey squirrel was introduced to Great Britain. The graph shows the red squirrel population in a British county between 1990 and 2000. Assume that the population continues to decrease exponentially.

(a) What was the red squirrel population in 1990?

(b) Find the ten-year decay factor a, and find an exponential decay model $P(x) = Ca^x$ for the red squirrel population, where x is the number of ten-year time periods since 1990.

(c) Use the model found in part (b) to predict the red squirrel population in 2010.



49. Algebra and Alcohol After alcohol is fully absorbed into the body, it is metabolized with a half-life of 1.5 hours. Suppose Thad consumes 45 mL of alcohol (ethanol).

(a) Find an exponential decay model for the amount of alcohol remaining after x 1.5-hour time intervals.

(b) Graph the function you found in part (a) for x between 0 and 4.

3.2 Exponential Models: Comparing Rates

- **Changing the Time Period**
- **Growth of an Investment: Compound Interest**

IN THIS SECTION... we find exponential models of growth and decay for different time periods. We also study compound interest as an example of exponential growth.

GET READY... by reviewing how to solve power equations in **Algebra Toolkit C.1.** Test your understanding by doing the **Algebra Checkpoint** on page 267.

To determine the growth rate of a certain type of bacteria, a biologist prepares a nutrient solution in which the bacteria can grow. The biologist introduces a number of bacteria into the solution and then estimates the bacteria count 40 minutes later. From this information the biologist can determine the 40-minute growth factor. But the biologist wants to report the hourly growth factor, not the 40-minute growth factor. This conforms to the standards for reporting bacteria growth rates

Algebra Checkpoint ✓

Check your knowledge of solving power equations by doing the following problems. You can review the rules of exponents in **Algebra Toolkit C.1** on page T47.

1–8 Solve the equation for x.

1. $x^4 = 16$ **2.** $x^3 = \frac{1}{27}$ **3.** $5x^3 = 40$ **4.** $2x^5 = 486$

5. $x^{1/2} = 5$ **6.** $x^{1/3} = 2$ **7.** $5x^{1/6} = 10$ **8.** $3x^{1/2} = 5$

9–12 Solve the equation for x; express your answer correct to two decimal places.

9. $x^4 = 5$ **10.** $2x^3 = 9$ **11.** $x^{1/5} = 7.35$ **12.** $3x^{1/4} = 34.12$

3.2 Exercises

CONCEPTS

Fundamentals

1. The bacteria population in a certain culture grows exponentially.

(a) If the 20-minute growth factor is a, then the one-hour growth factor is a^3.

(b) If the five-hour growth factor is b, then the one-hour growth factor is ________.

(c) If the half-life of a radioactive element is h years, then the yearly decay factor is ________.

2. In the compound interest formula $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$, the letters P, r, n, and t stand for ________, ________, ________, and ________, respectively, and $A(t)$ stands for ________.

Think About It

3. Biologists often report population growth rates in fixed time periods such as one hour, one day, one year, and so on. What are some of the advantages of this type of reporting (as opposed to reporting 38-minute growth rates or nine-day growth rates)?

4. The rate of decay of a radioactive element is usually expressed in terms of its half-life. Why do you think this is? How is this more useful than reporting yearly decay rates?

SKILLS

5–6 ■ A population P starts at 2000, and four years later the population reaches the given number.

(i) Find the four-year growth or decay factor.

(ii) Find the annual growth or decay factor.

5. **(a)** 8000 **(b)** 20,000 **(c)** 1500 **(d)** 1200

6. **(a)** 6000 **(b)** 3000 **(c)** 800 **(d)** 400

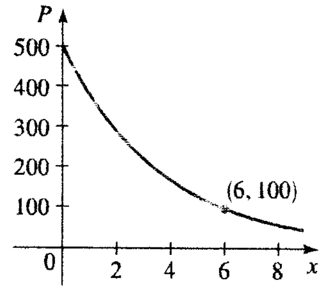
7–10 ■ The bacteria population in a certain culture grows exponentially. Find the one-hour growth rate if

7. the 10-minute growth rate is 0.02. **8.** the 15-minute growth rate is 0.09.

