

So at the low price of \$2.75 per bushel, the production of the wheat halts completely.

- (b) If no wheat is sold, then $y = 0$ in the demand equation.

$$0 = -0.99p + 25.34 \quad \text{Set } y = 0 \text{ in the demand equation}$$

$$p \approx 25.60$$

So at the high price of \$25.60 per bushel, no wheat is sold.

- (c) To find the equilibrium point, we set the supply and demand equations equal to each other and solve.

$$3.82p - 10.51 = -0.99p + 25.34 \quad \text{Set functions equal}$$

$$3.82p = -0.99p + 35.85$$

$$4.81p = 35.85$$

$$p \approx 7.45 \quad \text{Divide by 4.81}$$

So the equilibrium price is \$7.45. Evaluating the supply equation for $p = 7.45$, we get

$$y = 3.82(7.45) - 10.51 \approx 17.95$$

So for the equilibrium price of \$7.45 per bushel, about 18 million bushels of wheat are produced and sold.

NOW TRY EXERCISE 35

2.7 Exercises

CONCEPTS

Fundamentals

1. (a) To find where the graphs of the functions $y = m_1x + b_1$ and $y = m_2x + b_2$ intersect, we solve for x in the equation $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.
- (b) To find where the graphs of the functions $y = 2x + 7$ and $y = 3x - 10$ intersect, we solve for x in the equation $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$. So the graphs of these functions intersect when $x = \underline{\hspace{2cm}}$. Therefore the graphs intersect at the point $(\underline{\hspace{2cm}}, \underline{\hspace{2cm}})$.
2. (a) The point where the graphs of supply and demand equations intersect is called the $\underline{\hspace{2cm}}$ point.
- (b) The supply of a product is given by the equation $y = m_1p + b_1$, and the demand is given by the equation $y = m_2p + b_2$, where p is the price of the product. To find the price for which supply is equal to demand, we solve for p in the equation $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

Think About It

3. Muna's and Michael's distances from home are modeled by the following equations.

$$\text{Muna's equation: } y = 4 + 3t$$

$$\text{Michael's equation: } y = 2 + 3t$$

Explain why Michael will never catch up with Muna.

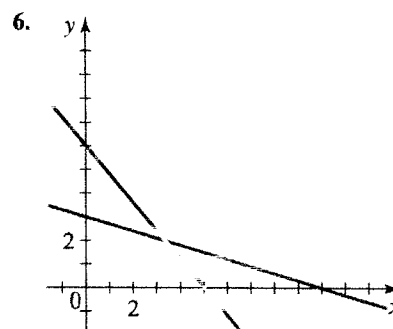
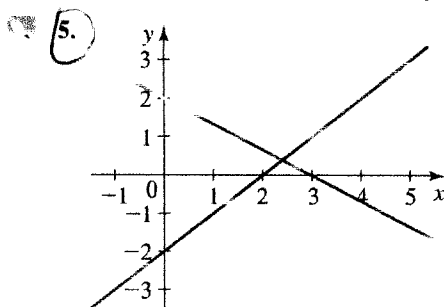
4. Udit is walking from school to his home 5 miles away, starting at time $t = 0$. Udit walks slowly at a speed of 2 miles per hour. If the time t is measured in hours, then

Udit's distance from school at time t is $y = \square + \square t$.

Udit's distance from home at time t is $y = \square + \square t$.

- 5–6** ■ A graph of two lines is given.

- (a) Use the graph to estimate the coordinates of the point of intersection.
 (b) Find an equation for each line.
 (c) Use the equations from part (b) to find the coordinates of the point of intersection. Compare with your answer to part (a).



- 7–10** ■ Linear functions f and g are given.

- (a) Graph f and g , and use the graphs to estimate the value of x where the graphs intersect.
 (b) Find algebraically the value of x where the graphs of f and g intersect.

7. $f(x) = 2x - 5$; $g(x) = -2x + 3$

8. $f(x) = x + 3$; $g(x) = -x - 1$

9. $f(x) = x - 4$; $g(x) = -2x + 2$

10. $f(x) = 2x - 4$; $g(x) = -3x + 6$

- 11–16** ■ Find the value of x for which the graphs of the two linear equations intersect.

