

CHAPTER 14 Applications of the Derivative

What is the difference between a relative extremum and an absolute extremum?

Can a relative extremum be an absolute extremum? Is a relative extremum necessarily an absolute extremum?

Find the absolute extrema if they exist, as well as all values of x where they occur, for each function, and specified domain. If you have one, use a graphing calculator to verify your answers.

$f(x) = x^3 - 6x^2 + 9x - 8; [0, 5]$

$f(x) = x^3 - 3x^2 - 24x + 5; [-3, 6]$

$f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 4x + 1; [-5, 2]$

$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 3; [-4, 4]$

$f(x) = x^4 - 18x^2 + 1; [-4, 4]$

$f(x) = x^4 - 32x^2 - 7; [-5, 6]$

$f(x) = \frac{1-x}{3+x}; [0, 3]$ 18. $f(x) = \frac{8+x}{8-x}; [4, 6]$

$f(x) = \frac{x-1}{x^2+1}; [1, 5]$ 20. $f(x) = \frac{x}{x^2+2}; [0, 4]$

$f(x) = (x^2 - 4)^{1/3}; [-2, 3]$

$f(x) = (x^2 - 16)^{2/3}; [-5, 8]$

$f(x) = 5x^{2/3} + 2x^{5/3}; [-2, 1]$

$f(x) = x + 3x^{2/3}; [-10, 1]$

$f(x) = x^2 - 8 \ln x; [1, 4]$ 26. $f(x) = \frac{\ln x}{x^2}; [1, 4]$

$f(x) = x + e^{-3x}; [-1, 3]$ 28. $f(x) = x^2 e^{-0.5x}; [2, 5]$

Graph each function on the indicated domain, and use the capabilities of your calculator to find the location and value of the absolute extrema.

$f(x) = \frac{-5x^4 + 2x^3 + 3x^2 + 9}{x^4 - x^3 + x^2 + 7}; [-1, 1]$

$f(x) = \frac{x^3 + 2x + 5}{x^4 + 3x^3 + 10}; [-3, 0]$

Find the absolute extrema if they exist, as well as all values of x where they occur.

$f(x) = 2x + \frac{8}{x^2} + 1, x > 0$

$f(x) = 12 - x - \frac{9}{x}, x > 0$

$f(x) = -3x^4 + 8x^3 + 18x^2 + 2$

$f(x) = x^4 - 4x^3 + 4x^2 + 1$

$f(x) = \frac{x-1}{x^2+2x+6}$ 36. $f(x) = \frac{x}{x^2+1}$

$f(x) = \frac{\ln x}{x^3}$ 38. $f(x) = x \ln x$

39. Find the absolute maximum and minimum of $f(x) = 2x - 3x^{2/3}$ (a) on the interval $[-1, 0.5]$; (b) on the interval $[0.5, 2]$.

40. Let $f(x) = e^{-2x}$. For $x > 0$, let $P(x)$ be the perimeter of the triangle with vertices $(0, 0)$, $(x, 0)$, $(x, f(x))$ and $(0, f(x))$. Which of the following statements is true? *Source: Society of Actuaries*

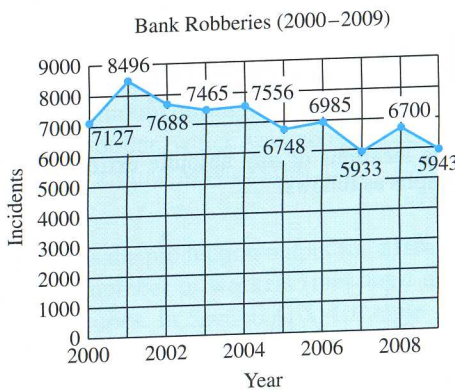
- a. The function P has an absolute minimum but not an absolute maximum on the interval $(0, \infty)$.
- b. The function P has an absolute maximum but not an absolute minimum on the interval $(0, \infty)$.
- c. The function P has both an absolute minimum and an absolute maximum on the interval $(0, \infty)$.
- d. The function P has neither an absolute maximum nor an absolute minimum on the interval $(0, \infty)$, but the graph of the function P does have an inflection point with positive x -coordinate.
- e. The function P has neither an absolute maximum nor an absolute minimum on the interval $(0, \infty)$, and the graph of the function P does not have an inflection point with positive x -coordinate.

APPLICATIONS

Business and Economics

41. **Bank Robberies** The number of bank robberies in the United States for the years 2000–2009 is given in the following figure. Consider the closed interval $[2000, 2009]$. *Source: FBI.*

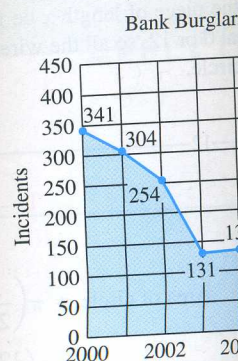
- a. Give all relative maxima and minima and when they occur on the interval.
- b. Give the absolute maxima and minima and when they occur on the interval. Interpret your results.



42. **Bank Burglaries** The number of bank burglaries (entry into a bank building for the purpose of theft from a bank during nonbusiness hours) in the United States for the years 2000–2009 is given in the figure on the following page. Consider the closed interval $[2000, 2009]$. *Source: FBI.*

- a. Give all relative maxima and minima and when they occur on the interval.

b. Give the absolute maxima and minima and when they occur on the interval. Interpret your results.



43. **Profit** The total profit $P(x)$ (in dollars) from the sale of x hundred thousand units is given by

$P(x) = -x^3 + 9x^2 + 10x$

Find the number of hundred thousand units that should be sold to maximize profit. Find the maximum profit.

44. **Profit** A company has found that the total profit $P(x)$ (in dollars) from the sale of x units of an auto part is given by

$P(x) = -0.02x^3 + 0.002x^4$

Production bottlenecks limit the number of units that can be made per week to no more than 1000. Find the maximum possible weekly profit.

Average Cost Find the minimum average cost for the given cost function on the given interval.

45. $C(x) = x^3 + 37x + 250$ on $[1, 10]$

a. $1 \leq x \leq 10$

46. $C(x) = 81x^2 + 17x + 324$ on $[1, 10]$

a. $1 \leq x \leq 10$

Cost Each graph gives the cost function $f(x)$ for the production level x . Use the method of graphing to find the production level that results in the minimum cost.