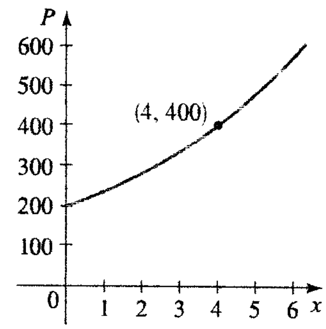
9. the three-hour growth rate is 57%. **10.** the four-hour growth rate is 72%.

11–12 ■ The graph of a function that models exponential growth or decay is shown. Find the initial population and the one-year growth factor.

11.



12.



13–16 ■ A bacterial infection starts with 2000 bacteria, and the bacteria count doubles in the given time period. Find an exponential growth model for the number of bacteria

(a) x time periods after infection.

(b) t hours after infection.

13. 20 minutes

14. 15 minutes

15. 90 minutes

16. 2 hours

17–24 ■ A population P is initially 1000. Find an exponential model (growth or decay) for the population after t years if the population P

17. doubles every year.

18. is multiplied by 2 every year.

19. increases by 35% every 5 years.

20. increases by 200% every 6 years.

21. decreases by $\frac{1}{2}$ every 6 months.

22. is multiplied by 0.75 every month.

23. decreases by 40% every 2 years.

24. decreases by 37% every 9 years.

25–28 ■ Information on two different bacteria populations is given.

(a) Find the one-hour growth rate for each type of bacteria.

(b) Which type of bacteria has the greater growth rate?

25. Type A: 20-minute growth factor is 1.19
Type B: 30-minute growth factor is 1.23

26. Type A: doubles every 20 minutes
Type B: triples every 30 minutes

27. Type A: triples every 5 hours
Type B: quadruples every 7 hours

28. Type A: increases by 30% every hour
Type B: increases by 60% every 2 hours

29–30 ■ **Compound Interest** An investment of $5000 is deposited into an account in which interest is compounded monthly. Complete the table by filling in the amount to which the investment grows at the indicated times or interest rates.

29. $r = 4\%$

Time (years)	Amount ($)
0	5000
1	
2	
3	
4	
5	

30. $t = 5$ years

Annual rate	Amount ($)
1%	
2%	
3%	
4%	
5%	
6%	

31. Compound Interest If $500 is invested at an interest rate of 3% per year, compounded quarterly, find the value of the investment after the given number of years.
(a) 1 year **(b)** 2 years **(c)** 5 years

32. Compound Interest If $2500 is invested at an interest rate of 2.5% per year, compounded daily, find the value of the investment after the given number of years.
(a) 2 years **(b)** 3 years **(c)** 6 years

33. Compound Interest If $4000 is invested at an interest rate of 1.6% per year, compounded quarterly, find the value of the investment after the given number of years.
(a) 4 years **(b)** 6 years **(c)** 8 years

34. Compound Interest If $10,000 is invested at an interest rate of 10% per year, compounded semiannually, find the value of the investment after the given number of years.
(a) 5 years **(b)** 10 years **(c)** 15 years

35. Compound Interest If $3000 is invested at an interest rate of 4% each year, find the amount of the investment at the end of 5 years for the following compounding methods.
(a) Annual **(b)** Semiannual **(c)** Monthly **(d)** Daily

36. Compound Interest If $4000 is invested in an account for which interest is compounded quarterly, find the amount of the investment at the end of 5 years for the following interest rates.
(a) 6% **(b)** $6\frac{1}{2}$% **(c)** 7% **(d)** 8%

37. Annual Percentage Yield Find the annual percentage yield for an investment that earns 2.5% each year, compounded daily.

38. Annual Percentage Yield Find the annual percentage yield for an investment that earns 4% each year, compounded monthly.

39. Annual Percentage Yield Find the annual percentage yield for an investment that earns 3.25% each year, compounded quarterly.

40. Annual Percentage Yield Find the annual percentage yield for an investment that earns 5.75% each year, compounded semiannually.

41. Compound Interest Kai wants to invest $5000, and he is comparing two different investment options:
(i) $3\frac{1}{4}$% interest each year, compounded semiannually
(ii) 3% interest each year, compounded daily
Which of the two options would provide the better investment?

