To use the Distributive Property in reverse, we look for common factors of each of the terms in the expression we are factoring.

example 4 Using the Distributive Property to Factor

Factor the following using the Distributive Property.

(a) $3x + 3$

(b) $ax + 2ay$

(c) $2bx + 4x$

(d) $3abc + 6ab + 3bc$

Solution

(a) The common factor in each term is 3. So we factor 3 from each term using the Distributive Property.

$$3x + 3 \cdot 1 = 3(x + 1)$$

(b) The common factor in each term is a. So we factor a from each term using the Distributive Property.

$ax + 2ay$

a is a factor of each term

$$ax + 2ay = a(x + 2y) \qquad \text{Distributive Property}$$

(c) The expression $2x$ is common to each term. So we factor $2x$ from each term using the Distributive Property.

$$2bx + 4x = 2x(b + 2)$$

$3abc + 6ab + 3bc$

$3b$ is a factor of each term

(d) The expression $3b$ is common to each term. So we factor $3b$ from each term using the Distributive Property (applied to the three terms).

$$3abc + 6ab + 3bc = 3b(ac + 2a + c) \qquad \text{Distributive Property}$$

■ **NOW TRY EXERCISES 35, 39, AND 43** ■

A.1 Exercises

1. Give an example of each of the following:
   (a) A natural number
   (b) An integer that is not a natural number
   (c) A rational number that is not an integer
   (d) An irrational number

2. Complete each statement and name the property of real numbers you used.
   (a) $ab =$ ____________; ____________ Property
   (b) $a + (b + c) =$ ____________; ____________ Property
   (c) $a(b + c) =$ ____________; ____________ Property

3. **(a)** When two numbers are multiplied together, each of the numbers is called a ____________ (term/factor). So in the product $3xy$ the numbers 3, x, and y are ____________.

(b) When two numbers are added together, each of the numbers is a ______________ (term/factor). So in the sum $3 + xy$ the expressions 3 and xy are ______________.

4. **(a)** We use the ______________ ______________ to expand expressions, so to expand the expression $a(b + c)$, we multiply each term inside the parentheses by ________, and get $a(b + c) =$ ________ + ________.

(b) We use the ______________ ______________ to factor. The expression $ab + ac$ has a common factor ________, so to factor the expression $ab + ac$, we factor ________ from each term and get $ab + ac =$ ______________.

SKILLS

5–6 ■ List the elements of the given set that are:

(a) Natural numbers

(b) Integers

(c) Rational numbers

(d) Irrational numbers

5. $\{0, -10, 50, \frac{22}{7}, 0.583, \sqrt{7}, 1.2\overline{3}, -\frac{1}{3}, \sqrt[3]{2}\}$

6. $\{1.001, 0.\overline{3}, -\pi, -11, 11, \frac{13}{15}, 3.14, \sqrt{16}, \frac{15}{3}\}$

7–10 ■ Evaluate the arithmetic expression.

7. $-2 + [4 \cdot 7 - 5(9 - \frac{8}{2})]$

8. $3(4 \cdot 6 - 2 \cdot 10) + 7(15 - 8 \cdot 2)$

9. $\dfrac{5 + 7}{3} - 6[12 - (17 - 2 \cdot 3)]$

10. $1 - 2[3 - 4(5 - 6 \cdot 7)]$

11–18 ■ State the property of real numbers being used.

11. $7 + 10 = 10 + 7$

12. $2(3 + 5) = (3 + 5)2$

13. $(x + 2y) + 3z = x + (2y + 3z)$

14. $2(A + B) = 2A + 2B$

15. $(5x + 1)3 = 15x + 3$

16. $(x + a)(x + b) = (x + a)x + (x + a)b$

17. $2x(3 + y) = (3 + y)2x$

18. $7(a + b + c) = 7(a + b) + 7c$

19–22 ■ Evaluate the given expression.

19. $\dfrac{10 - 4}{3}$

20. $\dfrac{4 - 9}{5}$

21. $\dfrac{10 - 4}{5 - 2}$

22. $\dfrac{6 - 16}{2 - 7}$

23–26 ■ Evaluate the given expression directly, then evaluate using the Distributive Property.

23. $3(10 + 2)$

24. $(20 + 14) \cdot 5$

25. $(13 - 10)(-10)$

26. $-0.3(30 - 20)$

27–34 ■ Expand the expression using the Distributive Property.

27. $3(x + 7)$

28. $8(a - 2)$

29. $(a - 3c)6$

30. $(x + 2y)7$

31. $-4(3ax - 2x)$

32. $-3c(6ab - 5bd)$

33. $4mn(2p + 3pq - 2q)$

34. $(3q - 2qr - 5r)(-2ps)$

35–42 ■ Factor the given expression using the Distributive Property.