11. $y = 4 - x$; $y = x$

12. $y = x - 3$; $y = 7 - x$

13. $y = 3 - \frac{4}{3}x$; $y = \frac{2}{3}x - 3$

14. $y = -\frac{3}{2}x$; $y = -4 - \frac{1}{2}x$

15. $y = \frac{5}{3} - \frac{1}{3}x$; $y = 2x - 3$

16. $y = 7 - x$; $y = \frac{2}{3}x + \frac{1}{3}$

- 17–22** ■ Find the point at which the graphs of the two linear equations intersect.

17. $y = 2 + x$; $y = \frac{4}{3}x + 3$

18. $y = \frac{4}{3}x - \frac{28}{3}$; $y = 9x + 6$

19. $y = \frac{7}{2} - \frac{1}{2}x$; $y = 5x - 2$

20. $y = \frac{1}{3}x$; $y = 40 - 3x$

21. $y = 6 - \frac{3}{2}x$; $y = \frac{3}{10}x - 12$

22. $y = 9 + x$; $y = 6.6 + 0.6x$

- 23–24** ■ Two linear functions are described verbally. Find equations for the functions, and find the point at which the graphs of the two functions intersect.

23. The graph of the linear function f has slope $\frac{2}{3}$ and y -intercept 5; the graph of the linear function g has slope 5 and y -intercept 2.

24. The graph of the linear function f has x -intercept 4 and y -intercept -1 ; the graph of the linear function g has x -intercept -2 and y -intercept 10.

- 25–26** ■ Equations for supply and demand are given. Find the price (in dollars) and the amount of the commodity produced and sold at equilibrium.

CONTEXTS

25. Supply: $y = 0.45p + 4$
Demand: $y = -0.65p + 28$

26. Supply: $y = 8.5p + 45$
Demand: $y = -0.6p + 300$

27. **Cell Phone Plan Comparison** Dietmar is in the process of choosing a cell phone and a cell phone plan. The first plan charges 20¢ per minute plus a monthly fee of \$10, and the second plan offers unlimited minutes for a monthly fee of \$100.

- Find a linear function f that models the monthly cost $f(x)$ of the first plan in terms of the number x of minutes used.
- Find a linear function g that models the monthly cost $g(x)$ of the second plan in terms of the number x of minutes used.
- Determine the number of minutes for which the two plans have the same monthly cost.

28. **Solar Power** Lina is considering installing solar panels on her house. Solar Advantage offers to install solar panels that generate 320 kWh of electricity per month for an installation fee of \$15,000. She uses 350 kWh of electricity per month, and her local utility company charges 20¢ per kWh.

- If Lina gets all her electrical power from the local utility company, find a linear function U that models the cost $U(x)$ of electricity for x months of service.
- If Lina has Solar Advantage install solar panels on her roof that generate 320 kWh of power per month, find a linear function S that models the cost $S(x)$ of electricity for x months of service.
- Determine the number of months it would take to reach the break-even point for installation of Solar Advantage's solar panels, that is, determine when $S(x) = U(x)$.

29. **Renting Versus Buying a Photocopier** A certain office can purchase a photocopier for \$5800 with a maintenance fee of \$25 a month. On the other hand, they can rent the photocopier for \$95 a month (including maintenance). If they purchase the photocopier, each copy would cost 3¢; if they rent, the cost is 6¢ per copy. The office manager estimates that they make 8000 copies a month.

- Find a linear function C that models the cost $C(x)$ of purchasing and using the copier for x months.
- Find a linear function S that models the cost $S(x)$ of renting and using the copier for x months.
- Make a table of the cost of each method for 1 year to 3 years of use, in 6-month increments.
- For how many months of use would the cost be the same for each method?

30. **Cost and Revenue** A tire company determines that to manufacture a certain type of tire, it costs \$8000 to set up the production process. Each tire that is produced costs \$22 in material and labor. The company sells this tire to wholesale distributors for \$49 each.

- Find a linear function C that models the total cost $C(x)$ of producing x tires.
- Find a linear function R that models the revenue $R(x)$ from selling x tires.
- Find a linear function P that models the profit $P(x)$ from selling x tires.
[Note: profit = revenue - cost.]
- How many tires must the company sell to break even (that is, when does revenue equal cost)?

31. **Car Rental** A businessman intends to rent a car for a 3-day business trip. The rental is \$35 a day and 15¢ per mile (Plan 1) or \$90 a day with unlimited mileage (Plan 2). He is not sure how many miles he will drive but estimates that it will be between 1000 and 1200 miles.