42. Compound Interest Sonya wants to invest \$3000, and she is comparing three different investment options:

(i) $4\frac{1}{2}\%$ interest each year, compounded semiannually
(ii) $4\frac{1}{4}\%$ interest each year, compounded quarterly
(iii) 4% interest each year, compounded daily

Which of the given interest rates and compounding periods would provide the best investment?

43. Bacteria Bacterioplankton that occur in large bodies of water are a powerful indicator of the status of aquatic life in the water. One team of biologists tests a certain location A, and their data indicate that bacterioplankton are increasing by 19% every 20 minutes. Another team of biologists test a different location B, and their data indicate that bacterioplankton are increasing by 40% every 3 hours.

(a) Find the one-hour growth factor for each location.
(b) Which location has the larger growth rate?


Tischenko Irina/Shutterstock.com 2009

B. melatensis

44. Bacteria The bacterium *Brucella melatensis* (*B. melatensis*) is one of the causes of mastitis infections in milking cows. A lab tests the growth rates of two strains of this bacterium:

Strain A increases by 30% every 4 hours
Strain B increases by 40% every 2 hours

(a) Find the one-hour growth factor for each strain.
(b) Which strain has the larger growth rate?

45. Population of India Although India occupies only a small portion of the world's land area, it is the second most populous country in the world, and its population is growing rapidly. The population was 846 million in 1990 and 1148 million in 2000. Assume that India's population grows exponentially.

(a) Find the 10-year growth factor and the annual growth factor for India's population.
(b) Find an exponential growth model P for the population t years after 1990.
(c) Use the model found in part (b) to predict the population of India in 2010.
(d) Graph the function P for t between 0 and 25.

46. U.S. National Debt The U.S. national debt was about \$5776 billion on January 1, 2000, and increased to about \$9229 billion on January 1, 2007. Assume that the U.S. national debt grows exponentially.

(a) Find the 7-year growth factor and the annual growth factor for national debt.
(b) Find an exponential growth model P for the national debt t years after January 1, 2000.
(c) Use the model found in part (b) to predict the national debt on January 1, 2009.
(d) Graph the function P for t between 0 and 15.

47. Radioactive Plutonium Nuclear power plants produce radioactive plutonium-239, which has a half-life of 24,360 years. A 700-gram sample of plutonium-239 is placed in an underground waste disposal facility.

(a) Find a function that models the mass $m(t)$ of plutonium-239 remaining in the sample after t years. What is the decay factor?
(b) Use the model to predict the amount of plutonium-239 remaining in the sample after 500 years.
(c) Make a table of values for $m(t)$, with t varying between 0 and 5000 years in 1000-year increments. Graph the entries in the table. What does the graph tell us about how plutonium-239 decays?

48. Radioactive Strontium One radioactive material that is produced in atomic bombs is the isotope strontium-90, with a half-life of 28 years. In 1986, a 20-gram sample of strontium-90 is taken from a site near Chernobyl.

(a) Find a function that models the mass $m(t)$ of strontium-90 remaining in the sample after t years. What is the decay factor?

(b) Use the model to predict the amount of strontium-90 remaining in the sample after 40 years.

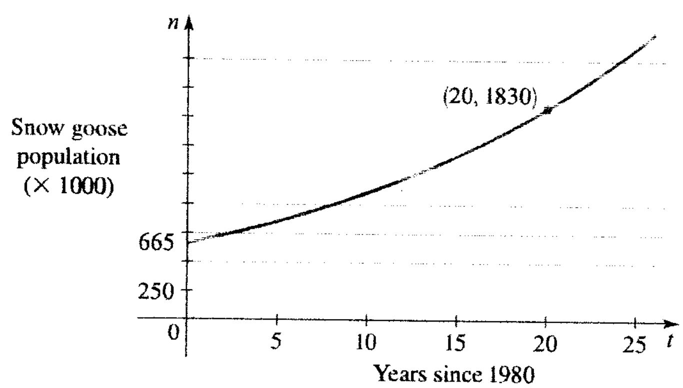
(c) Make a table of values for $m(t)$ with t varying between 0 and 100 years in 20-year increments. Make a plot of the entries in the table. What does the graph tell us about how strontium-90 decays?

49. Bird Population The snow goose population in the United States has become so large that food is becoming scarce for these birds. The graph shows the number of snow geese counted between 1980 and 2000, according to the Audubon Christmas Bird Count. Assume that the population grows exponentially.