35. $3x + 9$

36. $6y - 12$

37. $-2a - 2b$

38. $-20x + 40y$

39. $ab - 6b$

40. $2ax + 2ay$

41. $-5ab + 10bc$

42. $-30xz - 90yz$

43. $2qrs - 6qst + 12rst$

44. $\frac{1}{2}abx + \frac{1}{4}aby - \frac{1}{8}abz$

A.2 The Number Line and Intervals

- The Real Line
- Order on the Real Line
- Sets and Intervals
- Absolute Value and Distance

The Real Line

The real numbers can be represented by points on a line, as shown in Figure 1. The positive direction (toward the right) is indicated by an arrow. We choose an arbitrary reference point O, called the **origin**, which corresponds to the real number 0. Given any convenient unit of measurement, each positive number x is represented by the point on the line a distance of x units to the right of the origin, and each negative number $-x$ is represented by the point x units to the left of the origin. Thus every real number is represented by a point on the line, and every point P on the line corresponds to exactly one real number.

The number associated with the point P is called the *coordinate* of P, and the line is then called a **coordinate line**, or a **real number line**, or simply a **real line.** Often we identify the point with its coordinate and think of a number as being a point on the real line.

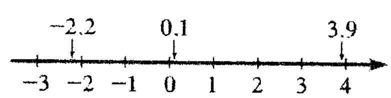


figure **1** The real line

Order on the Real Line

The real numbers are *ordered*. We say that ***a* is less than *b*** and write $a < b$ if $b - a$ is a positive number. Geometrically, this means that a lies to the left of b on the number line. (Equivalently, we can say that b is greater than a and write $b > a$.) The symbol $a \le b$ (or $b \ge a$) means that either $a < b$ or $a = b$ and is read "a is less than or equal to b." For instance, the following are true inequalities (see Figure 2):

$$-\pi < -3 \qquad \sqrt{2} < 2 \qquad 2 \le 2$$

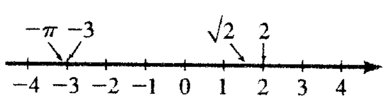


figure **2**

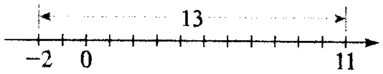


figure **8**

What is the distance on the real line between the numbers -2 and 11? From Figure 8 we see that the distance is 13. We arrive at this by finding either $|11 - (-2)| = 13$ or $|(-2) - 11| = 13$. From this observation we make the following definition (see Figure 9).

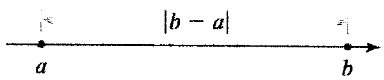


figure **9** Length of a line segment is $|b - a|$

Distance Between Points on the Real Line

If a and b are real numbers, then the distance between the points a and b on the real line is

$$d(a, b) = |b - a|$$

Note that $|b - a| = |a - b|$. This confirms that, as we would expect, the distance from a to b is the same as the distance from b to a.

example **7** Distance Between Points on the Real Line

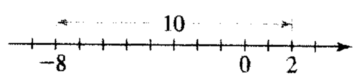


figure **10**

The distance between the numbers -8 and 2 is

$$d(a, b) = |-8 - 2| = |-10| = 10$$

We can check this calculation geometrically, as shown in Figure 10.

■ **NOW TRY EXERCISE 63** ■

A.2 Exercises

1. Explain how to graph numbers on a real number line.
2. If $a < b$, how are the points on a real line that correspond to the numbers a and b related to each other?
3. The set of numbers between but not including 2 and 7 can be written as follows:
   (a) ______________ in set builder notation
   (b) ______________ in interval notation
4. Explain the differences between the following two sets: $A = [-2, 5]$ and $B = (-2, 5)$
5. The symbol $|x|$ stands for the ______________ of the number x. If x is not 0, then the sign of $|x|$ is always ______________.
6. The absolute value of the difference between a and b is (geometrically) the ______________ between them on the real line.

7–8 ■ Place the correct symbol ($<$, $>$, or $=$) in the space.

7. (a) $3 \;\square\; \frac{7}{2}$ (b) $-3 \;\square\; -\frac{7}{2}$ (c) $3.5 \;\square\; \frac{7}{2}$
8. (a) $\frac{2}{3} \;\square\; 0.67$ (b) $\frac{2}{3} \;\square\; -0.67$ (c) $|0.67| \;\square\; |-0.67|$

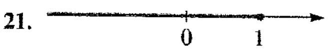
9–16 ■ State whether each inequality is true or false.