- For each plan, find a linear function C that models the cost $C(x)$ in terms of the number x of miles driven.
- Which rental plan is cheaper if the businessman drives 1000 miles? 1200 miles? At what mileage do the two plans cost the same?



- 32. Buying a Car** Kofi wants to buy a new car, and he has narrowed his choices to two models.

Model A sells for \$15,500, gets 25 mi/gal, and costs \$350 a year for insurance

Model B sells for \$19,100, gets 48 mi/gal, and costs \$425 a year for insurance

Kofi drives about 36,000 miles a year, and gas costs about \$4.50 a gallon.

- Find a linear function A that models the total cost $A(x)$ of owning Model A for x years.
- Find a linear function B that models the total cost $B(x)$ of owning Model B for x years.
- Find the number of years of ownership for which the cost to Kofi of owning Model A equals the cost of owning Model B.



- 33. Commute to Work** (See Exercise 59 in Section 2.2.) Jade and her roommate Jari live in a suburb of San Antonio, Texas, and both work at an elementary school in the city. Each morning they commute to work traveling west on I-10. One morning Jade leaves for work at 6:50 A.M., but Jari leaves 10 minutes later. On this trip Jade drives at an average speed of 65 mi/h, and Jari drives at an average speed of 72 mi/h.

- Find a linear equation that models the distance y Jari has traveled x hours after she leaves home.
- Find a linear equation that models the distance y Jade has traveled x hours after Jari leaves home.
- Determine how long it takes Jari to catch up with Jade. How far have they traveled at the time they meet?

- 34. Catching Up** Kumar leaves his house at 7:30 A.M. and cycles to school. Kumar's mother notices that he has left his lunch at home. She leaves the house by car 5 minutes after Kumar left to give him his lunch. Kumar cycles at an average speed of 8 mi/h, and his mother drives at an average speed of 24 mi/h.

- Find a linear equation that models the distance y Kumar's mother has traveled x hours after she left home.
- Find a linear equation that models the distance y Kumar has traveled x hours after his mother has left home.
- Determine how long it takes Kumar's mother to catch up with Kumar. How far have they traveled at the time they meet?



- 35. Supply and Demand for Corn** An economist models the market for corn by the following equations:

$$\text{Supply: } y = 4.18p - 11.5$$

$$\text{Demand: } y = -1.06p + 19.3$$

Here, p is the price per bushel (in dollars), and y is the number of bushels produced and sold (in billions).

- Use the model for supply to determine at what point the price is so low that no corn is produced.
- Use the model for demand to determine at what point the price is so high that no corn is sold.
- Find the equilibrium price and the quantities that are produced and sold at equilibrium.

- 36. Supply and Demand for Soybeans** An economist models the market for soybeans by the following equations:

$$\text{Supply: } y = 0.37p - 1.59$$

$$\text{Demand: } y = -0.23p + 5.22$$

Here p is the price per bushel (in dollars), and y is the number of bushels produced and sold (in billions).

- (a) Use the model for supply to determine at what point the price is so low that no soybeans are produced.
- (b) Use the model for demand to determine at what point the price is so high that no soybeans are sold.
- (c) Find the equilibrium price and the quantities that are produced and sold at equilibrium.

R 37. Median Incomes of Men and Women The gap between the median income of men and that of women has been slowly shrinking over the past 30 years. Search the Internet to find the median incomes of men and women for 1965 to 2005.

- (a) Find the regression line for the data found on the Internet for the median income of men.
- (b) Find the regression line for the data found on the Internet for the median income of women.
- (c) Use the regression lines found in parts (a) and (b) to predict the year when the median incomes of men and women will be the same.

R 38. Population The populations in many large metropolitan districts have recently been decreasing as residents move to the suburbs. Search the Internet for a city of your choice where there has been a decrease in the metropolitan population and an increase in the suburban population.

- (a) Find the regression line for the metropolitan population data for the past 40 years.
- (b) Find the regression line for the suburban population data for the past 40 years.
- (c) Use the regression lines found in parts (a) and (b) to estimate the year in which the suburban population will equal the metropolitan population.

R 39. Teacher Salaries The gap between the median salaries for men teachers and women teachers has recently been shrinking. Search the Internet to find a history for the past 30 years of men and women teachers' median salaries in your state.

