(e) If F is a field, show that π_F is isomorphic to the plane (P, L, I) constructed in d.

4. Observe that f_M holds the point $[x, y, z]$ fixed in $\pi_{\mathbb{R}}$ if and only if

$$M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = r\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

for some real number r. This equation is equivalent to the equation

$$(M - rI)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

where I is the 3×3 identity matrix and $M - rI$ is the diffcrebce between the to matrices M and rI. Recall that the latter equation implies the determinant of the matrix $(M - rI)$ is 0.

(a) Finds a collineation f_M on $\pi_{\mathbb{R}}$ that holds exactly three noncollinear points fixed; two points fixed; one point fixed.

(b) Show that every matrix-representable collineation on $\pi_{\mathbb{R}}$ holds at least one point fixed and one line fixed.

(c) State and prove a theorem giving necessary and sufficient conditions on a field F such that every matrix-representable collineation on π_F has at least one point fixed.

  i. Give examples of fields F such that there exist collineations f_M that hold no points fixed.

  ii. Give examples of collineations f_M that hold no points fixed.