9. $-6 < -10$
10. $\sqrt{2} > 1.41$
11. $\frac{10}{11} < \frac{12}{13}$
12. $-\frac{1}{2} < -1$
13. $-\pi > -3$
14. $8 \le 9$
15. $1.1 > 1.\overline{1}$
16. $8 \le 8$

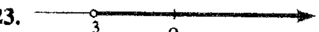
17–20 ■ On a real line, graph the numbers that satisfy the inequality.

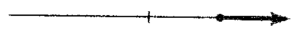
17. $x \geq 1$ **18.** $x > -4$ **19.** $x < -3$ **20.** $x \leq 0$

21–24 ■ Find the inequality whose graph is given.

21.


22.

23.


24.


25–26 ■ Write each statement in terms of inequalities.

25. **(a)** x is positive. **(b)** t is less than 4.
(c) a is greater than or equal to -1. **(d)** x is less than $\frac{1}{3}$ and is greater than -5.
(e) The distance from p to 3 is at most 5.

26. **(a)** y is negative. **(b)** z is greater than 1.
(c) b is at most 8. **(d)** w is positive and less than or equal to 17.
(e) y is at least 2 units away from 7.

27–30 ■ Find the indicated set if $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 4, 6, 8\}$, and $C = \{7, 8, 9, 10\}$.

27. **(a)** $A \cup B$ **(b)** $A \cap B$

28. **(a)** $B \cup C$ **(b)** $B \cap C$

29. **(a)** $A \cup C$ **(b)** $A \cap C$

30. **(a)** $A \cup B \cup C$ **(b)** $A \cap B \cap C$

31–32 ■ Find the indicated set if $A = \{x \mid x \geq -2\}$, $B = \{x \mid x < 4\}$, and $C = \{x \mid -1 < x \leq 5\}$.

31. **(a)** $B \cup C$ **(b)** $B \cap C$

32. **(a)** $A \cup C$ **(b)** $A \cap C$

33–40 ■ Consider the given interval.
(a) Give a verbal description of the interval.
(b) Express the interval in set builder notation.
(c) Graph the interval.

33. $(-3, 0)$ **34.** $(2, 8]$

35. $[2, 8)$ **36.** $[-6, -\frac{1}{2}]$

37. $[2, \infty)$ **38.** $(-\infty, 1)$

39. $(-\infty, -2)$ **40.** $[-3, \infty)$

41–48 ■ Express the inequality in interval notation, and then graph the corresponding interval.

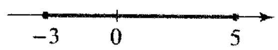
41. $1 \leq x \leq 2$ **42.** $3 < x < 5$

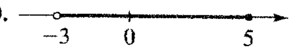
43. $-2 < x \leq 1$ **44.** $-5 < x < 2$

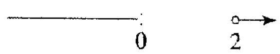
45. $x \geq -5$ **46.** $x \leq 1$

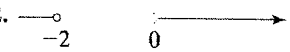
47. $x > -1$ **48.** $-5 \leq x < -2$

49–52 ■ The graph of an interval is given. Express the interval **(a)** in set-builder notation and **(b)** in interval notation.

49. 

50. 

51. 

52. 

53–58 ■ Graph the set.

53. $(-2, 0) \cup (-1, 1)$

54. $(-2, 0) \cap (-1, 1)$

55. $[-4, 6] \cap [0, 8)$

56. $[-4, 6) \cup [0, 8)$

57. $(-\infty, -4) \cup (4, \infty)$

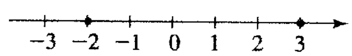
58. $(-\infty, 6) \cap (2, 10)$

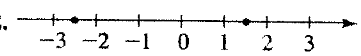
59–60 ■ Evaluate each expression.

59. **(a)** $|9|$ **(b)** $|-12|$ **(c)** $\left|\frac{-6}{24}\right|$ **(d)** $|\sqrt{5} - 5|$

60. **(a)** $|100|$ **(b)** $|-5|$ **(c)** $|(-2)\cdot 6|$ **(d)** $|10 - \pi|$

61–64 ■ Find the distance between the given numbers.

61. 

62. 

63. **(a)** 2 and 17 **(b)** -3 and 21 **(c)** $\frac{11}{8}$ and $-\frac{3}{8}$

64. **(a)** $\frac{7}{15}$ and $-\frac{1}{21}$ **(b)** -38 and -57 **(c)** -2.6 and -1.8

A.3 Integer Exponents

- Integer Exponents
- Rules for Working with Exponents

Integer Exponents

A product of identical numbers is usually written in exponential notation. For example,

$$9^4 = \underbrace{9 \times 9 \times 9 \times 9}_{4\text{ factors}}$$

We say that 9 is the *base* and 4 is the *exponent*. In general, we have the following definition.

If a is any real number and n is a positive integer, then the nth power of a is

$$a^n = \underbrace{a \times a \times \cdots \times a}_{n\text{ factors}}$$

The number a is called the base and n is called the exponent.

example 5 Using the Rules of Exponents in Different Ways

Show two different ways to simplify the expression $\left(\dfrac{x^4}{5x^3}\right)^2$.