- (a) Find the regression line for the data on the men teachers' median salary.
- (b) Find the regression line for the data on the women teachers' median salary.
- (c) Use the regression lines found in parts (a) and (b) to predict the year when the median teacher's salary will be equal for men and women.

CHAPTER 2 REVIEW

CONCEPT CHECK

Make sure you understand each of the ideas and concepts that you learned in this chapter, as detailed below section by section. If you need to review any of these ideas, reread the appropriate section, paying special attention to the examples.

2.1 Working with Functions: Average Rate of Change

The **average rate of change** of the function $y = f(x)$ between $x = a$ and $x = b$ is

$$\text{average rate of change} = \frac{\text{net change in } y}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$

Solution

(a) $(2\sqrt{x})(3\sqrt[3]{x}) = (2x^{1/2})(3x^{1/3})$ Definition of rational exponents
 $= 6^{1/2+1/3}$ Rule 1: $a^m a^n = a^{m+n}$
 $= 6x^{5/6}$ Rule 2: $a^m = a^{m/n} = \sqrt[n]{a^m}$

(b) $\sqrt{x\sqrt{x}} = (x \cdot x^{1/2})^{1/2}$ Definition of rational exponents
 $= (x^{3/2})^{1/2}$ Rule 1: $a^m a^n = a^{m+n}$
 $= x^{3/4}$ Rule 2: $a^m = a^{m/n} = \sqrt[n]{a^m}$

NOW TRY EXERCISES 41 AND 47

A.4 Exercises

1. Using exponential notation, we can write $\sqrt[3]{5}$ as _____.
2. Using radicals, we can write $5^{1/2}$ as _____.
3. Is there a difference between $\sqrt{5^2}$ and $(\sqrt{5})^2$? Explain.
4. Explain what $4^{3/2}$ means, and then calculate $4^{3/2}$ in two different ways:
 $4^{3/2} = (4^{1/2})^3 = \underline{\hspace{2cm}}$ $4^{3/2} = (4^3)^{1/2} = \underline{\hspace{2cm}}$
5. Find the missing power in the following calculation: $5^{1/3} \cdot 5^{\square} = 5$
6. Find the missing power in the following calculation: $5^{1/3} \cdot 5^{\square} = 1$

7–14 ■ Write each radical expression using exponents and each exponential expression using radicals.

Radical expression	Exponential expression
7. $\frac{1}{\sqrt{5}}$	_____
8. $\sqrt[3]{7^2}$	_____
9. _____	$4^{2/3}$
10. _____	$11^{-3/2}$
11. $\sqrt[5]{5^3}$	_____
12. _____	$2^{-1/2}$
13. _____	$a^{2/5}$
14. $\frac{1}{\sqrt{x^5}}$	_____

15–22 ■ Evaluate each expression.

15. (a) $\sqrt{16}$ (b) $\sqrt[3]{16}$ (c) $\sqrt[4]{16}$
 16. (a) $\sqrt{64}$ (b) $\sqrt[3]{-64}$ (c) $\sqrt[4]{32}$
 17. (a) $\sqrt[4]{9}$ (b) $\sqrt[3]{256}$ (c) $\sqrt[4]{\frac{1}{64}}$
 18. (a) $\sqrt{7}\sqrt{28}$ (b) $\frac{\sqrt{48}}{\sqrt{3}}$ (c) $\sqrt[3]{8}\sqrt[4]{32}$
 19. (a) $49^{1/2}$ (b) $(-32)^{1/5}$ (c) $(\frac{1}{125})^{1/3}$
 20. (a) $16^{1/4}$ (b) $(-\frac{1}{27})^{1/3}$ (c) $(\frac{81}{16})^{1/4}$
 21. (a) $(\frac{4}{9})^{-1/2}$ (b) $(-32)^{2/5}$ (c) $(\frac{25}{64})^{3/2}$
 22. (a) $(\frac{1}{32})^{2/5}$ (b) $(-27)^{-4/3}$ (c) $(\frac{1}{8})^{-2/3}$

23–34 ■ Simplify the expression and eliminate any negative exponent(s). Assume that all letters denote positive numbers.