(a) What was the snow goose population in 1980?

(b) Find the one-year growth factor a, and find an exponential growth model $n(t) = Ca^t$ for the snow goose population t years since 1980.

(c) Use the model to predict the number of snow geese in 2005.

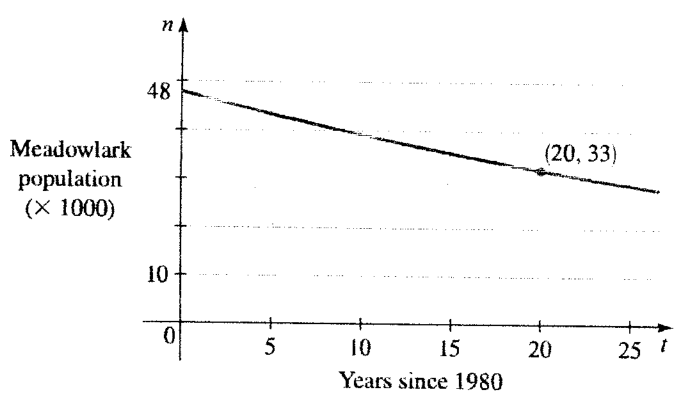


50. Bird Population Some common bird species are in decline as their habitat is lost to development. One of the most common such species is the eastern meadowlark. The graph shows the number of eastern meadowlarks in the United States between 1980 and 2005, according to the Audubon Christmas Bird Count. Assume that the population continues to decrease exponentially.

(a) What was the meadowlark population in 1980?

(b) Find the one-year decay factor a, and find an exponential decay model $n(t) = Ca^t$ for the meadowlark population t years since 1980.

(c) Use the model to predict the number of meadowlarks in 2010.



(a) What is the initial number of fish in the pond?

(b) Make a table of values of f for x between 0 and 21. From the table, what can you conclude happens to the fish population as x increases?

(c) Use a graphing calculator to draw a graph of the function f and the line $y = 100$. From the graph, what can you conclude about the fish population as x increases?

(d) What is the carrying capacity of the pond? Does this answer agree with the table in part (b) and the graph in part (c)?

Solution

(a) When x is 0 the population is

$$f(0) = \frac{100}{1 + 9 \cdot 1.5^0} = \frac{100}{1 + 9 \cdot 1} = 10$$

Initially, there were 10 fish in the pond.

(b) We use a calculator to find the values of the function f shown in the table below. It appears that the population stabilizes at 100 fish.

Year	Population	Year	Population
0	10	11	91
1	14	12	94
2	20	13	96
3	27	14	97
4	36	15	98
5	46	16	99
6	56	17	99
7	65	18	99
8	74	19	100
9	81	20	100
10	86	21	100

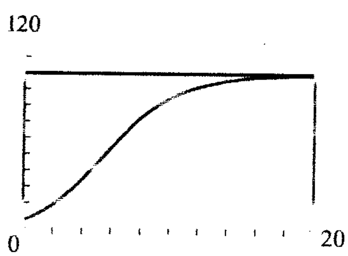


figure **4**

Graph of $f(x) = \dfrac{100}{1 + 9 \cdot 1.5^{-x}}$

(c) A graph of the function f and the line $y = 100$ are shown in Figure 4. It appears that the population gets closer and closer to 100 fish but will never exceed that number. The line $y = 100$ is a horizontal asymptote of f.

(d) Comparing the general form of the logistic equation with the model, we see that the carrying capacity C is 100. This means that, according to the model, the maximum number of fish the pond can support is 100, so the population never exceeds this capacity. The table of values and the graph of f confirm this property of the model.

■ **NOW TRY EXERCISE 31** ■

3.3 Exercises

CONCEPTS

Fundamentals

1. A population is modeled by $f(x) = 8 \cdot 5^x$.

(a) The *average rate of change* of f ________ (is constant/varies) on different intervals.

(b) The *percentage rate of change* of f ________ (is constant/varies) on all intervals of length 1.

(c) The percentage rate of change of f between $x = 3$ and $x = 4$ is ________

x	y	Percentage rate of change
0	6	—
1	18	300%
2	54	
3	162	
4	486	

2. A set of data with equally spaced inputs is given in the table in the margin. Complete the table for the percentage rate of change of the outputs y.