Solution 1

Here's one way: We start with Rule 5.

$$
\begin{aligned}
\left(\frac{x^4}{5x^3}\right)^2 &= \frac{(x^4)^2}{(5x^3)^2} && \text{Rule 5}\\
&= \frac{x^{4\cdot 2}}{5^2x^{3\cdot 2}} && \text{[illegible]}\\
&= \frac{x^8}{25x^6} && \text{Simplify}\\
&= \frac{x^{8-6}}{25} && \text{[illegible]}\\
&= \frac{x^2}{25} && \text{Simplify}
\end{aligned}
$$

Solution 2

Here's another way that starts with Rule 2.

$$
\begin{aligned}
\left(\frac{x^4}{5x^3}\right)^2 &= \left(\frac{x^{4-3}}{5}\right)^2 && \text{Rule 2}\\
&= \left(\frac{x}{5}\right)^2 && \text{Simplify}\\
&= \frac{x^2}{5^2} && \text{[illegible]}\\
&= \frac{x^2}{25} && \text{Simplify}
\end{aligned}
$$

▪ **NOW TRY EXERCISE 47** ▪

A.3 Exercises

CONCEPTS

1. Using exponential notation, we can write the product $5\cdot5\cdot5\cdot5\cdot5\cdot5$ as ________.
2. In the expression 3^4 the number 3 is called the ________ and the number 4 is called the ________.
3. When we multiply two powers with the same base, we ________ the exponents. So $3^4\cdot 3^5 =$ ________.
4. When we divide two powers with the same base, we ________ the exponents. So $\dfrac{3^5}{3^2} =$ ________.

5. When we raise a power, to a new power, we ________ the exponents. So $(3^4)^2 =$ ________.

6. Express the following numbers without using exponents.

(a) $2^{-1} =$ ________.

(b) $2^{-3} =$ ________.

(c) $\left(\frac{1}{2}\right)^{-1} =$ ________.

SKILLS

7–16 ■ Evaluate each expression.

7. (a) 3^4 (b) $\left(\frac{1}{3}\right)^5$

8. (a) 2^6 (b) $\left(\frac{1}{2}\right)^4$

9. (a) $(-5)^2$ (b) -5^2

10. (a) $(-10)^4$ (b) -10^4

11. (a) $(-6)^3$ (b) -6^3

12. (a) $(-3)^7$ (b) -3^7

13. (a) $\left(\frac{1}{3}\right)^0$ (b) -3^0

14. (a) 2^0 (b) $(-2)^0$

15. (a) 3^{-1} (b) 3^{-3}

16. (a) 10^{-1} (b) 10^{-6}

17–34 ■ Use the rules of exponents to write each expression in as simple a form as possible.

17. $5^2 \cdot 5$

18. $2^3 \cdot 2^2$

19. $(2^3)^2$

20. $(2^3)^0$

21. $(-6)^0$

22. -6^0

23. -3^2

24. $(-3)^2$

25. $\dfrac{10^7}{10^4}$

26. $\dfrac{3}{3^{-2}}$

27. $3^{-4} \cdot 3^2$

28. $5^4 \cdot 5^{-2}$

29. $\dfrac{7^5 \cdot 7^{-3}}{7^2}$

30. $\dfrac{2^{-3} \cdot 2^2}{3^0}$

31. $\left(\frac{3}{4}\right)^2$

32. $\left(\frac{2}{3}\right)^{-3}$

33. $\left(\frac{1}{2}\right)^4\left(\frac{5}{2}\right)^{-2}$

34. $\left(\frac{5}{3}\right)^0 2^{-1}$

35–52 ■ Simplify each expression, and eliminate any negative exponents.

35. x^8x^2

36. x^2x^{-6}

37. $(2y^2)^3$

38. $(8x)^2$

39. $(a^2a^4)^3$

40. $(2a^3a^2)^4$

41. $(-3a)^3$

42. $(-2w)^4$

43. $\dfrac{y^{10}y^0}{y^7}$

44. $\dfrac{x^6}{x^{10}}$

45. $\dfrac{a^9a^{-2}}{a}$

46. $\dfrac{z^2z^4}{z^3z^{-1}}$

47. $\left(\dfrac{a^2}{4a^3}\right)^3$

48. $\dfrac{3x^4}{4x^2}$

49. $(3z)^2(6z^2)^{-3}$

50. $(2z^2)^{-5}z^{10}$

51. $\left(\dfrac{3u^5}{2v^3}\right)^4$

52. $\left(\dfrac{-2x^2}{y^3}\right)^3$