[3.] Use computer software and simulate the solution of IVP $dx/dt = -x^3$ with $x_0 = y_0 = 10$ by the FEM, BEM (or Heun Method), linear-implicit Midpoint Method (LIMM)

$$y_{n+1} = y_n - y_n^2(y_n + y_{n+1})h_n/2$$

and linear-implicit backward Euler Method (LIBEM)

$$y_{n+1} = y_n - y_n^2 y_{n+1} h_n$$

for uniform step sizes $h = 0.1, 0.01, 0.001$ and plot the numerical solutions at times $t_n \in [0, 100]$. Furthermore, plot the global instantaneous error $\varepsilon_n = |x(t_n) - y_n|$ (i.e., the global absolute truncation error) for all these methods (you may linearly interpolate all numerical values between instants t_n). What are your findings and conclusions? Does the classical 4th order Runge-Kutta method behave better than those methods (e.g. with respect to errors, spurious oscillations and positivity)?

[4.] Prove local consistency of backward Euler method for ODEs (1): $dx/dt = f(x) \in \mathbb{R}^d$ with local order $p = 2$ and appropriate leading error constant K_C^0 whenever $f \in C^0_{OLip(L_2)} \cap C^0_{Lip(L_1)}$