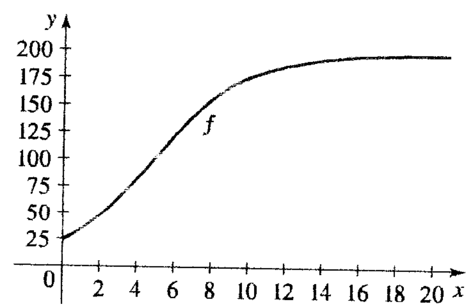
(a) Since the percentage rate of change ________ (is constant/varies), there is an exponential model $f(x) = C \cdot a^x$ that fits the data.

(b) The percentage rate of change is the constant ________%, so the growth rate r is ________, and the growth factor a is ________.

(c) An exponential model that fits the data is $f(x) = \square \cdot \square^x$.

3. A population with limited resources is modeled by the logistic growth function $f(x) = \dfrac{500}{1 + 24 \cdot a^{-x}}$. The initial population is ________, and the *carrying capacity* is ________.

4. The population of a species with limited resources is modeled by a logistic growth function f. Use the graph of f to estimate the initial population and the *carrying capacity*.



Think About It

5. True or false?

(a) The function $f(x) = 2x + 5$ has constant average rate of change on all intervals.

(b) The function $f(x) = 6^x$ has constant percentage rate of change on all intervals of length one.

6. Give some real-world examples of population growth with limited resources. For each example, explain how the population would grow if there were unlimited resources.

7. Identify each type of growth as linear or exponential.

(a) Doubling every 5 years (b) Adding 1000 units each year

(c) Increasing by 6% each year (d) Multiplying by 2 each year

8. Identify each type of decay as linear or exponential.

(a) Decreasing by 10% each year (b) Decreasing by 100 units each year

(c) Half remains each year (d) Multiplying by $\frac{1}{3}$ each year

9–12 ■ A population is modeled by the given function f in terms of time t, where t is measured in hours. Make a table of values for f, the average rate of change of f, and the percentage rate of change of f in each time period.

9. $f(t) = 5000(1.3)^t$

Hour t	Population $f(t)$	Average rate of change	Percentage rate of change
0	5000	—	—
1	6500	1500	30%
2			
3			
4			

10. $f(t) = 1000(0.95)^t$

Hour t	Population $f(t)$	Average rate of change	Percentage rate of change
0	1000	—	—
1	950	−50	−5%
2			
3			
4			

11. $f(t) = 30{,}000(0.80)^t$

Hour t	Population $f(t)$	Average rate of change	Percentage rate of change
0	30,000	—	—
1	24,000	−6000	−20%
2			
3			
4			

12. $f(t) = 20{,}000(1.7)^t$

Hour t	Population $f(t)$	Average rate of change	Percentage rate of change
0	2000	—	—
1	3400	1400	70%
2			
3			
4			

13–18 ■ A data set with equally spaced inputs is given.

(a) Find the average rate of change and the percentage rate of change for consecutive outputs.

(b) Is an exponential model appropriate? If so, find the model and sketch a graph of the model together with a scatter plot of the data.

13.

x	y
0	30,000
1	57,000
2	108,300
3	205,770
4	390,963

14.

x	y
0	500
1	450
2	405
3	364.5
4	328.05

15.

x	y
0	20,000
1	12,000
2	7200
3	4320
4	2592

16.

x	y
0	7000
1	8250
2	9500
3	10,750
4	12,000

17.

x	y
0	3000
1	2440
2	1880
3	1320
4	760

18.

x	y
0	40,000
1	84,000
2	176,400
3	370,440
4	777,924

19–22 ■ Suppose that the function f is a model for linear growth and that g is a model for exponential growth, with both f and g functions of time t.

(a) Fill in the table by evaluating the functions f and g at the given values of t.

(b) Construct equations for the functions f and g in terms t.

(c) Use a graphing calculator to graph the functions f and g on the same screen for $0 \le t \le 10$. What does the graph tell us about the differences between the two functions' behaviors?