23. $x^{3/4}x^{5/4}$ 24. $y^{2/3}y^{4/3}$ 25. $(4b)^{1/2}$ 26. $(3a^{2/3})^2$
 27. $\frac{w^{4/3}}{w^{1/3}}$ 28. $\frac{5^{5/2}}{5^{1/2}}$ 29. $(\frac{r^{1/2}}{16})^{1/4}$ 30. $(\frac{27}{k^3})^{-1/3}$
 31. $(x^6y^4)^{-1/2}$ 32. $(u^6v^3)^{1/3}$ 33. $(\frac{2q^{3/4}}{r^{3/2}})^{-4}$ 34. $(\frac{s^{2/3}}{2t^{1/6}})^6$

35–48 ■ Simplify the expression and express the answer using rational exponents. Assume that all letters denote positive numbers.

35. $\sqrt{x^3}\sqrt{x^5}$ 36. $\sqrt[3]{5y^4}$
 37. $\sqrt[3]{3s^3t^5}\sqrt[4]{27st^3}$ 38. $\sqrt[5]{4u^2v^6}\sqrt[5]{8u^3v^4}$
 39. $(\sqrt[6]{y^5})(\sqrt[3]{y^2})$ 40. $\sqrt[4]{b^3}\sqrt{b}$
 41. $(5\sqrt[3]{x})(2\sqrt[4]{x})$ 42. $(2\sqrt{a})(\sqrt[3]{a^2})$
 43. $\frac{\sqrt[3]{x^7}}{\sqrt{x^3}}$ 44. $\frac{\sqrt[3]{16x^3}}{\sqrt[3]{2x^5}}$
 45. $\frac{\sqrt[3]{8x^2}}{\sqrt{x}}$ 46. $\frac{\sqrt{xy}}{\sqrt[4]{16xy}}$
 47. $\sqrt[3]{y}\sqrt[4]{y}$ 48. $\sqrt{s}\sqrt[3]{s^3}$

example 6 Long Division of a Rational Expression

Divide $6x^2 - 26x + 12$ by $x - 4$.

Solution

We begin by arranging the expressions as follows.

$$x - 4 \overline{) 6x^2 - 26x + 12}$$

Next we divide the leading term in the dividend by the leading term in the divisor to get the first term of the quotient: $6x^2/x = 6x$. Then we multiply the divisor by $6x$ and subtract the result from the dividend.

$$\begin{array}{r} 6x \\ x - 4 \overline{) 6x^2 - 26x + 12} \\ \underline{6x^2 - 24x} \\ -2x + 12 \end{array}$$

Divide leading terms: $\frac{6x^2}{x} = 6x$
 Multiply: $6x(x - 4) = 6x^2 - 24x$
 Subtract and "bring down" 12

We repeat the process using the last line $-2x + 12$ as the dividend.

$$\begin{array}{r} 6x - 2 \\ x - 4 \overline{) 6x^2 - 26x + 12} \\ \underline{6x^2 - 24x} \\ -2x + 12 \\ \underline{-2x + 8} \\ 4 \end{array}$$

Divide leading terms: $\frac{-2x}{x} = -2$
 Multiply: $-2(x - 4) = -2x + 8$
 Subtract

The division process ends when the last line is of lesser degree than the divisor. The last line then contains the *remainder*, and the top line contains the *quotient*. The result of the division can be interpreted in the following way:

$$\begin{array}{ccc} \text{Dividend} & & \text{Quotient} \\ \frac{6x^2 - 26x + 12}{\text{Divisor } x - 4} = 6x - 2 + \frac{4}{x - 4} & \text{Remainder} \end{array}$$

NOW TRY EXERCISE 47

B.3 Exercises

CONCEPTS

1. Which of the following are rational expressions?

(a) $\frac{3x}{x^2 - 1}$ (b) $\frac{\sqrt{x+1}}{2x+3}$ (c) $\frac{x(x^2 - 1)}{x+3}$

2. To simplify a rational expression, we cancel *factors* that are common to the

_____ and _____. So the expression $\frac{(x+1)(x+2)}{(x+3)(x+2)}$ simplifies to _____.

3. True or false?

(a) $\frac{x^2 + 3}{x^2 + 5}$ simplifies to $\frac{3}{5}$ (b) $\frac{3x^2}{5x^2}$ simplifies to $\frac{3}{5}$

4. (a) To multiply two rational expressions, we multiply their _____ and multiply their _____. So $\frac{2}{x+1} \cdot \frac{x}{x+3}$ is the same as _____.