19.

t	$f(t)$
0	200
1	350
2	
3	
4	

t	$g(t)$
0	200
1	350
2	
3	
4	

20.

t	$f(t)$
0	90
1	10
2	
3	
4	

t	$g(t)$
0	90
1	10
2	
3	
4	

21.

t	$f(t)$
0	500
1	300
2	
3	
4	

t	$g(t)$
0	500
1	300
2	
3	
4	

22.

t	$f(t)$
0	40
1	70
2	
3	
4	

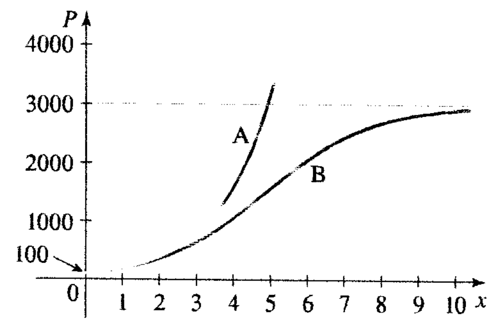
t	$g(t)$
0	40
1	70
2	
3	
4	

23. The graphs of two population models are shown. One grows exponentially and the other grows logistically.

(a) Which population grows exponentially?

(b) Which population grows logistically? What is the carrying capacity?

(c) What is the initial population for A and for B?



t	$f(t)$
0	
2	
4	
6	
8	
10	

t	$g(t)$
0	
2	
4	
6	
8	
10	

24. The function $f(t) = 100 \cdot 2^t$ is a model for a Population A, where t is measured in years. The function

$$g(t) = \frac{3000}{1 + 29 \cdot 2^{-t}}$$

is a model for a Population B, where t is measured in years.

(a) Fill in the tables in the margin for the functions f and g for the given values of t.

(b) Are the initial populations for A and B the same?

(c) Which population grows exponentially?

(d) Which population grows logistically? What is the carrying capacity?

CONTEXTS

25. World Population The population of the world was 6.454 billion in 2005 and 6.555 billion in 2006. Assume that the population grows exponentially.

(a) Find an exponential growth model $f(t) = Ca^t$ for the population t years since 2005. What is the growth rate?

(b) Sketch a graph of the function found in part (a), and plot the points $(0, f(0))$, $(1, f(1))$, $(3, f(3))$, and $(4, f(4))$.

(c) Use the model found in part (a) to find the average rate of change from 2005 to 2006 and from 2008 to 2009. Is the average rate of change the same on each of these time intervals?

(d) Use the model to find the percentage rate of change from 2005 to 2006 and from 2008 to 2009. Is the percentage change the same on each of these time intervals?

26. Population of California The population of California was 29.76 million in 1990 and 33.87 million in 2000. Assume that the population grows exponentially.

(a) Find an exponential growth model $f(t) = Ca^t$ for the population t years since 1990. What is the annual growth rate?

(b) Sketch a graph of the function found in part (a), and plot the points $(0, f(0))$, $(1, f(1))$, $(10, f(10))$, and $(11, f(11))$.

(c) Use the model found in part (a) to find the average rate of change from 1990 to 1991 and from 2000 to 2001. Is the average rate of change the same on each of these time intervals?

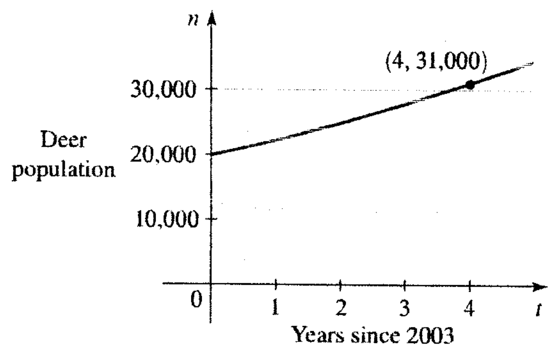
(d) Use the model to find the percentage rate of change from 1990 to 1991 and from 2000 to 2001. Is the percentage change the same on each of these time intervals?

27. Deer Population The graph shows the deer population in a Pennsylvania county between 2003 and 2007. Assume that the population grows exponentially.

(a) What was the deer population in 2003?

(b) Find an exponential growth model $n(t) = Ca^t$ for the population t years since 2003. What is the annual growth rate?

(c) Use the model to find the percentage rate of change from 2005 to 2006. Compare your answer to the growth rate from part (b).

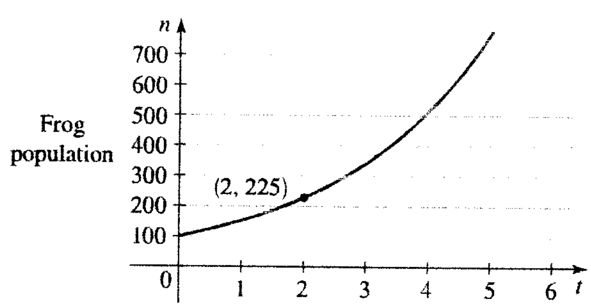


28. Frog Population Some bullfrogs were introduced into a small pond. The graph shows the bullfrog population for the next few years. Assume that the population grows exponentially.

(a) What was the initial bullfrog population?

(b) Find an exponential growth model $n(t) = Ca^t$ for the population t years since the bullfrogs were put into the pond. What is the annual growth rate?

(c) Use the model to find the percentage rate of change from $t = 2$ to $t = 3$. Compare your answer to the growth rate from part (b).



29. Salary Comparison Suppose you are offered a job that lasts three years and you are to be very well paid. Which of the following methods of payment is more profitable for you?

Offer A: You are paid \$10,000 in the starting month (month 0) and get a \$1000 raise each month.

Offer B: You are paid 2 cents in month 0, 4 cents in month 1, 8 cents in month 2, and so on, doubling your pay each month.

(d) Determine the amount of pesticide remaining after 96 hours for each method of removal. Which method would save the fish?

31. Limited Fox Population The fox population on a small island behaves according to the logistic growth model

$$n(t) = \frac{1200}{1 + 11\cdot(1.7)^{-t}}$$

where t is the number of years since the fox population was first observed.

(a) Find the initial fox population.

(b) Make a table of values of f for t between 0 and 20. From the table, what can you conclude happens to the fox population as t increases?

(c) Use a graphing calculator to draw a graph of the function $n(t)$. From the graph, what can you conclude happens to the fox population as t increases?

(d) What is the carrying capacity? Does this answer agree with the table in part (b) and the graph in part (c)?

32. Limited Bird Population The population of a certain species of bird is limited by the type of habitat required for nesting. The population behaves according to the logistic growth model

$$n(t) = \frac{11{,}200}{1 + 27\cdot(1.85)^{-t}}$$

where t is the number of years since the bird population was first observed.

(a) Find the initial bird population.

(b) Make a table of values of f for t between 0 and 20. From the table, what can you conclude happens to the bird population as t increases?

(c) Use a graphing calculator to draw a graph of the function $n(t)$. From the graph, what can you conclude happens to the bird population as t increases?

(d) What is the carrying capacity? Does this answer agree with the table in part (b) and the graph in part (c)?

33. Limited World Population The growth rate of world population has been decreasing steadily in recent years. On the basis of this, some population models predict that world population will eventually stabilize at a level that the planet can support. One such logistic model is

$$P(t) = \frac{12}{1 + 0.97\cdot(1.020214)^{-t}}$$

where $t = 0$ represents the year 2000 and population is measured in billions.

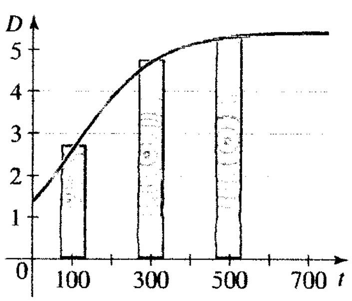
(a) What world population does this model predict for the year 2200? For 2300?

(b) Use a graphing calculator to draw a graph of the function P for the years 2000 to 2500.

(c) From the graph, what can you conclude happens to the world population as t increases?

(d) What is the carrying capacity? Does this answer agree with the graph in part (c)?

34. Tree Diameter For a certain type of tree the diameter D (in feet) depends on the tree's age t (in years) according to the logistic growth model



$$D(t) = \frac{5.4}{1 + 2.9\cdot(1.01009)^{-t}}$$

(a) What tree diameter does this model predict for a 150-year-old tree?

(b) Use the graph of D shown in the margin to determine what happens to the tree diameter as t increases.

(c) What is the carrying capacity? Does this answer agree with the graph?