(b) To divide two rational expressions, we _____ the divisor, then multiply. So $\frac{3}{x+5} \div \frac{x}{x+2}$ is the same as _____.

5. Consider the expression $\frac{1}{x} - \frac{2}{x+1} - \frac{x}{(x+1)^2}$

(a) How many terms does this expression have?

(b) Find the least common denominator of all the terms.

(c) Perform the addition and simplify.

6. True or false?

(a) $\frac{1}{2} + \frac{1}{x}$ is the same as $\frac{1}{2+x}$

(b) $\frac{1}{2} + \frac{1}{x}$ is the same as $\frac{x+2}{2x}$

7–8 ■ An expression is given. Evaluate it at the given value.

7. $\frac{2s+1}{s-4}$, 7

8. $\frac{2t^2-5}{3t+6}$, 1

9–20 ■ Simplify the rational expression.

9. $\frac{12x}{6x^2}$

10. $\frac{81x^3}{18x}$

11. $\frac{5y^2}{10y+y^2}$

12. $\frac{14t^2-t}{7t}$

13. $\frac{3(x+2)(x-1)}{6(x-1)^2}$

14. $\frac{4(x^2-1)}{12(x-2)(x-1)}$

15. $\frac{x-2}{x^2-4}$

16. $\frac{x^2-x-2}{x^2-1}$

17. $\frac{x^2+6x+8}{x^2+5x+4}$

18. $\frac{x^2+x-12}{x^2-5x+6}$

19. $\frac{y^2+y}{y^2-1}$

20. $\frac{y^2-3y-18}{y^2+4y+3}$

21–30 ■ Perform the multiplication or division and simplify.

21. $\frac{4x}{x^2-4} \cdot \frac{x+2}{16x}$

22. $\frac{x^2-25}{x^2-16} \cdot \frac{x+4}{x+5}$

23. $\frac{x^2-2x-15}{x^2-9} \cdot \frac{x+3}{x-5}$

24. $\frac{x^2+2x-3}{x^2-2x-3} \cdot \frac{3-x}{3+x}$

25. $\frac{t-3}{t^2+9} \cdot \frac{t+3}{t^2-9}$

26. $\frac{x^2-x-6}{x^2+2x} \cdot \frac{x^3+x^2}{x^2-2x-3}$

27. $\frac{x+3}{4x^2-9} \div \frac{x^2+7x+12}{2x^2+7x-15}$

28. $\frac{2x+1}{2x^2+x-15} \div \frac{6x^2-x-2}{x+3}$

29. $\frac{2x^2+3x+1}{x^2+2x-15} \div \frac{x^2+6x+5}{2x^2-7x+3}$

30. $\frac{4y^2-9}{2y^2+9y-18} \div \frac{2y^2+y-3}{y^2+5y-6}$

31–40 ■ Perform the addition or subtraction and simplify.

31. $2 + \frac{x}{x+3}$

32. $\frac{2x-1}{x+4} - 1$

33. $\frac{1}{x+5} + \frac{2}{x-3}$

34. $\frac{1}{x+1} + \frac{1}{x-1}$

35. $\frac{1}{x+1} - \frac{1}{x+2}$

36. $\frac{x}{x-4} - \frac{3}{x+6}$

37. $\frac{x}{(x+1)^2} + \frac{2}{x+1}$

38. $\frac{5}{2x-3} - \frac{3}{(2x-3)^2}$

39. $\frac{1}{x^2} + \frac{1}{x^2+x}$

40. $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

41–46 ■ Rationalize the denominator.

41. $\frac{1}{2-\sqrt{3}}$

42. $\frac{2}{3-\sqrt{5}}$

43. $\frac{2}{\sqrt{2}+\sqrt{7}}$

44. $\frac{1}{\sqrt{x}+1}$

45. $\frac{y}{\sqrt{3}+\sqrt{y}}$

46. $\frac{2(x-y)}{\sqrt{x}-\sqrt{y}}$

47–52 ■ Find the quotient and remainder using long division.

47. $\frac{x^2-6x-8}{x-4}$

48. $\frac{x^3-x^2-2x+6}{x-2}$

49. $\frac{4x^3+2x^2-2x-3}{2x+1}$

50. $\frac{x^3+3x^2+4x+3}{3x+6}$

51. $\frac{x^6+x^4+x^2+1}{x^2+1}$

52. $\frac{6x^3+2x^2+20x}{2x^2+5}$